

Pensieve Header: Solving the two F equations with perturbative hair, where F is written as an exponential. This time with a corrected F21.

It seems that F is not unique.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\ScatterAndGlow"];
<< ScatterAndGlow.m

$CutoffDegree = 6;
H[h_] := PH[(h /. x[i_] => z * x[i]) + O[z]^$CutoffDegree];

f1coeffs = DeclareSeries[f1[x[1], x[2]], $CutoffDegree];
f2coeffs = DeclareSeries[f2[x[1], x[2]], $CutoffDegree];
PH[f1]

PH[f1[0, 0] + (f1[1, 0] x[1] + f1[0, 1] x[2]) z +
  (1/2 f1[2, 0] x[1]^2 + f1[1, 1] x[1] x[2] + 1/2 f1[0, 2] x[2]^2) z^2 +
  (1/6 f1[3, 0] x[1]^3 + 1/2 f1[2, 1] x[1]^2 x[2] + 1/2 f1[1, 2] x[1] x[2]^2 + 1/6 f1[0, 3] x[2]^3) z^3 +
  (1/24 f1[4, 0] x[1]^4 + 1/6 f1[3, 1] x[1]^3 x[2] +
  1/4 f1[2, 2] x[1]^2 x[2]^2 + 1/6 f1[1, 3] x[1] x[2]^3 + 1/24 f1[0, 4] x[2]^4) z^4 +
  (1/120 f1[5, 0] x[1]^5 + 1/24 f1[4, 1] x[1]^4 x[2] + 1/12 f1[3, 2] x[1]^3 x[2]^2 + 1/12 f1[2, 3] x[1]^2 x[2]^3 +
  1/24 f1[1, 4] x[1] x[2]^4 + 1/120 f1[0, 5] x[2]^5) z^5 + O[z]^6]

F = S[Exp[a1 Ar[2, 1] + a2 Ar[1, 2] + Y[1, 2, 1, PH[f1]] + Y[1, 2, 2, PH[f2]]]];
F21 = S[Exp[(
  a1 Ar[1, 2] + a2 Ar[2, 1] + Y[2, 1, 2, PH[f1]] + Y[2, 1, 1, PH[f2]]
) /. {x[1] -> x[2], x[2] -> x[1]}
]];

lhs1 = Ar[3, 0] // S[Exp[Ar[1, 3] + Ar[2, 3]]] // F

Ar[3, 0] +
Y[1, 2, 0, PH[(
  -x[3]/2 + a1 x[3] - a2 x[3]
) z + (
  1/2 a1 x[1] x[3] - 1/2 a2 x[1] x[3] + 1/2 a1 a2 x[1] x[3] -
  1/2 a2^2 x[1] x[3] + f1[0, 0] x[1] x[3] + 1/2 a1 x[2] x[3] + 1/2 a1^2 x[2] x[3] - 1/2 a2 x[2] x[3] -

