

Pensieve Header: A failed attempt to solve the two F equations with analytic hair using Tail Scattering, where F is written as an exponential. The equations are derived but not solved.

Ouch! f1 and f2 were treated as constants rather than functions!

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\ScatterAndGlow"];
<< ScatterAndGlow.m
```

```
F = TS[Exp[
  -1/4 Ar[2, 1] + 1/4 Ar[1, 2] + Y[1, 2, 1, AH[f1]] + Y[1, 2, 2, AH[f2]]
]]
```

```
TS[Ar[1, 0] →
```

$$\begin{aligned} & \text{Ar}[1, 0] + Y[1, 2, 0, \text{AH}\left[\left(e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]}\right) (-1 + 4 f1 x[1])\right)\right] / \\ & \left(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]\right)]], \text{Ar}[2, 0] \rightarrow \\ & \text{Ar}[2, 0] + Y[1, 2, 0, \text{AH}\left[-\left(e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]}\right) (-1 + 4 f2 x[2])\right)\right] / \\ & \left(-x[1] + x[2] - 4 f1 x[1] x[2] + 4 f2 x[1] x[2]\right)]]] \end{aligned}$$

```
Ar[1, 0] + Ar[2, 0] // F
```

```
Ar[1, 0] + Ar[2, 0] +
```

$$\begin{aligned} & Y[1, 2, 0, \text{AH}\left[\left(2 e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]}\right) (-1 + 2 f1 x[1] + 2 f2 x[2])\right)\right] / \\ & \left(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]\right)]] \end{aligned}$$

$$\begin{aligned} & \text{Simplify}\left[\left(2 e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]}\right) (-1 + 2 f1 x[1] + 2 f2 x[2])\right)\right] / \\ & \left(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]\right) \end{aligned}$$

$$\begin{aligned} & \left(2 e^{-\frac{1}{4} (1+4 f2 x[1]) x[2]} \left(e^{\frac{x[1]}{4} + f1 x[1] x[2]} - e^{\frac{x[2]}{4} + f2 x[1] x[2]}\right) (-1 + 2 f1 x[1] + 2 f2 x[2])\right) / \\ & \left(x[2] + x[1] (-1 - 4 f1 x[2] + 4 f2 x[2])\right) \end{aligned}$$

```
F21 = TS[Exp[
```

```
  -1/4 Ar[1, 2] + 1/4 Ar[2, 1] + Y[2, 1, 2, AH[f1]] + Y[2, 1, 1, AH[f2]]
]]
```