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$$\begin{aligned}
& \left. \frac{1}{2} a_1 a_2 x[2] x[3] + f_2[0, 0] x[2] x[3] + \frac{1}{6} (-x[2] x[3] - (x[1] + x[2]) x[3]) \right) z^2 + \\
& \left( \frac{1}{6} a_1 x[1]^2 x[3] - \frac{1}{6} a_2 x[1]^2 x[3] + \frac{1}{4} a_1 a_2 x[1]^2 x[3] - \frac{1}{4} a_2^2 x[1]^2 x[3] + \right. \\
& \frac{1}{6} a_1 a_2^2 x[1]^2 x[3] - \frac{1}{6} a_2^3 x[1]^2 x[3] + \frac{1}{2} f_1[0, 0] x[1]^2 x[3] + \frac{1}{2} a_2 f_1[0, 0] x[1]^2 x[3] + \\
& f_1[1, 0] x[1]^2 x[3] + \frac{1}{3} a_1 x[1] x[2] x[3] + \frac{1}{4} a_1^2 x[1] x[2] x[3] - \frac{1}{3} a_2 x[1] x[2] x[3] + \\
& \frac{1}{3} a_1^2 a_2 x[1] x[2] x[3] - \frac{1}{4} a_2^2 x[1] x[2] x[3] - \frac{1}{3} a_1 a_2^2 x[1] x[2] x[3] + \\
& \frac{1}{2} f_1[0, 0] x[1] x[2] x[3] + a_1 f_1[0, 0] x[1] x[2] x[3] - \frac{1}{2} a_2 f_1[0, 0] x[1] x[2] x[3] + \\
& f_1[0, 1] x[1] x[2] x[3] + \frac{1}{2} f_2[0, 0] x[1] x[2] x[3] - \frac{1}{2} a_1 f_2[0, 0] x[1] x[2] x[3] + \\
& a_2 f_2[0, 0] x[1] x[2] x[3] + f_2[1, 0] x[1] x[2] x[3] + \frac{1}{6} a_1 x[2]^2 x[3] + \\
& \frac{1}{4} a_1^2 x[2]^2 x[3] + \frac{1}{6} a_1^3 x[2]^2 x[3] - \frac{1}{6} a_2 x[2]^2 x[3] - \frac{1}{4} a_1 a_2 x[2]^2 x[3] - \\
& \frac{1}{6} a_1^2 a_2 x[2]^2 x[3] + \frac{1}{2} f_2[0, 0] x[2]^2 x[3] + \frac{1}{2} a_1 f_2[0, 0] x[2]^2 x[3] + \\
& \left. f_2[0, 1] x[2]^2 x[3] + \frac{1}{24} (-x[2]^2 x[3] - (x[1] + x[2]) (x[1] + 2x[2]) x[3]) \right) z^3 + \\
& \left( \frac{1}{24} a_1 x[1]^3 x[3] - \frac{1}{24} a_2 x[1]^3 x[3] + \frac{1}{12} a_1 a_2 x[1]^3 x[3] - \frac{1}{12} a_2^2 x[1]^3 x[3] + \right. \\
& \frac{1}{12} a_1 a_2^2 x[1]^3 x[3] - \frac{1}{12} a_2^3 x[1]^3 x[3] + \frac{1}{24} a_1 a_2^3 x[1]^3 x[3] - \frac{1}{24} a_2^4 x[1]^3 x[3] + \\
& \frac{1}{6} f_1[0, 0] x[1]^3 x[3] + \frac{1}{4} a_2 f_1[0, 0] x[1]^3 x[3] + \frac{1}{6} a_2^2 f_1[0, 0] x[1]^3 x[3] + \\
& \frac{1}{2} f_1[1, 0] x[1]^3 x[3] + \frac{1}{2} a_2 f_1[1, 0] x[1]^3 x[3] + \frac{1}{2} f_1[2, 0] x[1]^3 x[3] + \\
& \frac{1}{8} a_1 x[1]^2 x[2] x[3] + \frac{1}{12} a_1^2 x[1]^2 x[2] x[3] - \frac{1}{8} a_2 x[1]^2 x[2] x[3] + \\
& \frac{1}{12} a_1 a_2 x[1]^2 x[2] x[3] + \frac{1}{6} a_1^2 a_2 x[1]^2 x[2] x[3] - \frac{1}{6} a_2^2 x[1]^2 x[2] x[3] - \\
& \left. \frac{1}{12} a_1 a_2^2 x[1]^2 x[2] x[3] + \frac{1}{8} a_1^2 a_2^2 x[1]^2 x[2] x[3] - \frac{1}{12} a_2^3 x[1]^2 x[2] x[3] - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{8} a_1 a_2^3 x[1]^2 x[2] x[3] + \frac{1}{3} f_1[0, 0] x[1]^2 x[2] x[3] + \frac{1}{2} a_1 f_1[0, 0] x[1]^2 x[2] x[3] + \\
& \frac{2}{3} a_1 a_2 f_1[0, 0] x[1]^2 x[2] x[3] - \frac{1}{3} a_2^2 f_1[0, 0] x[1]^2 x[2] x[3] + \\
& \frac{1}{2} f_1[0, 0]^2 x[1]^2 x[2] x[3] + \frac{1}{2} f_1[0, 1] x[1]^2 x[2] x[3] + \frac{1}{2} a_2 f_1[0, 1] x[1]^2 x[2] x[3] + \\
& \frac{1}{2} f_1[1, 0] x[1]^2 x[2] x[3] + a_1 f_1[1, 0] x[1]^2 x[2] x[3] - \frac{1}{2} a_2 f_1[1, 0] x[1]^2 x[2] x[3] + \\
& f_1[1, 1] x[1]^2 x[2] x[3] + \frac{1}{6} f_2[0, 0] x[1]^2 x[2] x[3] - \frac{1}{4} a_1 f_2[0, 0] x[1]^2 x[2] x[3] + \\
& \frac{1}{2} a_2 f_2[0, 0] x[1]^2 x[2] x[3] - \frac{1}{3} a_1 a_2 f_2[0, 0] x[1]^2 x[2] x[3] + \\
& \frac{1}{2} a_2^2 f_2[0, 0] x[1]^2 x[2] x[3] - \frac{1}{2} f_1[0, 0] f_2[0, 0] x[1]^2 x[2] x[3] + \\
& \frac{1}{2} f_2[1, 0] x[1]^2 x[2] x[3] - \frac{1}{2} a_1 f_2[1, 0] x[1]^2 x[2] x[3] + a_2 f_2[1, 0] x[1]^2 x[2] x[3] + \\
& \frac{1}{2} f_2[2, 0] x[1]^2 x[2] x[3] + \frac{1}{8} a_1 x[1] x[2]^2 x[3] + \frac{1}{6} a_1^2 x[1] x[2]^2 x[3] + \\
& \frac{1}{12} a_1^3 x[1] x[2]^2 x[3] - \frac{1}{8} a_2 x[1] x[2]^2 x[3] - \frac{1}{12} a_1 a_2 x[1] x[2]^2 x[3] + \\
& \frac{1}{12} a_1^2 a_2 x[1] x[2]^2 x[3] + \frac{1}{8} a_1^3 a_2 x[1] x[2]^2 x[3] - \frac{1}{12} a_2^2 x[1] x[2]^2 x[3] - \\
& \frac{1}{6} a_1 a_2^2 x[1] x[2]^2 x[3] - \frac{1}{8} a_1^2 a_2^2 x[1] x[2]^2 x[3] + \frac{1}{6} f_1[0, 0] x[1] x[2]^2 x[3] + \\
& \frac{1}{2} a_1 f_1[0, 0] x[1] x[2]^2 x[3] + \frac{1}{2} a_1^2 f_1[0, 0] x[1] x[2]^2 x[3] - \\
& \frac{1}{4} a_2 f_1[0, 0] x[1] x[2]^2 x[3] - \frac{1}{3} a_1 a_2 f_1[0, 0] x[1] x[2]^2 x[3] + \\
& \frac{1}{2} f_1[0, 1] x[1] x[2]^2 x[3] + a_1 f_1[0, 1] x[1] x[2]^2 x[3] - \frac{1}{2} a_2 f_1[0, 1] x[1] x[2]^2 x[3] + \\
& \frac{1}{2} f_1[0, 2] x[1] x[2]^2 x[3] + \frac{1}{3} f_2[0, 0] x[1] x[2]^2 x[3] - \frac{1}{3} a_1^2 f_2[0, 0] x[1] x[2]^2 x[3] + \\
& \frac{1}{2} a_2 f_2[0, 0] x[1] x[2]^2 x[3] + \frac{2}{3} a_1 a_2 f_2[0, 0] x[1] x[2]^2 x[3] + \\
& \frac{1}{2} f_1[0, 0] f_2[0, 0] x[1] x[2]^2 x[3] - \frac{1}{2} f_2[0, 0]^2 x[1] x[2]^2 x[3] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} f2[0, 1] x[1] x[2]^2 x[3] - \frac{1}{2} a1 f2[0, 1] x[1] x[2]^2 x[3] + a2 f2[0, 1] x[1] x[2]^2 x[3] + \\
& \frac{1}{2} f2[1, 0] x[1] x[2]^2 x[3] + \frac{1}{2} a1 f2[1, 0] x[1] x[2]^2 x[3] + f2[1, 1] x[1] x[2]^2 x[3] + \\
& \frac{1}{24} a1 x[2]^3 x[3] + \frac{1}{12} a1^2 x[2]^3 x[3] + \frac{1}{12} a1^3 x[2]^3 x[3] + \frac{1}{24} a1^4 x[2]^3 x[3] - \\
& \frac{1}{24} a2 x[2]^3 x[3] - \frac{1}{12} a1 a2 x[2]^3 x[3] - \frac{1}{12} a1^2 a2 x[2]^3 x[3] - \frac{1}{24} a1^3 a2 x[2]^3 x[3] + \\
& \frac{1}{6} f2[0, 0] x[2]^3 x[3] + \frac{1}{4} a1 f2[0, 0] x[2]^3 x[3] + \frac{1}{6} a1^2 f2[0, 0] x[2]^3 x[3] + \\
& \frac{1}{2} f2[0, 1] x[2]^3 x[3] + \frac{1}{2} a1 f2[0, 1] x[2]^3 x[3] + \frac{1}{2} f2[0, 2] x[2]^3 x[3] + \\
& \frac{1}{120} \left( -x[2]^3 x[3] - (x[1] + x[2])^3 x[3] - (x[1] + x[2]) (x[2]^2 + x[2] (x[1] + x[2])) x[3] \right) z^4 + \\
& \left( \frac{1}{120} a1 x[1]^4 x[3] - \frac{1}{120} a2 x[1]^4 x[3] + \frac{1}{48} a1 a2 x[1]^4 x[3] - \frac{1}{48} a2^2 x[1]^4 x[3] + \right. \\
& \frac{1}{36} a1 a2^2 x[1]^4 x[3] - \frac{1}{36} a2^3 x[1]^4 x[3] + \frac{1}{48} a1 a2^3 x[1]^4 x[3] - \frac{1}{48} a2^4 x[1]^4 x[3] + \\
& \frac{1}{120} a1 a2^4 x[1]^4 x[3] - \frac{1}{120} a2^5 x[1]^4 x[3] + \frac{1}{24} f1[0, 0] x[1]^4 x[3] + \\
& \frac{1}{12} a2 f1[0, 0] x[1]^4 x[3] + \frac{1}{12} a2^2 f1[0, 0] x[1]^4 x[3] + \frac{1}{24} a2^3 f1[0, 0] x[1]^4 x[3] + \\
& \frac{1}{6} f1[1, 0] x[1]^4 x[3] + \frac{1}{4} a2 f1[1, 0] x[1]^4 x[3] + \frac{1}{6} a2^2 f1[1, 0] x[1]^4 x[3] + \\
& \frac{1}{4} f1[2, 0] x[1]^4 x[3] + \frac{1}{4} a2 f1[2, 0] x[1]^4 x[3] + \frac{1}{6} f1[3, 0] x[1]^4 x[3] + \\
& \frac{1}{30} a1 x[1]^3 x[2] x[3] + \frac{1}{48} a1^2 x[1]^3 x[2] x[3] - \frac{1}{30} a2 x[1]^3 x[2] x[3] + \\
& \frac{1}{24} a1 a2 x[1]^3 x[2] x[3] + \frac{1}{18} a1^2 a2 x[1]^3 x[2] x[3] - \frac{1}{16} a2^2 x[1]^3 x[2] x[3] + \\
& \frac{1}{16} a1^2 a2^2 x[1]^3 x[2] x[3] - \frac{1}{18} a2^3 x[1]^3 x[2] x[3] - \frac{1}{24} a1 a2^3 x[1]^3 x[2] x[3] + \\
& \frac{1}{30} a1^2 a2^3 x[1]^3 x[2] x[3] - \frac{1}{48} a2^4 x[1]^3 x[2] x[3] - \frac{1}{30} a1 a2^4 x[1]^3 x[2] x[3] + \\
& \left. \frac{1}{8} f1[0, 0] x[1]^3 x[2] x[3] + \frac{1}{6} a1 f1[0, 0] x[1]^3 x[2] x[3] + \frac{1}{12} a2 f1[0, 0] x[1]^3 x[2] x[3] + \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} a_1 a_2 f_1[0, 0] x[1]^3 x[2] x[3] - \frac{1}{12} a_2^2 f_1[0, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{4} a_1 a_2^2 f_1[0, 0] x[1]^3 x[2] x[3] - \frac{1}{8} a_2^3 f_1[0, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{4} f_1[0, 0]^2 x[1]^3 x[2] x[3] + \frac{1}{3} a_2 f_1[0, 0]^2 x[1]^3 x[2] x[3] + \frac{1}{6} f_1[0, 1] x[1]^3 x[2] x[3] + \\
& \frac{1}{4} a_2 f_1[0, 1] x[1]^3 x[2] x[3] + \frac{1}{6} a_2^2 f_1[0, 1] x[1]^3 x[2] x[3] + \frac{1}{3} f_1[1, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{2} a_1 f_1[1, 0] x[1]^3 x[2] x[3] + \frac{2}{3} a_1 a_2 f_1[1, 0] x[1]^3 x[2] x[3] - \\
& \frac{1}{3} a_2^2 f_1[1, 0] x[1]^3 x[2] x[3] + f_1[0, 0] f_1[1, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{2} f_1[1, 1] x[1]^3 x[2] x[3] + \frac{1}{2} a_2 f_1[1, 1] x[1]^3 x[2] x[3] + \frac{1}{4} f_1[2, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{2} a_1 f_1[2, 0] x[1]^3 x[2] x[3] - \frac{1}{4} a_2 f_1[2, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{2} f_1[2, 1] x[1]^3 x[2] x[3] + \frac{1}{24} f_2[0, 0] x[1]^3 x[2] x[3] - \frac{1}{12} a_1 f_2[0, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{6} a_2 f_2[0, 0] x[1]^3 x[2] x[3] - \frac{1}{6} a_1 a_2 f_2[0, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{4} a_2^2 f_2[0, 0] x[1]^3 x[2] x[3] - \frac{1}{8} a_1 a_2^2 f_2[0, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{6} a_2^3 f_2[0, 0] x[1]^3 x[2] x[3] - \frac{1}{4} f_1[0, 0] f_2[0, 0] x[1]^3 x[2] x[3] - \\
& \frac{1}{3} a_2 f_1[0, 0] f_2[0, 0] x[1]^3 x[2] x[3] - \frac{1}{2} f_1[1, 0] f_2[0, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{6} f_2[1, 0] x[1]^3 x[2] x[3] - \frac{1}{4} a_1 f_2[1, 0] x[1]^3 x[2] x[3] + \frac{1}{2} a_2 f_2[1, 0] x[1]^3 x[2] x[3] - \\
& \frac{1}{3} a_1 a_2 f_2[1, 0] x[1]^3 x[2] x[3] + \frac{1}{2} a_2^2 f_2[1, 0] x[1]^3 x[2] x[3] - \\
& \frac{1}{2} f_1[0, 0] f_2[1, 0] x[1]^3 x[2] x[3] + \frac{1}{4} f_2[2, 0] x[1]^3 x[2] x[3] - \\
& \frac{1}{4} a_1 f_2[2, 0] x[1]^3 x[2] x[3] + \frac{1}{2} a_2 f_2[2, 0] x[1]^3 x[2] x[3] + \\
& \frac{1}{6} f_2[3, 0] x[1]^3 x[2] x[3] + \frac{1}{20} a_1 x[1]^2 x[2]^2 x[3] + \frac{1}{16} a_1^2 x[1]^2 x[2]^2 x[3] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{36} a_1^3 x[1]^2 x[2]^2 x[3] - \frac{1}{20} a_2 x[1]^2 x[2]^2 x[3] + \frac{1}{12} a_1^2 a_2 x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{16} a_1^3 a_2 x[1]^2 x[2]^2 x[3] - \frac{1}{16} a_2^2 x[1]^2 x[2]^2 x[3] - \frac{1}{12} a_1 a_2^2 x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{20} a_1^3 a_2^2 x[1]^2 x[2]^2 x[3] - \frac{1}{36} a_2^3 x[1]^2 x[2]^2 x[3] - \frac{1}{16} a_1 a_2^3 x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{20} a_1^2 a_2^3 x[1]^2 x[2]^2 x[3] + \frac{1}{8} f_1[0, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{3} a_1 f_1[0, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{4} a_1^2 f_1[0, 0] x[1]^2 x[2]^2 x[3] - \frac{1}{12} a_2 f_1[0, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{6} a_1 a_2 f_1[0, 0] x[1]^2 x[2]^2 x[3] + \frac{3}{8} a_1^2 a_2 f_1[0, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{6} a_2^2 f_1[0, 0] x[1]^2 x[2]^2 x[3] - \frac{1}{4} a_1 a_2^2 f_1[0, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{4} f_1[0, 0]^2 x[1]^2 x[2]^2 x[3] + \frac{1}{2} a_1 f_1[0, 0]^2 x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{6} a_2 f_1[0, 0]^2 x[1]^2 x[2]^2 x[3] + \frac{1}{3} f_1[0, 1] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{2} a_1 f_1[0, 1] x[1]^2 x[2]^2 x[3] + \frac{2}{3} a_1 a_2 f_1[0, 1] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{3} a_2^2 f_1[0, 1] x[1]^2 x[2]^2 x[3] + f_1[0, 0] f_1[0, 1] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{4} f_1[0, 2] x[1]^2 x[2]^2 x[3] + \frac{1}{4} a_2 f_1[0, 2] x[1]^2 x[2]^2 x[3] + \frac{1}{6} f_1[1, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{2} a_1 f_1[1, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{2} a_1^2 f_1[1, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{4} a_2 f_1[1, 0] x[1]^2 x[2]^2 x[3] - \frac{1}{3} a_1 a_2 f_1[1, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{2} f_1[1, 1] x[1]^2 x[2]^2 x[3] + a_1 f_1[1, 1] x[1]^2 x[2]^2 x[3] - \frac{1}{2} a_2 f_1[1, 1] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{2} f_1[1, 2] x[1]^2 x[2]^2 x[3] + \frac{1}{8} f_2[0, 0] x[1]^2 x[2]^2 x[3] - \frac{1}{12} a_1 f_2[0, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{6} a_1^2 f_2[0, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{3} a_2 f_2[0, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{6} a_1 a_2 f_2[0, 0] x[1]^2 x[2]^2 x[3] - \frac{1}{4} a_1^2 a_2 f_2[0, 0] x[1]^2 x[2]^2 x[3] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} a^2 f_2[0, 0] x[1]^2 x[2]^2 x[3] + \frac{3}{8} a_1 a^2 f_2[0, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{2}{3} a_1 f_1[0, 0] f_2[0, 0] x[1]^2 x[2]^2 x[3] + \frac{2}{3} a_2 f_1[0, 0] f_2[0, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{2} f_1[0, 1] f_2[0, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{2} f_1[1, 0] f_2[0, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{4} f_2[0, 0]^2 x[1]^2 x[2]^2 x[3] + \frac{1}{6} a_1 f_2[0, 0]^2 x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{2} a_2 f_2[0, 0]^2 x[1]^2 x[2]^2 x[3] + \frac{1}{6} f_2[0, 1] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{4} a_1 f_2[0, 1] x[1]^2 x[2]^2 x[3] + \frac{1}{2} a_2 f_2[0, 1] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{3} a_1 a_2 f_2[0, 1] x[1]^2 x[2]^2 x[3] + \frac{1}{2} a^2 f_2[0, 1] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{2} f_1[0, 0] f_2[0, 1] x[1]^2 x[2]^2 x[3] + \frac{1}{3} f_2[1, 0] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{3} a_1^2 f_2[1, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{2} a_2 f_2[1, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{2}{3} a_1 a_2 f_2[1, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{2} f_1[0, 0] f_2[1, 0] x[1]^2 x[2]^2 x[3] - \\
& f_2[0, 0] f_2[1, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{2} f_2[1, 1] x[1]^2 x[2]^2 x[3] - \\
& \frac{1}{2} a_1 f_2[1, 1] x[1]^2 x[2]^2 x[3] + a_2 f_2[1, 1] x[1]^2 x[2]^2 x[3] + \frac{1}{4} f_2[2, 0] x[1]^2 x[2]^2 x[3] + \\
& \frac{1}{4} a_1 f_2[2, 0] x[1]^2 x[2]^2 x[3] + \frac{1}{2} f_2[2, 1] x[1]^2 x[2]^2 x[3] + \frac{1}{30} a_1 x[1] x[2]^3 x[3] + \\
& \frac{1}{16} a_1^2 x[1] x[2]^3 x[3] + \frac{1}{18} a_1^3 x[1] x[2]^3 x[3] + \frac{1}{48} a_1^4 x[1] x[2]^3 x[3] - \\
& \frac{1}{30} a_2 x[1] x[2]^3 x[3] - \frac{1}{24} a_1 a_2 x[1] x[2]^3 x[3] + \frac{1}{24} a_1^3 a_2 x[1] x[2]^3 x[3] + \\
& \frac{1}{30} a_1^4 a_2 x[1] x[2]^3 x[3] - \frac{1}{48} a_2^2 x[1] x[2]^3 x[3] - \frac{1}{18} a_1 a_2^2 x[1] x[2]^3 x[3] - \\
& \frac{1}{16} a_1^2 a_2^2 x[1] x[2]^3 x[3] - \frac{1}{30} a_1^3 a_2^2 x[1] x[2]^3 x[3] + \frac{1}{24} f_1[0, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{6} a_1 f_1[0, 0] x[1] x[2]^3 x[3] + \frac{1}{4} a_1^2 f_1[0, 0] x[1] x[2]^3 x[3] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{6} a1^3 f1[0, 0] x[1] x[2]^3 x[3] - \frac{1}{12} a2 f1[0, 0] x[1] x[2]^3 x[3] - \\
& \frac{1}{6} a1 a2 f1[0, 0] x[1] x[2]^3 x[3] - \frac{1}{8} a1^2 a2 f1[0, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{6} f1[0, 1] x[1] x[2]^3 x[3] + \frac{1}{2} a1 f1[0, 1] x[1] x[2]^3 x[3] + \frac{1}{2} a1^2 f1[0, 1] x[1] x[2]^3 x[3] - \\
& \frac{1}{4} a2 f1[0, 1] x[1] x[2]^3 x[3] - \frac{1}{3} a1 a2 f1[0, 1] x[1] x[2]^3 x[3] + \\
& \frac{1}{4} f1[0, 2] x[1] x[2]^3 x[3] + \frac{1}{2} a1 f1[0, 2] x[1] x[2]^3 x[3] - \frac{1}{4} a2 f1[0, 2] x[1] x[2]^3 x[3] + \\
& \frac{1}{6} f1[0, 3] x[1] x[2]^3 x[3] + \frac{1}{8} f2[0, 0] x[1] x[2]^3 x[3] + \frac{1}{12} a1 f2[0, 0] x[1] x[2]^3 x[3] - \\
& \frac{1}{12} a1^2 f2[0, 0] x[1] x[2]^3 x[3] - \frac{1}{8} a1^3 f2[0, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{6} a2 f2[0, 0] x[1] x[2]^3 x[3] + \frac{1}{3} a1 a2 f2[0, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{4} a1^2 a2 f2[0, 0] x[1] x[2]^3 x[3] + \frac{1}{4} f1[0, 0] f2[0, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{3} a1 f1[0, 0] f2[0, 0] x[1] x[2]^3 x[3] + \frac{1}{2} f1[0, 1] f2[0, 0] x[1] x[2]^3 x[3] - \\
& \frac{1}{4} f2[0, 0]^2 x[1] x[2]^3 x[3] - \frac{1}{3} a1 f2[0, 0]^2 x[1] x[2]^3 x[3] + \frac{1}{3} f2[0, 1] x[1] x[2]^3 x[3] - \\
& \frac{1}{3} a1^2 f2[0, 1] x[1] x[2]^3 x[3] + \frac{1}{2} a2 f2[0, 1] x[1] x[2]^3 x[3] + \\
& \frac{2}{3} a1 a2 f2[0, 1] x[1] x[2]^3 x[3] + \frac{1}{2} f1[0, 0] f2[0, 1] x[1] x[2]^3 x[3] - \\
& f2[0, 0] f2[0, 1] x[1] x[2]^3 x[3] + \frac{1}{4} f2[0, 2] x[1] x[2]^3 x[3] - \\
& \frac{1}{4} a1 f2[0, 2] x[1] x[2]^3 x[3] + \frac{1}{2} a2 f2[0, 2] x[1] x[2]^3 x[3] + \frac{1}{6} f2[1, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{4} a1 f2[1, 0] x[1] x[2]^3 x[3] + \frac{1}{6} a1^2 f2[1, 0] x[1] x[2]^3 x[3] + \\
& \frac{1}{2} f2[1, 1] x[1] x[2]^3 x[3] + \frac{1}{2} a1 f2[1, 1] x[1] x[2]^3 x[3] + \frac{1}{2} f2[1, 2] x[1] x[2]^3 x[3] + \\
& \frac{1}{120} a1 x[2]^4 x[3] + \frac{1}{48} a1^2 x[2]^4 x[3] + \frac{1}{36} a1^3 x[2]^4 x[3] + \frac{1}{48} a1^4 x[2]^4 x[3] +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{120} a1^5 x[2]^4 x[3] - \frac{1}{120} a2 x[2]^4 x[3] - \frac{1}{48} a1 a2 x[2]^4 x[3] - \frac{1}{36} a1^2 a2 x[2]^4 x[3] - \\
& \frac{1}{48} a1^3 a2 x[2]^4 x[3] - \frac{1}{120} a1^4 a2 x[2]^4 x[3] + \frac{1}{24} f2[0, 0] x[2]^4 x[3] + \\
& \frac{1}{12} a1 f2[0, 0] x[2]^4 x[3] + \frac{1}{12} a1^2 f2[0, 0] x[2]^4 x[3] + \frac{1}{24} a1^3 f2[0, 0] x[2]^4 x[3] + \\
& \frac{1}{6} f2[0, 1] x[2]^4 x[3] + \frac{1}{4} a1 f2[0, 1] x[2]^4 x[3] + \frac{1}{6} a1^2 f2[0, 1] x[2]^4 x[3] + \\
& \frac{1}{4} f2[0, 2] x[2]^4 x[3] + \frac{1}{4} a1 f2[0, 2] x[2]^4 x[3] + \frac{1}{6} f2[0, 3] x[2]^4 x[3] + \\
& \frac{1}{720} \left( - (x[1] + x[2])^2 (x[2]^2 + x[2] (x[1] + x[2]) + (x[1] + x[2])^2) x[3] + \right. \\
& \quad \left. x[2]^2 (-x[2]^2 x[3] - x[2] (x[1] + x[2]) x[3]) \right) z^5 + O[z]^6 \Big] + \\
Y[1, 3, 0, \text{PH}\left[1 + \frac{1}{2} (x[1] + 2 x[2]) z + \frac{1}{6} (x[2]^2 + x[2] (x[1] + x[2]) + (x[1] + x[2])^2) z^2 + \right. \\
& \frac{1}{24} \left( (x[1] + x[2])^2 (x[1] + 2 x[2]) + x[2] (x[2]^2 + x[2] (x[1] + x[2])) \right) z^3 + \\
& \frac{1}{120} \left( (x[1] + x[2])^4 + x[2]^2 (x[2]^2 + x[2] (x[1] + x[2])) + \right. \\
& \quad \left. (x[1] + x[2])^2 (x[2]^2 + x[2] (x[1] + x[2])) \right) z^4 + \\
& \frac{1}{720} \left( (x[1] + x[2])^3 (x[2]^2 + x[2] (x[1] + x[2]) + (x[1] + x[2])^2) + \right. \\
& \quad \left. x[2]^2 (x[2] (x[1] + x[2])^2 + x[2] (x[2]^2 + x[2] (x[1] + x[2]))) \right) z^5 + O[z]^6 \Big] + \\
Y[2, 3, 0, \text{PH}\left[1 + \frac{1}{2} x[2] z + \frac{1}{6} x[2]^2 z^2 + \frac{1}{24} x[2]^3 z^3 + \frac{1}{120} x[2]^4 z^4 + \right. \\
& \left. \frac{1}{720} x[2]^5 z^5 + O[z]^6 \right]
\end{aligned}$$

**rhs1 = Ar[3, 0] // F // S[sigma[1, 3], sigma[2, 3]]**

$$\begin{aligned} & \text{Ar}[3, 0] + \text{Y}\left[1, 2, 0, \text{PH}\left[ \right. \right. \\ & \quad \left. \left. -\text{x}[3] z + \left(-\frac{1}{2} \text{x}[1] \text{x}[3] - \frac{1}{2} \text{x}[2] \text{x}[3]\right) z^2 + \left(-\frac{1}{6} \text{x}[1]^2 \text{x}[3] - \frac{1}{4} \text{x}[1] \text{x}[2] \text{x}[3] - \frac{1}{6} \text{x}[2]^2 \text{x}[3]\right) z^3 + \right. \right. \\ & \quad \left. \left. \left(-\frac{1}{24} \text{x}[1]^3 \text{x}[3] - \frac{1}{12} \text{x}[1]^2 \text{x}[2] \text{x}[3] - \frac{1}{12} \text{x}[1] \text{x}[2]^2 \text{x}[3] - \frac{1}{24} \text{x}[2]^3 \text{x}[3]\right) z^4 + \right. \right. \\ & \quad \left. \left. \left(-\frac{1}{120} \text{x}[1]^4 \text{x}[3] - \frac{1}{48} \text{x}[1]^3 \text{x}[2] \text{x}[3] - \frac{1}{36} \text{x}[1]^2 \text{x}[2]^2 \text{x}[3] - \right. \right. \\ & \quad \left. \left. \frac{1}{48} \text{x}[1] \text{x}[2]^3 \text{x}[3] - \frac{1}{120} \text{x}[2]^4 \text{x}[3]\right) z^5 + \text{O}[z]^6 \right] \right] + \end{aligned}$$

$$\begin{aligned} & \text{Y}\left[1, 3, 0, \text{PH}\left[1 + \left(\frac{\text{x}[1]}{2} + \text{x}[2]\right) z + \left(\frac{\text{x}[1]^2}{6} + \frac{1}{2} \text{x}[1] \text{x}[2] + \frac{\text{x}[2]^2}{2}\right) z^2 + \right. \right. \\ & \quad \left. \left. \left(\frac{\text{x}[1]^3}{24} + \frac{1}{6} \text{x}[1]^2 \text{x}[2] + \frac{1}{4} \text{x}[1] \text{x}[2]^2 + \frac{\text{x}[2]^3}{6}\right) z^3 + \right. \right. \\ & \quad \left. \left. \left(\frac{\text{x}[1]^4}{120} + \frac{1}{24} \text{x}[1]^3 \text{x}[2] + \frac{1}{12} \text{x}[1]^2 \text{x}[2]^2 + \frac{1}{12} \text{x}[1] \text{x}[2]^3 + \frac{\text{x}[2]^4}{24}\right) z^4 + \right. \right. \\ & \quad \left. \left. \left(\frac{\text{x}[1]^5}{720} + \frac{1}{120} \text{x}[1]^4 \text{x}[2] + \frac{1}{48} \text{x}[1]^3 \text{x}[2]^2 + \frac{1}{36} \text{x}[1]^2 \text{x}[2]^3 + \frac{1}{48} \text{x}[1] \text{x}[2]^4 + \frac{\text{x}[2]^5}{120}\right) z^5 + \text{O}[z]^6 \right] \right] + \\ & \text{Y}\left[2, 3, 0, \text{PH}\left[1 + \frac{1}{2} \text{x}[2] z + \frac{1}{6} \text{x}[2]^2 z^2 + \frac{1}{24} \text{x}[2]^3 z^3 + \frac{1}{120} \text{x}[2]^4 z^4 + \frac{1}{720} \text{x}[2]^5 z^5 + \text{O}[z]^6 \right] \right] \end{aligned}$$