```
TS[Ar[1, 0] → Ar[1, 0] +
```

$$\begin{aligned} & Y[1, 2, 0, \text{AH}\left[\frac{e^{-\frac{x[1]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[2]}{4} + f1 x[1] x[2]} + e^{\frac{x[1]}{4} + f2 x[1] x[2]}\right) (-1 + 4 f2 x[1])}{x[1] - x[2] - 4 f1 x[1] x[2] + 4 f2 x[1] x[2]}\right] \right], \text{Ar}[2, 0] \rightarrow \\ & \text{Ar}[2, 0] + Y[1, 2, 0, \text{AH}\left[-\frac{e^{-\frac{x[1]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[2]}{4} + f1 x[1] x[2]} + e^{\frac{x[1]}{4} + f2 x[1] x[2]}\right) (-1 + 4 f1 x[2])}{-x[1] + x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]}\right] \right] \end{aligned}$$

lhs1 = Ar[3, 0] // S[Exp[Ar[1, 3] + Ar[2, 3]]] // F

Ar[3, 0] + Y[1, 2, 0,

$$\text{AH} \left[- \left(e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(e^{\frac{x[2]}{4} + f2 x[1] x[2]} x[1]^2 - e^{\frac{5x[2]}{4} + f2 x[1] x[2]} x[1]^2 - 2 e^{\frac{x[1]}{4} + f1 x[1] x[2]} x[1] x[2] + 2 e^{\frac{5x[1]}{4} + x[2] + f1 x[1] x[2]} x[1] x[2] + e^{\frac{x[2]}{4} + f2 x[1] x[2]} x[1] x[2] - e^{x[1] + \frac{5x[2]}{4} + f2 x[1] x[2]} x[1] x[2] + 4 e^{\frac{x[1]}{4} + f1 x[1] x[2]} f1 x[1]^2 x[2] - 4 e^{\frac{5x[1]}{4} + x[2] + f1 x[1] x[2]} f1 x[1]^2 x[2] - 4 e^{\frac{5x[2]}{4} + f2 x[1] x[2]} f1 x[1]^2 x[2] + 4 e^{x[1] + \frac{5x[2]}{4} + f2 x[1] x[2]} f1 x[1]^2 x[2] - 4 e^{\frac{x[2]}{4} + f2 x[1] x[2]} f2 x[1]^2 x[2] + 4 e^{\frac{5x[2]}{4} + f2 x[1] x[2]} f2 x[1]^2 x[2] + e^{\frac{5x[2]}{4} + f2 x[1] x[2]} x[2]^2 - e^{x[1] + \frac{5x[2]}{4} + f2 x[1] x[2]} x[2]^2 - 4 e^{\frac{5x[2]}{4} + f2 x[1] x[2]} f1 x[1] x[2]^2 + 4 e^{x[1] + \frac{5x[2]}{4} + f2 x[1] x[2]} f1 x[1] x[2]^2 + 4 e^{\frac{x[1]}{4} + f1 x[1] x[2]} f2 x[1] x[2]^2 - 4 e^{\frac{5x[1]}{4} + x[2] + f1 x[1] x[2]} f2 x[1] x[2]^2 - 4 e^{\frac{x[2]}{4} + f2 x[1] x[2]} f2 x[1] x[2]^2 + 4 e^{\frac{5x[2]}{4} + f2 x[1] x[2]} f2 x[1] x[2]^2 \right) x[3] \right) / \left(x[1] x[2] (x[1] + x[2]) (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]) \right) \right] +$$

$$Y[1, 3, 0, \text{AH} \left[\frac{e^{x[2]} (-1 + e^{x[1]})}{x[1]} \right]] +$$

Y[
2,
3,
0,

$$\text{AH} \left[\frac{-1 + e^{x[2]}}{x[2]} \right]]$$

rhs1 = Ar[3, 0] // F // S[sigma[1, 3], sigma[2, 3]]

$$\text{Ar}[3, 0] + Y[1, 2, 0, \text{AH} \left[- \frac{(-1 + e^{x[1]}) (-1 + e^{x[2]}) x[3]}{x[1] x[2]} \right]] +$$

$$Y[1, 3, 0, \text{AH} \left[\frac{e^{x[2]} (-1 + e^{x[1]})}{x[1]} \right]] + Y[2, 3, 0, \text{AH} \left[\frac{-1 + e^{x[2]}}{x[2]} \right]]$$

lhs2 = Ar[1, 0] // F21 // S[sigma[1, 2]] // S[Exp[1/2 Ar[1, 1]]] // S[Exp[1/2 Ar[2, 2]]]

$$\text{Ar}[1, 0] + Y[1, 2, 0, \text{AH} \left[- \frac{e^{\frac{3x[1]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[2]}{4} + f1 x[1] x[2]} + e^{\frac{x[1]}{4} + f2 x[1] x[2]} \right) (-1 + 4 f2 x[1])}{-x[1] + x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]} \right]]$$

rhs2 = Ar[1, 0] // S[Exp[Expand[1/2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] // F

Ar[1, 0] +

$$Y[1, 2, 0, \text{AH} \left[\left(e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(2 e^{\frac{x[1]}{4} + f1 x[1] x[2]} x[1] - e^{\frac{3x[1]}{4} + \frac{x[2]}{2} + f1 x[1] x[2]} x[1] - e^{\frac{x[2]}{4} + f2 x[1] x[2]} x[1] - 4 e^{\frac{x[1]}{4} + f1 x[1] x[2]} f1 x[1]^2 + 4 e^{\frac{x[2]}{4} + f2 x[1] x[2]} f1 x[1]^2 + e^{\frac{3x[1]}{4} + \frac{x[2]}{2} + f1 x[1] x[2]} x[2] - e^{\frac{x[2]}{4} + f2 x[1] x[2]} x[2] - 4 e^{\frac{3x[1]}{4} + \frac{x[2]}{2} + f1 x[1] x[2]} f1 x[1] x[2] + 4 e^{\frac{x[2]}{4} + f2 x[1] x[2]} f1 x[1] x[2] - 4 e^{\frac{x[1]}{4} + f1 x[1] x[2]} f2 x[1] x[2] + 4 e^{\frac{3x[1]}{4} + \frac{x[2]}{2} + f1 x[1] x[2]} f2 x[1] x[2] \right) \right) / \left((x[1] + x[2]) (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]) \right) \right]]$$