**lhs2 = Ar[1, 0] // F21 // S[sigma[1, 2]] // S[Exp[1/2 Ar[1, 1]]] // S[Exp[1/2 Ar[2, 2]]]**

$$\begin{aligned} & \text{Ar}[1, 0] + \text{Y}\left[1, 2, 0, \text{PH}\left[-\text{a2} + \left(-\text{a2} \text{x}[1] + \text{f2}[0, 0] \text{x}[1] - \frac{1}{2} \text{a2} (\text{a1} \text{x}[1] + \text{a2} \text{x}[2])\right) z + \right. \right. \\ & \quad \left. \left. \left(-\frac{1}{2} \text{a2} \text{x}[1]^2 - \frac{1}{2} \text{a1} \text{a2} \text{x}[1]^2 + \text{f2}[0, 0] \text{x}[1]^2 + \text{f2}[0, 1] \text{x}[1]^2 - \right. \right. \\ & \quad \left. \left. \frac{1}{2} \text{a2}^2 \text{x}[1] \text{x}[2] + \text{f2}[1, 0] \text{x}[1] \text{x}[2] - \frac{1}{6} \text{a2} (\text{a1} \text{x}[1] + \text{a2} \text{x}[2])^2 + \right. \right. \\ & \quad \left. \left. \frac{1}{2} (\text{f2}[0, 0] \text{x}[1] (\text{a1} \text{x}[1] + \text{a2} \text{x}[2]) - \text{a2} (\text{f1}[0, 0] \text{x}[1] \text{x}[2] - \text{f2}[0, 0] \text{x}[1] \text{x}[2]))\right) z^2 + \right. \right. \\ & \quad \left. \left. \left(-\frac{1}{6} \text{a2} \text{x}[1]^3 - \frac{1}{4} \text{a1} \text{a2} \text{x}[1]^3 - \frac{1}{6} \text{a1}^2 \text{a2} \text{x}[1]^3 + \frac{1}{2} \text{f2}[0, 0] \text{x}[1]^3 + \frac{1}{2} \text{a1} \text{f2}[0, 0] \text{x}[1]^3 + \text{f2}[0, 1] \right. \right. \\ & \quad \left. \left. \text{x}[1]^3 + \frac{1}{2} \text{f2}[0, 2] \text{x}[1]^3 - \frac{1}{4} \text{a2}^2 \text{x}[1]^2 \text{x}[2] - \frac{1}{3} \text{a1} \text{a2}^2 \text{x}[1]^2 \text{x}[2] - \frac{1}{2} \text{a2} \text{f1}[0, 0] \text{x}[1]^2 \text{x}[2] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& a2 f2[0, 0] x[1]^2 x[2] + f2[1, 0] x[1]^2 x[2] + f2[1, 1] x[1]^2 x[2] - \frac{1}{6} a2^3 x[1] x[2]^2 + \\
& \frac{1}{2} f2[2, 0] x[1] x[2]^2 - \frac{1}{24} a2 (a1 x[1] + a2 x[2])^3 + \frac{1}{6} (f2[0, 0] x[1] (a1 x[1] + a2 x[2])^2 - \\
& \quad 2 a2 (a1 x[1] + a2 x[2]) (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2])) + \\
& \frac{1}{2} (f2[0, 0] x[1] (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) + \\
& \quad (a1 x[1] + a2 x[2]) (f2[0, 1] x[1]^2 + f2[1, 0] x[1] x[2]) - a2 \\
& \quad (f1[0, 1] x[1]^2 x[2] - f2[0, 1] x[1]^2 x[2] + f1[1, 0] x[1] x[2]^2 - f2[1, 0] x[1] x[2]^2)) \Big) \\
z^3 + & \left( -\frac{1}{24} a2 x[1]^4 - \frac{1}{12} a1 a2 x[1]^4 - \frac{1}{12} a1^2 a2 x[1]^4 - \frac{1}{24} a1^3 a2 x[1]^4 + \frac{1}{6} f2[0, 0] x[1]^4 + \right. \\
& \frac{1}{4} a1 f2[0, 0] x[1]^4 + \frac{1}{6} a1^2 f2[0, 0] x[1]^4 + \frac{1}{2} f2[0, 1] x[1]^4 + \\
& \frac{1}{2} a1 f2[0, 1] x[1]^4 + \frac{1}{2} f2[0, 2] x[1]^4 + \frac{1}{6} f2[0, 3] x[1]^4 - \frac{1}{12} a2^2 x[1]^3 x[2] - \\
& \frac{1}{6} a1 a2^2 x[1]^3 x[2] - \frac{1}{8} a1^2 a2^2 x[1]^3 x[2] - \frac{1}{4} a2 f1[0, 0] x[1]^3 x[2] - \\
& \frac{1}{3} a1 a2 f1[0, 0] x[1]^3 x[2] - \frac{1}{2} a2 f1[0, 1] x[1]^3 x[2] + \frac{1}{2} a2 f2[0, 0] x[1]^3 x[2] + \\
& \frac{2}{3} a1 a2 f2[0, 0] x[1]^3 x[2] + \frac{1}{2} f1[0, 0] f2[0, 0] x[1]^3 x[2] - \frac{1}{2} f2[0, 0]^2 x[1]^3 x[2] + \\
& a2 f2[0, 1] x[1]^3 x[2] + \frac{1}{2} f2[1, 0] x[1]^3 x[2] + \frac{1}{2} a1 f2[1, 0] x[1]^3 x[2] + \\
& f2[1, 1] x[1]^3 x[2] + \frac{1}{2} f2[1, 2] x[1]^3 x[2] - \frac{1}{12} a2^3 x[1]^2 x[2]^2 - \\
& \frac{1}{8} a1 a2^3 x[1]^2 x[2]^2 - \frac{1}{3} a2^2 f1[0, 0] x[1]^2 x[2]^2 - \frac{1}{2} a2 f1[1, 0] x[1]^2 x[2]^2 + \\
& \frac{1}{2} a2^2 f2[0, 0] x[1]^2 x[2]^2 + a2 f2[1, 0] x[1]^2 x[2]^2 + \frac{1}{2} f2[2, 0] x[1]^2 x[2]^2 + \\
& \frac{1}{2} f2[2, 1] x[1]^2 x[2]^2 - \frac{1}{24} a2^4 x[1] x[2]^3 + \frac{1}{6} f2[3, 0] x[1] x[2]^3 - \\
& \left. \frac{1}{120} a2 (a1 x[1] + a2 x[2])^4 + \frac{1}{2} \left( (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) \right. \right. \\
& \quad (f2[0, 1] x[1]^2 + f2[1, 0] x[1] x[2]) + f2[0, 0] x[1] \\
& \quad \left. \left. (f1[0, 1] x[1]^2 x[2] - f2[0, 1] x[1]^2 x[2] + f1[1, 0] x[1] x[2]^2 - f2[1, 0] x[1] x[2]^2) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a_1 x[1] + a_2 x[2]) \left( \frac{1}{2} f_2[0, 2] x[1]^3 + f_2[1, 1] x[1]^2 x[2] + \frac{1}{2} f_2[2, 0] x[1] x[2]^2 \right) - \\
& a_2 \left( \frac{1}{2} f_1[0, 2] x[1]^3 x[2] - \frac{1}{2} f_2[0, 2] x[1]^3 x[2] + f_1[1, 1] x[1]^2 x[2]^2 - \right. \\
& \quad \left. f_2[1, 1] x[1]^2 x[2]^2 + \frac{1}{2} f_1[2, 0] x[1] x[2]^3 - \frac{1}{2} f_2[2, 0] x[1] x[2]^3 \right) + \frac{1}{24} \\
& (-2 a_2 (a_1 x[1] + a_2 x[2])^2 (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) + (a_1 x[1] + a_2 x[2])^2 \\
& \quad (f_2[0, 0] x[1] (a_1 x[1] + a_2 x[2]) - a_2 (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]))) + \\
& \frac{1}{6} (2 f_2[0, 0] x[1] (a_1 x[1] + a_2 x[2]) (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) + \\
& \quad (a_1 x[1] + a_2 x[2])^2 (f_2[0, 1] x[1]^2 + f_2[1, 0] x[1] x[2]) - \\
& \quad a_2 ((f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])^2 + 2 (a_1 x[1] + a_2 x[2]) (f_1[0, 1] x[1]^2 \\
& \quad x[2] - f_2[0, 1] x[1]^2 x[2] + f_1[1, 0] x[1] x[2]^2 - f_2[1, 0] x[1] x[2]^2))) \Big) z^4 + \\
& \left( -\frac{1}{120} a_2 x[1]^5 - \frac{1}{48} a_1 a_2 x[1]^5 - \frac{1}{36} a_1^2 a_2 x[1]^5 - \frac{1}{48} a_1^3 a_2 x[1]^5 - \frac{1}{120} a_1^4 a_2 x[1]^5 + \right. \\
& \quad \frac{1}{24} f_2[0, 0] x[1]^5 + \frac{1}{12} a_1 f_2[0, 0] x[1]^5 + \frac{1}{12} a_1^2 f_2[0, 0] x[1]^5 + \\
& \quad \frac{1}{24} a_1^3 f_2[0, 0] x[1]^5 + \frac{1}{6} f_2[0, 1] x[1]^5 + \frac{1}{4} a_1 f_2[0, 1] x[1]^5 + \frac{1}{6} a_1^2 f_2[0, 1] x[1]^5 + \\
& \quad \frac{1}{4} f_2[0, 2] x[1]^5 + \frac{1}{4} a_1 f_2[0, 2] x[1]^5 + \frac{1}{6} f_2[0, 3] x[1]^5 + \frac{1}{24} f_2[0, 4] x[1]^5 - \\
& \quad \frac{1}{48} a_2^2 x[1]^4 x[2] - \frac{1}{18} a_1 a_2^2 x[1]^4 x[2] - \frac{1}{16} a_1^2 a_2^2 x[1]^4 x[2] - \\
& \quad \frac{1}{30} a_1^3 a_2^2 x[1]^4 x[2] - \frac{1}{12} a_2 f_1[0, 0] x[1]^4 x[2] - \frac{1}{6} a_1 a_2 f_1[0, 0] x[1]^4 x[2] - \\
& \quad \frac{1}{8} a_1^2 a_2 f_1[0, 0] x[1]^4 x[2] - \frac{1}{4} a_2 f_1[0, 1] x[1]^4 x[2] - \frac{1}{3} a_1 a_2 f_1[0, 1] x[1]^4 x[2] - \\
& \quad \frac{1}{4} a_2 f_1[0, 2] x[1]^4 x[2] + \frac{1}{6} a_2 f_2[0, 0] x[1]^4 x[2] + \frac{1}{3} a_1 a_2 f_2[0, 0] x[1]^4 x[2] + \\
& \quad \frac{1}{4} a_1^2 a_2 f_2[0, 0] x[1]^4 x[2] + \frac{1}{4} f_1[0, 0] f_2[0, 0] x[1]^4 x[2] + \\
& \quad \frac{1}{3} a_1 f_1[0, 0] f_2[0, 0] x[1]^4 x[2] + \frac{1}{2} f_1[0, 1] f_2[0, 0] x[1]^4 x[2] - \\
& \quad \left. \frac{1}{4} f_2[0, 0]^2 x[1]^4 x[2] - \frac{1}{3} a_1 f_2[0, 0]^2 x[1]^4 x[2] + \frac{1}{2} a_2 f_2[0, 1] x[1]^4 x[2] + \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{2}{3} a_1 a_2 f_2[0, 1] x[1]^4 x[2] + \frac{1}{2} f_1[0, 0] f_2[0, 1] x[1]^4 x[2] - \\
& f_2[0, 0] f_2[0, 1] x[1]^4 x[2] + \frac{1}{2} a_2 f_2[0, 2] x[1]^4 x[2] + \frac{1}{6} f_2[1, 0] x[1]^4 x[2] + \\
& \frac{1}{4} a_1 f_2[1, 0] x[1]^4 x[2] + \frac{1}{6} a_1^2 f_2[1, 0] x[1]^4 x[2] + \frac{1}{2} f_2[1, 1] x[1]^4 x[2] + \\
& \frac{1}{2} a_1 f_2[1, 1] x[1]^4 x[2] + \frac{1}{2} f_2[1, 2] x[1]^4 x[2] + \frac{1}{6} f_2[1, 3] x[1]^4 x[2] - \\
& \frac{1}{36} a_2^3 x[1]^3 x[2]^2 - \frac{1}{16} a_1 a_2^3 x[1]^3 x[2]^2 - \frac{1}{20} a_1^2 a_2^3 x[1]^3 x[2]^2 - \\
& \frac{1}{6} a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{4} a_1 a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{6} a_2 f_1[0, 0]^2 x[1]^3 x[2]^2 - \\
& \frac{1}{3} a_2^2 f_1[0, 1] x[1]^3 x[2]^2 - \frac{1}{4} a_2 f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{3} a_1 a_2 f_1[1, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{2} a_2 f_1[1, 1] x[1]^3 x[2]^2 + \frac{1}{4} a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{3}{8} a_1 a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{2}{3} a_2 f_1[0, 0] f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{2} f_1[1, 0] f_2[0, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{2} a_2 f_2[0, 0]^2 x[1]^3 x[2]^2 + \frac{1}{2} a_2^2 f_2[0, 1] x[1]^3 x[2]^2 + \frac{1}{2} a_2 f_2[1, 0] x[1]^3 x[2]^2 + \\
& \frac{2}{3} a_1 a_2 f_2[1, 0] x[1]^3 x[2]^2 + \frac{1}{2} f_1[0, 0] f_2[1, 0] x[1]^3 x[2]^2 - \\
& f_2[0, 0] f_2[1, 0] x[1]^3 x[2]^2 + a_2 f_2[1, 1] x[1]^3 x[2]^2 + \frac{1}{4} f_2[2, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{4} a_1 f_2[2, 0] x[1]^3 x[2]^2 + \frac{1}{2} f_2[2, 1] x[1]^3 x[2]^2 + \frac{1}{4} f_2[2, 2] x[1]^3 x[2]^2 - \\
& \frac{1}{48} a_2^4 x[1]^2 x[2]^3 - \frac{1}{30} a_1 a_2^4 x[1]^2 x[2]^3 - \frac{1}{8} a_2^3 f_1[0, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{3} a_2^2 f_1[1, 0] x[1]^2 x[2]^3 - \frac{1}{4} a_2 f_1[2, 0] x[1]^2 x[2]^3 + \frac{1}{6} a_2^3 f_2[0, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{2} a_2^2 f_2[1, 0] x[1]^2 x[2]^3 + \frac{1}{2} a_2 f_2[2, 0] x[1]^2 x[2]^3 + \frac{1}{6} f_2[3, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{6} f_2[3, 1] x[1]^2 x[2]^3 - \frac{1}{120} a_2^5 x[1] x[2]^4 + \frac{1}{24} f_2[4, 0] x[1] x[2]^4 - \\
& \frac{1}{720} a_2 (a_1 x[1] + a_2 x[2])^5 + \frac{1}{120} (f_2[0, 0] x[1] (a_1 x[1] + a_2 x[2]))^4 -
\end{aligned}$$

$$\begin{aligned}
& 4 a_2 (a_1 x[1] + a_2 x[2])^3 (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) + \\
& \frac{1}{2} \left( (f_2[0, 1] x[1]^2 + f_2[1, 0] x[1] x[2]) (f_1[0, 1] x[1]^2 x[2] - f_2[0, 1] x[1]^2 x[2] + \right. \\
& \quad \left. f_1[1, 0] x[1] x[2]^2 - f_2[1, 0] x[1] x[2]^2) + (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) \right. \\
& \quad \left. \left( \frac{1}{2} f_2[0, 2] x[1]^3 + f_2[1, 1] x[1]^2 x[2] + \frac{1}{2} f_2[2, 0] x[1] x[2]^2 \right) + \right. \\
& f_2[0, 0] x[1] \left( \frac{1}{2} f_1[0, 2] x[1]^3 x[2] - \frac{1}{2} f_2[0, 2] x[1]^3 x[2] + f_1[1, 1] x[1]^2 x[2]^2 - \right. \\
& \quad \left. f_2[1, 1] x[1]^2 x[2]^2 + \frac{1}{2} f_1[2, 0] x[1] x[2]^3 - \frac{1}{2} f_2[2, 0] x[1] x[2]^3 \right) + \\
& (a_1 x[1] + a_2 x[2]) \left( \frac{1}{6} f_2[0, 3] x[1]^4 + \frac{1}{2} f_2[1, 2] x[1]^3 x[2] + \frac{1}{2} f_2[2, 1] x[1]^2 x[2]^2 + \right. \\
& \quad \left. \frac{1}{6} f_2[3, 0] x[1] x[2]^3 \right) - a_2 \left( \frac{1}{6} f_1[0, 3] x[1]^4 x[2] - \frac{1}{6} f_2[0, 3] x[1]^4 x[2] + \right. \\
& \quad \left. \frac{1}{2} f_1[1, 2] x[1]^3 x[2]^2 - \frac{1}{2} f_2[1, 2] x[1]^3 x[2]^2 + \frac{1}{2} f_1[2, 1] x[1]^2 x[2]^3 - \right. \\
& \quad \left. \frac{1}{2} f_2[2, 1] x[1]^2 x[2]^3 + \frac{1}{6} f_1[3, 0] x[1] x[2]^4 - \frac{1}{6} f_2[3, 0] x[1] x[2]^4 \right) \Big) + \\
& \frac{1}{24} \left( 2 (a_1 x[1] + a_2 x[2]) (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) \right. \\
& \quad (f_2[0, 0] x[1] (a_1 x[1] + a_2 x[2]) - a_2 (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])) + \\
& (a_1 x[1] + a_2 x[2])^2 (f_2[0, 0] x[1] (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) + (a_1 x[1] + \\
& \quad a_2 x[2]) (f_2[0, 1] x[1]^2 + f_2[1, 0] x[1] x[2]) - a_2 (f_1[0, 1] x[1]^2 x[2] - f_2[0, 1] \\
& \quad x[1]^2 x[2] + f_1[1, 0] x[1] x[2]^2 - f_2[1, 0] x[1] x[2]^2)) - a_2 (a_1 x[1] + a_2 x[2]) \\
& \quad \left. \left( (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])^2 + 2 (a_1 x[1] + a_2 x[2]) (f_1[0, 1] \right. \right. \\
& \quad \left. \left. x[1]^2 x[2] - f_2[0, 1] x[1]^2 x[2] + f_1[1, 0] x[1] x[2]^2 - f_2[1, 0] x[1] x[2]^2) \right) \right) + \\
& \frac{1}{6} \left( 2 (a_1 x[1] + a_2 x[2]) (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) \right. \\
& \quad (f_2[0, 1] x[1]^2 + f_2[1, 0] x[1] x[2]) + (a_1 x[1] + a_2 x[2])^2 \\
& \quad \left. \left( \frac{1}{2} f_2[0, 2] x[1]^3 + f_2[1, 1] x[1]^2 x[2] + \frac{1}{2} f_2[2, 0] x[1] x[2]^2 \right) + f_2[0, 0] x[1] \right. \\
& \quad \left. \left( (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])^2 + 2 (a_1 x[1] + a_2 x[2]) (f_1[0, 1] x[1]^2 x[2] - \right. \right. \\
& \quad \left. \left. f_2[0, 1] x[1]^2 x[2] + f_1[1, 0] x[1] x[2]^2 - f_2[1, 0] x[1] x[2]^2) \right) - \right. \\
& a_2 \left( 2 (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) (f_1[0, 1] x[1]^2 x[2] - f_2[0, 1] \right.
\end{aligned}$$

$$\begin{aligned} & x[1]^2 x[2] + f1[1, 0] x[1] x[2]^2 - f2[1, 0] x[1] x[2]^2) + 2 (a1 x[1] + a2 x[2]) \\ & \left( \frac{1}{2} f1[0, 2] x[1]^3 x[2] - \frac{1}{2} f2[0, 2] x[1]^3 x[2] + f1[1, 1] x[1]^2 x[2]^2 - f2[1, 1] \right. \\ & \left. x[1]^2 x[2]^2 + \frac{1}{2} f1[2, 0] x[1] x[2]^3 - \frac{1}{2} f2[2, 0] x[1] x[2]^3 \right) \Big) \Big) z^5 + O[z]^6 \Big] \end{aligned}$$