```
(eqns1 = Simplify[{
  Coefficient[lhs2, Y[1, 2, 0]] == Coefficient[rhs2, Y[1, 2, 0]],
  Coefficient[lhs1, Y[1, 2, 0]] == Coefficient[rhs1, Y[1, 2, 0]]
}]) // ColumnForm
```

$$\frac{e^{\frac{1}{4}x[1] (3-4f2x[2])} \left(-e^{\frac{x[2]}{4}+f1x[1]x[2]} + e^{\frac{x[1]}{4}+f2x[1]x[2]} \right) (-1+4f2x[1])}{-x[2]+x[1] (1-4f1x[2]+4f2x[2])} = \frac{e^{-\frac{1}{4}(1+4f2x[1])x[2]} \left(-e^{\frac{x[2]}{4}+f2x[1]x[2]} (-1+4f1x[1]) (x[1]+x[2]) + 2e^{\frac{x[1]}{4}+f1x[1]x[2]} \right) (x[1]+x[2]) (x[2])}{e^{-\frac{1}{4}(1+4f2x[1])x[2]} \left(-e^{\frac{x[2]}{4}+f2x[1]x[2]} (-1+4f1x[1]) x[2] (x[1]+x[2]) + 2e^{\frac{x[1]}{4}+f1x[1]x[2]} x[1]x[2] (-1+2f1x[1]+2f2x[2]) - 2e^{x[2]+x[1] \left(\frac{5}{4}+f1x[2] \right)} x[1]x[2] \right) x[1]x[2] (x[1]+x[2]) (x[2])}$$

```
(eqns2 = Simplify[{
  e^{f2x[1]x[2]} (Coefficient[lhs2, Y[1, 2, 0]] - Coefficient[rhs2, Y[1, 2, 0]]),
  e^{\frac{1}{4}(1+4f2x[1])x[2]} x[1]x[2] (x[1]+x[2]) (x[2]+x[1] (-1-4f1x[2]+4f2x[2])) / x[3]
  (Coefficient[lhs1, Y[1, 2, 0]] - Coefficient[rhs1, Y[1, 2, 0]])
}]) /. {x[1] -> x1, x[2] -> x2} // ColumnForm
```

$$e^{3x1/4} \left(-e^{\frac{x2}{4}+f1x1x2} + e^{\frac{x1}{4}+f2x1x2} \right) (-1+4f2x1) \frac{e^{-x2/4} \left(-e^{\frac{x2}{4}+f2x1x2} (-1+4f1x1) (x1+x2) + 2e^{\frac{x1}{4}+f1x1x2} x1 (-1+2f1x1+2f2x2) + e^{\frac{x2}{2}+x1 \left(\frac{3}{4}+f1x2 \right)} (-x2+x1 (1-x2+x1 (1-4f1x2+4f2x2)) \right)}{-x2+x1 (1-4f1x2+4f2x2)} + \frac{e^{-x2/4} \left(-e^{\frac{x2}{4}+f2x1x2} (-1+4f1x1) (x1+x2) + 2e^{\frac{x1}{4}+f1x1x2} x1 (-1+2f1x1+2f2x2) - 2e^{x2+x1 \left(\frac{5}{4}+f1x2 \right)} x1x2 (-1-x2+x1 (1-4f1x2+4f2x2)) \right)}{(x1+x2) (-x2+x1 (1+4f1x2-4f2x2))}$$

```
(eqns3 = Simplify[eqns2 /. {x1 -> Random[], x2 -> Random[]}]) // ColumnForm
```

$$\frac{e^{0.26421f1} (-3.08431f1^2+f1 (3.00559-0.0942192f2)+3.17853 (-0.959865+f2) (-0.0146104+f2)) + e^{0.26421f2} (2.3068f1^2-3.98104 (-0.767188+f2) (-0.0615005+1.05684f1-1.05684f2) (0.0615005-1.05684f1+1.05684f2))}{e^{0.26421f1} (1.0906-1.19024f1-1.0561f2) + e^{0.26421f2} (-1.09028+0.891596f1+1.32047f2)}$$

The following crashed after a few hours

```
Solve[ (# == 0) & /@ eqns3, {f1, f2}]
```

No more memory available.
 Mathematica kernel has shut down.
 Try quitting other applications and then retry.

F

```
TS[Ar[1, 0] -> Ar[1, 0] +
  Y[1, 2, 0, AH[
    e^{-\frac{x[2]}{4}-f2x[1]x[2]} \left( -e^{\frac{x[1]}{4}+f1x[1]x[2]} + e^{\frac{x[2]}{4}+f2x[1]x[2]} \right) (-1+4f1x[1])
    x[1] - x[2] + 4f1x[1]x[2] - 4f2x[1]x[2]
  ]], Ar[2, 0] ->
  Ar[2, 0] + Y[1, 2, 0, AH[
    e^{-\frac{x[2]}{4}-f2x[1]x[2]} \left( -e^{\frac{x[1]}{4}+f1x[1]x[2]} + e^{\frac{x[2]}{4}+f2x[1]x[2]} \right) (-1+4f2x[2])
    -x[1] + x[2] - 4f1x[1]x[2] + 4f2x[1]x[2]
  ]]]]
```

```

Reduce [
{

$$g1 == \frac{e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left( -e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]} \right) (-1 + 4 f1 x[1])}{x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]},$$


$$g2 == - \frac{e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left( -e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]} \right) (-1 + 4 f2 x[2])}{-x[1] + x[2] - 4 f1 x[1] x[2] + 4 f2 x[1] x[2]}$$

}, {f1, f2}
]
$Aborted

```