**rhs2 = Ar[1, 0] // S[Exp[Expand[1/2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] // F**

$$\begin{aligned} & \text{Ar}[1, 0] + Y[1, 2, 0, \text{PH}\left[\left(-\frac{1}{2} - a1\right) + \right. \\ & \left. \left(-\frac{1}{2} a2 x[1] - f1[0, 0] x[1] + \frac{1}{4} \left(-\frac{x[1]}{2} - \frac{x[2]}{2}\right) - \frac{1}{2} a1 x[2] - \frac{1}{2} a1 (a2 x[1] + a1 x[2])\right) z + \right. \\ & \left. \left(-\frac{1}{8} a2 x[1]^2 - \frac{1}{4} a2^2 x[1]^2 - f1[1, 0] x[1]^2 - \frac{1}{12} \left(\frac{x[1]}{2} + \frac{x[2]}{2}\right)^2 - \frac{1}{8} a1 x[1] x[2] - \right. \right. \\ & \left. \left. \frac{1}{8} a2 x[1] x[2] - \frac{1}{2} a1 a2 x[1] x[2] - \frac{1}{2} f1[0, 0] x[1] x[2] - f1[0, 1] x[1] x[2] + \right. \right. \\ & \left. \left. \frac{1}{2} f2[0, 0] x[1] x[2] - \frac{1}{8} a1 x[2]^2 - \frac{1}{4} a1^2 x[2]^2 - \frac{1}{6} a1 (a2 x[1] + a1 x[2])^2 + \right. \right. \\ & \left. \left. \frac{1}{2} (-f1[0, 0] x[1] (a2 x[1] + a1 x[2]) - a1 (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]))\right) z^2 + \right. \\ & \left. \left(-\frac{1}{48} a2 x[1]^3 - \frac{1}{16} a2^2 x[1]^3 - \frac{1}{12} a2^3 x[1]^3 - \frac{1}{2} f1[2, 0] x[1]^3 + \right. \right. \\ & \left. \left. \frac{1}{48} \left(-\frac{x[1]}{2} - \frac{x[2]}{2}\right) \left(\frac{x[1]}{2} + \frac{x[2]}{2}\right)^2 - \frac{1}{48} a1 x[1]^2 x[2] - \frac{1}{24} a2 x[1]^2 x[2] - \right. \right. \\ & \left. \left. \frac{1}{8} a1 a2 x[1]^2 x[2] - \frac{1}{16} a2^2 x[1]^2 x[2] - \frac{1}{4} a1 a2^2 x[1]^2 x[2] - \frac{1}{8} f1[0, 0] x[1]^2 x[2] - \right. \right. \\ & \left. \left. \frac{1}{2} a2 f1[0, 0] x[1]^2 x[2] - \frac{1}{2} f1[1, 0] x[1]^2 x[2] - f1[1, 1] x[1]^2 x[2] + \right. \right. \\ & \left. \left. \frac{1}{8} f2[0, 0] x[1]^2 x[2] + \frac{1}{2} a2 f2[0, 0] x[1]^2 x[2] + \frac{1}{2} f2[1, 0] x[1]^2 x[2] - \right. \right. \\ & \left. \left. \frac{1}{24} a1 x[1] x[2]^2 - \frac{1}{16} a1^2 x[1] x[2]^2 - \frac{1}{48} a2 x[1] x[2]^2 - \frac{1}{8} a1 a2 x[1] x[2]^2 - \right. \right. \\ & \left. \left. \frac{1}{4} a1^2 a2 x[1] x[2]^2 - \frac{1}{8} f1[0, 0] x[1] x[2]^2 - \frac{1}{2} a1 f1[0, 0] x[1] x[2]^2 - \right. \right. \\ & \left. \left. \frac{1}{2} f1[0, 1] x[1] x[2]^2 - \frac{1}{2} f1[0, 2] x[1] x[2]^2 + \frac{1}{8} f2[0, 0] x[1] x[2]^2 + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} a_1 f_2[0, 0] x[1] x[2]^2 + \frac{1}{2} f_2[0, 1] x[1] x[2]^2 - \frac{1}{48} a_1 x[2]^3 - \frac{1}{16} a_1^2 x[2]^3 - \\
& \frac{1}{12} a_1^3 x[2]^3 - \frac{1}{24} a_1 (a_2 x[1] + a_1 x[2])^3 + \frac{1}{6} (-f_1[0, 0] x[1] (a_2 x[1] + a_1 x[2])^2 - \\
& \quad 2 a_1 (a_2 x[1] + a_1 x[2]) (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])) + \\
& \frac{1}{2} ((a_2 x[1] + a_1 x[2]) (-f_1[1, 0] x[1]^2 - f_1[0, 1] x[1] x[2]) - \\
& \quad f_1[0, 0] x[1] (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])) - a_1 \\
& \quad (f_1[1, 0] x[1]^2 x[2] - f_2[1, 0] x[1]^2 x[2] + f_1[0, 1] x[1] x[2]^2 - f_2[0, 1] x[1] x[2]^2)) \Big) \\
z^3 + & \left( -\frac{1}{384} a_2 x[1]^4 - \frac{1}{96} a_2^2 x[1]^4 - \frac{1}{48} a_2^3 x[1]^4 - \frac{1}{48} a_2^4 x[1]^4 - \frac{1}{6} f_1[3, 0] x[1]^4 - \right. \\
& \frac{1}{240} \left( \frac{x[1]}{2} + \frac{x[2]}{2} \right)^4 - \frac{1}{384} a_1 x[1]^3 x[2] - \frac{1}{128} a_2 x[1]^3 x[2] - \frac{1}{48} a_1 a_2 x[1]^3 x[2] - \\
& \frac{1}{48} a_2^2 x[1]^3 x[2] - \frac{1}{16} a_1 a_2^2 x[1]^3 x[2] - \frac{1}{48} a_2^3 x[1]^3 x[2] - \frac{1}{12} a_1 a_2^3 x[1]^3 x[2] - \\
& \frac{1}{48} f_1[0, 0] x[1]^3 x[2] - \frac{1}{8} a_2 f_1[0, 0] x[1]^3 x[2] - \frac{1}{4} a_2^2 f_1[0, 0] x[1]^3 x[2] - \\
& \frac{1}{8} f_1[1, 0] x[1]^3 x[2] - \frac{1}{2} a_2 f_1[1, 0] x[1]^3 x[2] - \frac{1}{4} f_1[2, 0] x[1]^3 x[2] - \\
& \frac{1}{2} f_1[2, 1] x[1]^3 x[2] + \frac{1}{48} f_2[0, 0] x[1]^3 x[2] + \frac{1}{8} a_2 f_2[0, 0] x[1]^3 x[2] + \\
& \frac{1}{4} a_2^2 f_2[0, 0] x[1]^3 x[2] + \frac{1}{8} f_2[1, 0] x[1]^3 x[2] + \frac{1}{2} a_2 f_2[1, 0] x[1]^3 x[2] + \\
& \frac{1}{4} f_2[2, 0] x[1]^3 x[2] - \frac{1}{128} a_1 x[1]^2 x[2]^2 - \frac{1}{96} a_1^2 x[1]^2 x[2]^2 - \frac{1}{128} a_2 x[1]^2 x[2]^2 - \\
& \frac{1}{24} a_1 a_2 x[1]^2 x[2]^2 - \frac{1}{16} a_1^2 a_2 x[1]^2 x[2]^2 - \frac{1}{96} a_2^2 x[1]^2 x[2]^2 - \frac{1}{16} a_1 a_2^2 x[1]^2 x[2]^2 - \\
& \frac{1}{8} a_1^2 a_2^2 x[1]^2 x[2]^2 - \frac{1}{24} f_1[0, 0] x[1]^2 x[2]^2 - \frac{1}{8} a_1 f_1[0, 0] x[1]^2 x[2]^2 - \\
& \frac{1}{8} a_2 f_1[0, 0] x[1]^2 x[2]^2 - \frac{1}{2} a_1 a_2 f_1[0, 0] x[1]^2 x[2]^2 - \frac{1}{4} f_1[0, 0]^2 x[1]^2 x[2]^2 - \\
& \frac{1}{8} f_1[0, 1] x[1]^2 x[2]^2 - \frac{1}{2} a_2 f_1[0, 1] x[1]^2 x[2]^2 - \frac{1}{8} f_1[1, 0] x[1]^2 x[2]^2 - \\
& \frac{1}{2} a_1 f_1[1, 0] x[1]^2 x[2]^2 - \frac{1}{2} f_1[1, 1] x[1]^2 x[2]^2 - \frac{1}{2} f_1[1, 2] x[1]^2 x[2]^2 +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{24} f2[0, 0] x[1]^2 x[2]^2 + \frac{1}{8} a1 f2[0, 0] x[1]^2 x[2]^2 + \frac{1}{8} a2 f2[0, 0] x[1]^2 x[2]^2 + \\
& \frac{1}{2} a1 a2 f2[0, 0] x[1]^2 x[2]^2 + \frac{1}{2} f1[0, 0] f2[0, 0] x[1]^2 x[2]^2 - \frac{1}{4} f2[0, 0]^2 x[1]^2 x[2]^2 + \\
& \frac{1}{8} f2[0, 1] x[1]^2 x[2]^2 + \frac{1}{2} a2 f2[0, 1] x[1]^2 x[2]^2 + \frac{1}{8} f2[1, 0] x[1]^2 x[2]^2 + \\
& \frac{1}{2} a1 f2[1, 0] x[1]^2 x[2]^2 + \frac{1}{2} f2[1, 1] x[1]^2 x[2]^2 - \frac{1}{128} a1 x[1] x[2]^3 - \\
& \frac{1}{48} a1^2 x[1] x[2]^3 - \frac{1}{48} a1^3 x[1] x[2]^3 - \frac{1}{384} a2 x[1] x[2]^3 - \frac{1}{48} a1 a2 x[1] x[2]^3 - \\
& \frac{1}{16} a1^2 a2 x[1] x[2]^3 - \frac{1}{12} a1^3 a2 x[1] x[2]^3 - \frac{1}{48} f1[0, 0] x[1] x[2]^3 - \\
& \frac{1}{8} a1 f1[0, 0] x[1] x[2]^3 - \frac{1}{4} a1^2 f1[0, 0] x[1] x[2]^3 - \frac{1}{8} f1[0, 1] x[1] x[2]^3 - \\
& \frac{1}{2} a1 f1[0, 1] x[1] x[2]^3 - \frac{1}{4} f1[0, 2] x[1] x[2]^3 - \frac{1}{6} f1[0, 3] x[1] x[2]^3 + \\
& \frac{1}{48} f2[0, 0] x[1] x[2]^3 + \frac{1}{8} a1 f2[0, 0] x[1] x[2]^3 + \frac{1}{4} a1^2 f2[0, 0] x[1] x[2]^3 + \\
& \frac{1}{8} f2[0, 1] x[1] x[2]^3 + \frac{1}{2} a1 f2[0, 1] x[1] x[2]^3 + \frac{1}{4} f2[0, 2] x[1] x[2]^3 - \\
& \frac{1}{384} a1 x[2]^4 - \frac{1}{96} a1^2 x[2]^4 - \frac{1}{48} a1^3 x[2]^4 - \frac{1}{48} a1^4 x[2]^4 - \frac{1}{120} a1 (a2 x[1] + a1 x[2])^4 + \\
& \frac{1}{2} \left( (-f1[1, 0] x[1]^2 - f1[0, 1] x[1] x[2]) (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) + \right. \\
& \quad (a2 x[1] + a1 x[2]) \left( -\frac{1}{2} f1[2, 0] x[1]^3 - f1[1, 1] x[1]^2 x[2] - \frac{1}{2} f1[0, 2] x[1] x[2]^2 \right) - \\
& \quad f1[0, 0] x[1] (f1[1, 0] x[1]^2 x[2] - f2[1, 0] x[1]^2 x[2] + f1[0, 1] x[1] x[2]^2 - f2[0, 1] \\
& \quad x[1] x[2]^2) - a1 \left( \frac{1}{2} f1[2, 0] x[1]^3 x[2] - \frac{1}{2} f2[2, 0] x[1]^3 x[2] + f1[1, 1] x[1]^2 \right. \\
& \quad \left. x[2]^2 - f2[1, 1] x[1]^2 x[2]^2 + \frac{1}{2} f1[0, 2] x[1] x[2]^3 - \frac{1}{2} f2[0, 2] x[1] x[2]^3 \right) \left. \right) + \frac{1}{24} \\
& \quad (-2 a1 (a2 x[1] + a1 x[2])^2 (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) + (a2 x[1] + a1 x[2])^2 \\
& \quad (-f1[0, 0] x[1] (a2 x[1] + a1 x[2]) - a1 (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]))) + \\
& \frac{1}{6} \left( (a2 x[1] + a1 x[2])^2 (-f1[1, 0] x[1]^2 - f1[0, 1] x[1] x[2]) - \right. \\
& \quad \left. 2 f1[0, 0] x[1] (a2 x[1] + a1 x[2]) (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) - \right.
\end{aligned}$$

$$\begin{aligned}
& a1 \left( (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2])^2 + 2 (a2 x[1] + a1 x[2]) (f1[1, 0] x[1]^2 \right. \\
& \quad \left. x[2] - f2[1, 0] x[1]^2 x[2] + f1[0, 1] x[1] x[2]^2 - f2[0, 1] x[1] x[2]^2) \right) z^4 + \\
& \left( -\frac{a2 x[1]^5}{3840} - \frac{1}{768} a2^2 x[1]^5 - \frac{1}{288} a2^3 x[1]^5 - \frac{1}{192} a2^4 x[1]^5 - \frac{1}{240} a2^5 x[1]^5 - \right. \\
& \quad \frac{1}{24} f1[4, 0] x[1]^5 - \frac{\left(\frac{x[1]}{2} + \frac{x[2]}{2}\right)^5}{1440} - \frac{a1 x[1]^4 x[2]}{3840} - \frac{1}{960} a2 x[1]^4 x[2] - \\
& \quad \frac{1}{384} a1 a2 x[1]^4 x[2] - \frac{1}{256} a2^2 x[1]^4 x[2] - \frac{1}{96} a1 a2^2 x[1]^4 x[2] - \\
& \quad \frac{1}{144} a2^3 x[1]^4 x[2] - \frac{1}{48} a1 a2^3 x[1]^4 x[2] - \frac{1}{192} a2^4 x[1]^4 x[2] - \frac{1}{48} a1 a2^4 x[1]^4 x[2] - \\
& \quad \frac{1}{384} f1[0, 0] x[1]^4 x[2] - \frac{1}{48} a2 f1[0, 0] x[1]^4 x[2] - \frac{1}{16} a2^2 f1[0, 0] x[1]^4 x[2] - \\
& \quad \frac{1}{12} a2^3 f1[0, 0] x[1]^4 x[2] - \frac{1}{48} f1[1, 0] x[1]^4 x[2] - \frac{1}{8} a2 f1[1, 0] x[1]^4 x[2] - \\
& \quad \frac{1}{4} a2^2 f1[1, 0] x[1]^4 x[2] - \frac{1}{16} f1[2, 0] x[1]^4 x[2] - \frac{1}{4} a2 f1[2, 0] x[1]^4 x[2] - \\
& \quad \frac{1}{12} f1[3, 0] x[1]^4 x[2] - \frac{1}{6} f1[3, 1] x[1]^4 x[2] + \frac{1}{384} f2[0, 0] x[1]^4 x[2] + \\
& \quad \frac{1}{48} a2 f2[0, 0] x[1]^4 x[2] + \frac{1}{16} a2^2 f2[0, 0] x[1]^4 x[2] + \frac{1}{12} a2^3 f2[0, 0] x[1]^4 x[2] + \\
& \quad \frac{1}{48} f2[1, 0] x[1]^4 x[2] + \frac{1}{8} a2 f2[1, 0] x[1]^4 x[2] + \frac{1}{4} a2^2 f2[1, 0] x[1]^4 x[2] + \\
& \quad \frac{1}{16} f2[2, 0] x[1]^4 x[2] + \frac{1}{4} a2 f2[2, 0] x[1]^4 x[2] + \frac{1}{12} f2[3, 0] x[1]^4 x[2] - \\
& \quad \frac{1}{960} a1 x[1]^3 x[2]^2 - \frac{1}{768} a1^2 x[1]^3 x[2]^2 - \frac{1}{640} a2 x[1]^3 x[2]^2 - \frac{1}{128} a1 a2 x[1]^3 x[2]^2 - \\
& \quad \frac{1}{96} a1^2 a2 x[1]^3 x[2]^2 - \frac{1}{256} a2^2 x[1]^3 x[2]^2 - \frac{1}{48} a1 a2^2 x[1]^3 x[2]^2 - \\
& \quad \frac{1}{32} a1^2 a2^2 x[1]^3 x[2]^2 - \frac{1}{288} a2^3 x[1]^3 x[2]^2 - \frac{1}{48} a1 a2^3 x[1]^3 x[2]^2 - \\
& \quad \left. \frac{1}{24} a1^2 a2^3 x[1]^3 x[2]^2 - \frac{1}{128} f1[0, 0] x[1]^3 x[2]^2 - \frac{1}{48} a1 f1[0, 0] x[1]^3 x[2]^2 - \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{24} a_2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{8} a_1 a_2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{16} a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{4} a_1 a_2^2 f_1[0, 0] x[1]^3 x[2]^2 - \frac{1}{16} f_1[0, 0]^2 x[1]^3 x[2]^2 - \frac{1}{4} a_2 f_1[0, 0]^2 x[1]^3 x[2]^2 - \\
& \frac{1}{48} f_1[0, 1] x[1]^3 x[2]^2 - \frac{1}{8} a_2 f_1[0, 1] x[1]^3 x[2]^2 - \frac{1}{4} a_2^2 f_1[0, 1] x[1]^3 x[2]^2 - \\
& \frac{1}{24} f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{8} a_1 f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{8} a_2 f_1[1, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{2} a_1 a_2 f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{2} f_1[0, 0] f_1[1, 0] x[1]^3 x[2]^2 - \frac{1}{8} f_1[1, 1] x[1]^3 x[2]^2 - \\
& \frac{1}{2} a_2 f_1[1, 1] x[1]^3 x[2]^2 - \frac{1}{16} f_1[2, 0] x[1]^3 x[2]^2 - \frac{1}{4} a_1 f_1[2, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{4} f_1[2, 1] x[1]^3 x[2]^2 - \frac{1}{4} f_1[2, 2] x[1]^3 x[2]^2 + \frac{1}{128} f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{48} a_1 f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{24} a_2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{8} a_1 a_2 f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{16} a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{4} a_1 a_2^2 f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{8} f_1[0, 0] f_2[0, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{2} a_2 f_1[0, 0] f_2[0, 0] x[1]^3 x[2]^2 + \frac{1}{2} f_1[1, 0] f_2[0, 0] x[1]^3 x[2]^2 - \\
& \frac{1}{16} f_2[0, 0]^2 x[1]^3 x[2]^2 - \frac{1}{4} a_2 f_2[0, 0]^2 x[1]^3 x[2]^2 + \frac{1}{48} f_2[0, 1] x[1]^3 x[2]^2 + \\
& \frac{1}{8} a_2 f_2[0, 1] x[1]^3 x[2]^2 + \frac{1}{4} a_2^2 f_2[0, 1] x[1]^3 x[2]^2 + \frac{1}{24} f_2[1, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{8} a_1 f_2[1, 0] x[1]^3 x[2]^2 + \frac{1}{8} a_2 f_2[1, 0] x[1]^3 x[2]^2 + \frac{1}{2} a_1 a_2 f_2[1, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{2} f_1[0, 0] f_2[1, 0] x[1]^3 x[2]^2 - \frac{1}{2} f_2[0, 0] f_2[1, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{8} f_2[1, 1] x[1]^3 x[2]^2 + \frac{1}{2} a_2 f_2[1, 1] x[1]^3 x[2]^2 + \frac{1}{16} f_2[2, 0] x[1]^3 x[2]^2 + \\
& \frac{1}{4} a_1 f_2[2, 0] x[1]^3 x[2]^2 + \frac{1}{4} f_2[2, 1] x[1]^3 x[2]^2 - \frac{1}{640} a_1 x[1]^2 x[2]^3 - \\
& \frac{1}{256} a_1^2 x[1]^2 x[2]^3 - \frac{1}{288} a_1^3 x[1]^2 x[2]^3 - \frac{1}{960} a_2 x[1]^2 x[2]^3 - \frac{1}{128} a_1 a_2 x[1]^2 x[2]^3 - \\
& \frac{1}{48} a_1^2 a_2 x[1]^2 x[2]^3 - \frac{1}{48} a_1^3 a_2 x[1]^2 x[2]^3 - \frac{1}{768} a_2^2 x[1]^2 x[2]^3 - \frac{1}{96} a_1 a_2^2 x[1]^2 x[2]^3 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{32} a_1^2 a_2^2 x[1]^2 x[2]^3 - \frac{1}{24} a_1^3 a_2^2 x[1]^2 x[2]^3 - \frac{1}{128} f_1[0, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{24} a_1 f_1[0, 0] x[1]^2 x[2]^3 - \frac{1}{16} a_1^2 f_1[0, 0] x[1]^2 x[2]^3 - \frac{1}{48} a_2 f_1[0, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{8} a_1 a_2 f_1[0, 0] x[1]^2 x[2]^3 - \frac{1}{4} a_1^2 a_2 f_1[0, 0] x[1]^2 x[2]^3 - \frac{1}{16} f_1[0, 0]^2 x[1]^2 x[2]^3 - \\
& \frac{1}{4} a_1 f_1[0, 0]^2 x[1]^2 x[2]^3 - \frac{1}{24} f_1[0, 1] x[1]^2 x[2]^3 - \frac{1}{8} a_1 f_1[0, 1] x[1]^2 x[2]^3 - \\
& \frac{1}{8} a_2 f_1[0, 1] x[1]^2 x[2]^3 - \frac{1}{2} a_1 a_2 f_1[0, 1] x[1]^2 x[2]^3 - \frac{1}{2} f_1[0, 0] f_1[0, 1] x[1]^2 x[2]^3 - \\
& \frac{1}{16} f_1[0, 2] x[1]^2 x[2]^3 - \frac{1}{4} a_2 f_1[0, 2] x[1]^2 x[2]^3 - \frac{1}{48} f_1[1, 0] x[1]^2 x[2]^3 - \\
& \frac{1}{8} a_1 f_1[1, 0] x[1]^2 x[2]^3 - \frac{1}{4} a_1^2 f_1[1, 0] x[1]^2 x[2]^3 - \frac{1}{8} f_1[1, 1] x[1]^2 x[2]^3 - \\
& \frac{1}{2} a_1 f_1[1, 1] x[1]^2 x[2]^3 - \frac{1}{4} f_1[1, 2] x[1]^2 x[2]^3 - \frac{1}{6} f_1[1, 3] x[1]^2 x[2]^3 + \\
& \frac{1}{128} f_2[0, 0] x[1]^2 x[2]^3 + \frac{1}{24} a_1 f_2[0, 0] x[1]^2 x[2]^3 + \frac{1}{16} a_1^2 f_2[0, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{48} a_2 f_2[0, 0] x[1]^2 x[2]^3 + \frac{1}{8} a_1 a_2 f_2[0, 0] x[1]^2 x[2]^3 + \frac{1}{4} a_1^2 a_2 f_2[0, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{8} f_1[0, 0] f_2[0, 0] x[1]^2 x[2]^3 + \frac{1}{2} a_1 f_1[0, 0] f_2[0, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{2} f_1[0, 1] f_2[0, 0] x[1]^2 x[2]^3 - \frac{1}{16} f_2[0, 0]^2 x[1]^2 x[2]^3 - \frac{1}{4} a_1 f_2[0, 0]^2 x[1]^2 x[2]^3 + \\
& \frac{1}{24} f_2[0, 1] x[1]^2 x[2]^3 + \frac{1}{8} a_1 f_2[0, 1] x[1]^2 x[2]^3 + \frac{1}{8} a_2 f_2[0, 1] x[1]^2 x[2]^3 + \\
& \frac{1}{2} a_1 a_2 f_2[0, 1] x[1]^2 x[2]^3 + \frac{1}{2} f_1[0, 0] f_2[0, 1] x[1]^2 x[2]^3 - \\
& \frac{1}{2} f_2[0, 0] f_2[0, 1] x[1]^2 x[2]^3 + \frac{1}{16} f_2[0, 2] x[1]^2 x[2]^3 + \frac{1}{4} a_2 f_2[0, 2] x[1]^2 x[2]^3 + \\
& \frac{1}{48} f_2[1, 0] x[1]^2 x[2]^3 + \frac{1}{8} a_1 f_2[1, 0] x[1]^2 x[2]^3 + \frac{1}{4} a_1^2 f_2[1, 0] x[1]^2 x[2]^3 + \\
& \frac{1}{8} f_2[1, 1] x[1]^2 x[2]^3 + \frac{1}{2} a_1 f_2[1, 1] x[1]^2 x[2]^3 + \frac{1}{4} f_2[1, 2] x[1]^2 x[2]^3 - \\
& \frac{1}{960} a_1 x[1] x[2]^4 - \frac{1}{256} a_1^2 x[1] x[2]^4 - \frac{1}{144} a_1^3 x[1] x[2]^4 - \frac{1}{192} a_1^4 x[1] x[2]^4 -
\end{aligned}$$

$$\begin{aligned}
& \frac{a_2 x[1] x[2]^4}{3840} - \frac{1}{384} a_1 a_2 x[1] x[2]^4 - \frac{1}{96} a_1^2 a_2 x[1] x[2]^4 - \frac{1}{48} a_1^3 a_2 x[1] x[2]^4 - \\
& \frac{1}{48} a_1^4 a_2 x[1] x[2]^4 - \frac{1}{384} f_1[0, 0] x[1] x[2]^4 - \frac{1}{48} a_1 f_1[0, 0] x[1] x[2]^4 - \\
& \frac{1}{16} a_1^2 f_1[0, 0] x[1] x[2]^4 - \frac{1}{12} a_1^3 f_1[0, 0] x[1] x[2]^4 - \frac{1}{48} f_1[0, 1] x[1] x[2]^4 - \\
& \frac{1}{8} a_1 f_1[0, 1] x[1] x[2]^4 - \frac{1}{4} a_1^2 f_1[0, 1] x[1] x[2]^4 - \frac{1}{16} f_1[0, 2] x[1] x[2]^4 - \\
& \frac{1}{4} a_1 f_1[0, 2] x[1] x[2]^4 - \frac{1}{12} f_1[0, 3] x[1] x[2]^4 - \frac{1}{24} f_1[0, 4] x[1] x[2]^4 + \\
& \frac{1}{384} f_2[0, 0] x[1] x[2]^4 + \frac{1}{48} a_1 f_2[0, 0] x[1] x[2]^4 + \frac{1}{16} a_1^2 f_2[0, 0] x[1] x[2]^4 + \\
& \frac{1}{12} a_1^3 f_2[0, 0] x[1] x[2]^4 + \frac{1}{48} f_2[0, 1] x[1] x[2]^4 + \frac{1}{8} a_1 f_2[0, 1] x[1] x[2]^4 + \\
& \frac{1}{4} a_1^2 f_2[0, 1] x[1] x[2]^4 + \frac{1}{16} f_2[0, 2] x[1] x[2]^4 + \frac{1}{4} a_1 f_2[0, 2] x[1] x[2]^4 + \\
& \frac{1}{12} f_2[0, 3] x[1] x[2]^4 - \frac{a_1 x[2]^5}{3840} - \frac{1}{768} a_1^2 x[2]^5 - \frac{1}{288} a_1^3 x[2]^5 - \frac{1}{192} a_1^4 x[2]^5 - \\
& \frac{1}{240} a_1^5 x[2]^5 - \frac{1}{720} a_1 (a_2 x[1] + a_1 x[2])^5 + \frac{1}{120} (-f_1[0, 0] x[1] (a_2 x[1] + a_1 x[2])^4 - \\
& 4 a_1 (a_2 x[1] + a_1 x[2])^3 (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2])) + \\
& \frac{1}{2} \left( (f_1[0, 0] x[1] x[2] - f_2[0, 0] x[1] x[2]) \left( -\frac{1}{2} f_1[2, 0] x[1]^3 - f_1[1, 1] x[1]^2 x[2] - \right. \right. \\
& \left. \left. \frac{1}{2} f_1[0, 2] x[1] x[2]^2 \right) + (-f_1[1, 0] x[1]^2 - f_1[0, 1] x[1] x[2]) \right. \\
& \left. (f_1[1, 0] x[1]^2 x[2] - f_2[1, 0] x[1]^2 x[2] + f_1[0, 1] x[1] x[2]^2 - f_2[0, 1] x[1] x[2]^2) + \right. \\
& \left. (a_2 x[1] + a_1 x[2]) \left( -\frac{1}{6} f_1[3, 0] x[1]^4 - \frac{1}{2} f_1[2, 1] x[1]^3 x[2] - \frac{1}{2} f_1[1, 2] x[1]^2 x[2]^2 - \right. \right. \\
& \left. \left. \frac{1}{6} f_1[0, 3] x[1] x[2]^3 \right) - f_1[0, 0] x[1] \left( \frac{1}{2} f_1[2, 0] x[1]^3 x[2] - \frac{1}{2} f_2[2, 0] x[1]^3 \right. \right. \\
& \left. \left. x[2] + f_1[1, 1] x[1]^2 x[2]^2 - f_2[1, 1] x[1]^2 x[2]^2 + \frac{1}{2} f_1[0, 2] x[1] x[2]^3 - \right. \right. \\
& \left. \left. \frac{1}{2} f_2[0, 2] x[1] x[2]^3 \right) - a_1 \left( \frac{1}{6} f_1[3, 0] x[1]^4 x[2] - \frac{1}{6} f_2[3, 0] x[1]^4 x[2] + \right. \right. \\
& \left. \left. \frac{1}{2} f_1[2, 1] x[1]^3 x[2]^2 - \frac{1}{2} f_2[2, 1] x[1]^3 x[2]^2 + \frac{1}{2} f_1[1, 2] x[1]^2 x[2]^3 - \right. \right.
\end{aligned}$$

$$\left. \left. \left. \left. \frac{1}{2} f2[1, 2] x[1]^2 x[2]^3 + \frac{1}{6} f1[0, 3] x[1] x[2]^4 - \frac{1}{6} f2[0, 3] x[1] x[2]^4 \right) \right) + \right. \\ \left. \frac{1}{24} \left( 2 (a2 x[1] + a1 x[2]) (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) \right. \right. \\ \left. \left. (-f1[0, 0] x[1] (a2 x[1] + a1 x[2]) - a1 (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2])) + \right. \right. \\ \left. \left. (a2 x[1] + a1 x[2])^2 (a2 x[1] + a1 x[2]) (-f1[1, 0] x[1]^2 - f1[0, 1] x[1] x[2]) - \right. \right. \\ \left. \left. f1[0, 0] x[1] (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) - a1 (f1[1, 0] x[1]^2 x[2] - \right. \right. \\ \left. \left. f2[1, 0] x[1]^2 x[2] + f1[0, 1] x[1] x[2]^2 - f2[0, 1] x[1] x[2]^2) \right) - a1 (a2 x[1] + \right. \\ \left. a1 x[2]) ((f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2])^2 + 2 (a2 x[1] + a1 x[2]) (f1[1, 0] \right. \\ \left. x[1]^2 x[2] - f2[1, 0] x[1]^2 x[2] + f1[0, 1] x[1] x[2]^2 - f2[0, 1] x[1] x[2]^2)) \right) \left. \right) + \\ \frac{1}{6} \left( 2 (a2 x[1] + a1 x[2]) (-f1[1, 0] x[1]^2 - f1[0, 1] x[1] x[2]) \right. \\ \left. (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) + (a2 x[1] + a1 x[2])^2 \right. \\ \left. \left( -\frac{1}{2} f1[2, 0] x[1]^3 - f1[1, 1] x[1]^2 x[2] - \frac{1}{2} f1[0, 2] x[1] x[2]^2 \right) - f1[0, 0] x[1] \right. \\ \left. \left( (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2])^2 + 2 (a2 x[1] + a1 x[2]) (f1[1, 0] x[1]^2 x[2] - \right. \right. \\ \left. \left. f2[1, 0] x[1]^2 x[2] + f1[0, 1] x[1] x[2]^2 - f2[0, 1] x[1] x[2]^2) \right) - \right. \\ \left. a1 \left( 2 (f1[0, 0] x[1] x[2] - f2[0, 0] x[1] x[2]) (f1[1, 0] x[1]^2 x[2] - f2[1, 0] \right. \right. \\ \left. \left. x[1]^2 x[2] + f1[0, 1] x[1] x[2]^2 - f2[0, 1] x[1] x[2]^2) + 2 (a2 x[1] + a1 x[2]) \right. \right. \\ \left. \left. \left( \frac{1}{2} f1[2, 0] x[1]^3 x[2] - \frac{1}{2} f2[2, 0] x[1]^3 x[2] + f1[1, 1] x[1]^2 x[2]^2 - f2[1, 1] \right. \right. \right. \\ \left. \left. \left. x[1]^2 x[2]^2 + \frac{1}{2} f1[0, 2] x[1] x[2]^3 - \frac{1}{2} f2[0, 2] x[1] x[2]^3 \right) \right) \right) \right) z^5 + O[z]^6 \Bigg] \Bigg]$$

```
eqs = {
  Coefficient[lhs1, Y[1, 2, 0]] == Coefficient[rhs1, Y[1, 2, 0]],
  Coefficient[lhs2, Y[1, 2, 0]] == Coefficient[rhs2, Y[1, 2, 0]]
}
```

A very large output was generated. Here is a sample of it:

$$\left\{ \left( -\frac{x[3]}{2} + a1 x[3] - a2 x[3] \right) z + \right. \\ \left( \frac{1}{2} a1 x[1] x[3] - \frac{1}{2} a2 x[1] x[3] + \frac{1}{2} a1 a2 x[1] x[3] - \frac{1}{2} a2^2 x[1] x[3] + f1[0, 0] x[1] x[3] + \right. \\ \left. \frac{1}{2} a1 x[2] x[3] + \frac{1}{2} a1^2 x[2] x[3] - \frac{1}{2} a2 x[2] x[3] - \frac{1}{2} a1 a2 x[2] x[3] + f2[0, 0] x[2] x[3] + \right. \\ \left. \frac{1}{6} (-x[2] x[3] - (x[1] + x[2]) x[3]) \right) z^2 + \left( \ll 43 \gg + \frac{1}{\ll 2 \gg} \ll 1 \gg \right) z^3 + \left( \ll 1 \gg \right) z^4 + \\ \left( \ll 298 \gg + \frac{1}{720} (- (x[1] + x[2])^2 (x[2]^2 + x[2] (x[1] + x[2]) + (x[1] + x[2])^2) x[3] + \right. \\ \left. x[2]^2 (\ll 1 \gg)) \right) z^5 + O[z]^6 = \ll 1 \gg, \ll 1 \gg = \ll 1 \gg \}$$

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$$\left\{ \begin{aligned}
f1[4, 0] &\rightarrow \frac{-5 + 24 a^2 - 24 a^2 + 32 a^3 - 5760 f2[0, 4]}{5760}, \\
f1[3, 1] &\rightarrow \frac{1}{23040} (-83 + 164 a^2 - 184 a^2 + 176 a^3 + 144 a^4 + 11520 f1[0, 2] - \\
&\quad 11520 a^2 f1[0, 2] + 960 f2[1, 0] + 5760 a^2 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[1, 3]), \\
f1[2, 2] &\rightarrow \frac{1}{8640} (-21 + 62 a^2 - 106 a^2 + 84 a^3 + 36 a^4 + 3600 f1[0, 2] - \\
&\quad 5760 a^2 f1[0, 2] + 1440 a^2 f2[1, 0] - 8640 f2[2, 1] - 8640 f2[2, 2]), \\
f1[1, 3] &\rightarrow \frac{1}{23040} (-83 + 164 a^2 - 184 a^2 + 176 a^3 + 144 a^4 + 11520 f1[0, 2] - \\
&\quad 11520 a^2 f1[0, 2] + 960 f2[1, 0] + 5760 a^2 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[3, 1]), \\
f1[0, 4] &\rightarrow \frac{-5 + 24 a^2 - 24 a^2 + 32 a^3 - 5760 f2[4, 0]}{5760}, \\
f1[1, 2] &\rightarrow \frac{1}{34560} (-79 + 120 a^2 - 288 a^2 + 192 a^3 + 288 a^4 + \\
&\quad 17280 f1[0, 2] - 2880 f2[1, 0] + 11520 a^2 f2[1, 0] - 34560 f2[2, 1]), \\
f1[0, 3] &\rightarrow \frac{-35 + 28 a^2 + 12 a^2 - 24 a^3 + 96 a^4 + 4320 f1[0, 2] + 2880 a^2 f2[1, 0] - 17280 f2[1, 2]}{5760}, \\
f1[3, 0] &\rightarrow \frac{1}{240} a^2 (-1 + a^3), \\
f1[2, 1] &\rightarrow \frac{1}{34560} (-79 + 120 a^2 - 288 a^2 + 192 a^3 + 288 a^4 + \\
&\quad 17280 f1[0, 2] - 2880 f2[1, 0] + 11520 a^2 f2[1, 0] - 34560 f2[1, 2]), \\
f2[3, 0] &\rightarrow \frac{49 - 80 a^2 + 48 a^2 - 48 a^3 - 96 a^4 - 8640 f1[0, 2] - 5760 a^2 f2[1, 0] + 34560 f2[1, 2]}{11520}, \\
f2[0, 3] &\rightarrow \frac{-7 + 8 a^2 + 24 a^2 - 32 a^3 + 16 a^4}{3840}, \\
f1[1, 1] &\rightarrow \frac{3 + 10 a^2 - 25 a^2 + 720 f1[0, 2] - 360 f2[1, 0]}{1440}, \\
f2[1, 1] &\rightarrow \frac{9 - 20 a^2 - 10 a^2 - 1440 f1[0, 2] - 720 f2[1, 0]}{2880}, \\
f2[2, 0] &\rightarrow \frac{1}{288} (1 - 4 a^2 - 288 f1[0, 2]), \quad f1[2, 0] \rightarrow \frac{1}{360} (1 - 5 a^2), \\
f2[0, 2] &\rightarrow \frac{1 - 20 a^2 + 20 a^2}{1440}, \quad f1[0, 1] \rightarrow \frac{1}{96} (1 + 4 a^2 - 8 a^2 - 96 f2[1, 0]),
\end{aligned} \right.$$



$$\begin{aligned}
f1[1, 0] &\rightarrow \frac{1}{24} (a2 - a2^2), \quad f2[0, 1] \rightarrow \frac{1}{96} (1 - 4 a2^2), \\
f1[0, 0] &\rightarrow \frac{1}{12} (-1 + 3 a2), \quad f2[0, 0] \rightarrow \frac{1}{24} (-1 + 6 a2), \\
a1 &\rightarrow \frac{1}{2} (-1 + 2 a2), \quad \left\{ f1[4, 0] \rightarrow \frac{-1 - 1152 f2[0, 4]}{1152}, \right. \\
f1[3, 1] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[1, 3]}{23040}, \\
f1[2, 2] &\rightarrow \frac{-7 + 1200 f1[0, 2] - 2880 f2[2, 1] - 2880 f2[2, 2]}{2880}, \\
f1[1, 3] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[3, 1]}{23040}, \\
f1[0, 4] &\rightarrow \frac{-1 - 1152 f2[4, 0]}{1152}, \quad f1[1, 2] \rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[2, 1]}{34560}, \\
f1[0, 3] &\rightarrow \frac{-7 + 864 f1[0, 2] - 3456 f2[1, 2]}{1152}, \quad f1[3, 0] \rightarrow 0, \\
f1[2, 1] &\rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[1, 2]}{34560}, \\
f2[3, 0] &\rightarrow \frac{49 - 8640 f1[0, 2] + 34560 f2[1, 2]}{11520}, \quad f2[0, 3] \rightarrow -\frac{7}{3840}, \\
f1[1, 1] &\rightarrow \frac{1}{480} (1 + 240 f1[0, 2] - 120 f2[1, 0]), \\
f2[1, 1] &\rightarrow \frac{1}{320} (1 - 160 f1[0, 2] - 80 f2[1, 0]), \quad f2[2, 0] \rightarrow \frac{1}{288} (1 - 288 f1[0, 2]), \\
f1[2, 0] &\rightarrow \frac{1}{360}, \quad f2[0, 2] \rightarrow \frac{1}{1440}, \quad f1[0, 1] \rightarrow \frac{1}{96} (1 - 96 f2[1, 0]), \quad f1[1, 0] \rightarrow 0, \\
f2[0, 1] &\rightarrow \frac{1}{96}, \quad f1[0, 0] \rightarrow -\frac{1}{12}, \quad f2[0, 0] \rightarrow -\frac{1}{24}, \quad a1 \rightarrow -\frac{1}{2}, \quad a2 \rightarrow 0 \left. \right\} \}
\end{aligned}$$

Union[sol][[1]] /. a2 → 0

$$\left\{ \begin{aligned}
f1[4, 0] &\rightarrow \frac{-5 - 5760 f2[0, 4]}{5760}, \\
f1[3, 1] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[1, 3]}{23040}, \\
f1[2, 2] &\rightarrow \frac{-21 + 3600 f1[0, 2] - 8640 f2[2, 1] - 8640 f2[2, 2]}{8640}, \\
f1[1, 3] &\rightarrow \frac{-83 + 11520 f1[0, 2] + 960 f2[1, 0] - 34560 f2[1, 2] - 23040 f2[3, 1]}{23040}, \\
f1[0, 4] &\rightarrow \frac{-5 - 5760 f2[4, 0]}{5760}, f1[1, 2] \rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[2, 1]}{34560}, \\
f1[0, 3] &\rightarrow \frac{-35 + 4320 f1[0, 2] - 17280 f2[1, 2]}{5760}, f1[3, 0] \rightarrow 0, \\
f1[2, 1] &\rightarrow \frac{-79 + 17280 f1[0, 2] - 2880 f2[1, 0] - 34560 f2[1, 2]}{34560}, \\
f2[3, 0] &\rightarrow \frac{49 - 8640 f1[0, 2] + 34560 f2[1, 2]}{11520}, \\
f2[0, 3] &\rightarrow -\frac{7}{3840}, f1[1, 1] \rightarrow \frac{3 + 720 f1[0, 2] - 360 f2[1, 0]}{1440}, \\
f2[1, 1] &\rightarrow \frac{9 - 1440 f1[0, 2] - 720 f2[1, 0]}{2880}, f2[2, 0] \rightarrow \frac{1}{288} (1 - 288 f1[0, 2]), \\
f1[2, 0] &\rightarrow \frac{1}{360}, f2[0, 2] \rightarrow \frac{1}{1440}, f1[0, 1] \rightarrow \frac{1}{96} (1 - 96 f2[1, 0]), \\
f1[1, 0] &\rightarrow 0, f2[0, 1] \rightarrow \frac{1}{96}, f1[0, 0] \rightarrow -\frac{1}{12}, f2[0, 0] \rightarrow -\frac{1}{24}, a1 \rightarrow -\frac{1}{2} \}
\end{aligned} \right.$$