

Pensieve Header: A failed attempt to solve the two F equations with analytic hair using Full Scattering, where F is written as an exponential.

Ouch! f1 and f2 were treated as constants rather than functions!

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\ScatterAndGlow"];
<< ScatterAndGlow-090103.m
```

```
F = S[Exp[-1/4 Ar[2, 1] + 1/4 Ar[1, 2] + Y[1, 2, 1, AH[f1]] + Y[1, 2, 2, AH[f2]]]]
```

```
S[Ar[0, 1] -> Ar[0, 1] +
```

$$\begin{aligned}
& Y\left[0, 1, 1, \text{AH}\left[\left(e^{-\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}\left(-1+e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}\right)^2\right.\right.\right. \\
& \quad \left.\left.\left.(-1+4f1x[1])x[2](-1+4f2x[2])\right)\right]/\left.(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2\right)\right] + \\
& Y\left[0, 1, 2, \text{AH}\left[\left(e^{-\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}\left(-1+e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}\right)\right.\right.\right. \\
& \quad \left.\left.\left.(-1+4f2x[2])\left(x[1]^2+e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}x[1]^2-2x[1]x[2]-2e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}x[1]x[2]+8f1x[1]^2x[2]+8e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f1x[1]^2x[2]-8f2x[1]^2x[2]-8e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f2x[1]^2x[2]+x[2]^2+e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}x[2]^2-8f1x[1]x[2]^2-8e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f1x[1]x[2]^2+8f2x[1]x[2]^2+8e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f2x[1]x[2]^2+16f1^2x[1]^2x[2]^2+16e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f1^2x[1]^2x[2]^2-32f1f2x[1]^2x[2]^2-32e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f1f2x[1]^2x[2]^2+16f2^2x[1]^2x[2]^2+16e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}f2^2x[1]^2x[2]^2+x[1]\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}-e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}x[1]\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}+x[2]\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}-e^{\frac{1}{4}\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}}x[2]\sqrt{(x[1]-x[2]+4f1x[1]x[2]-4f2x[1]x[2])^2}-4f1x[1]x[2]\right)\right]\right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + 4 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} \right. \right. \right. \right. \\
& f1 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - 4 f2 x[1] x[2] \\
& \left. \left. \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + 4 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} \right. \right. \right. \right. \\
& f2 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \left. \right) \left. \right) \left. \right) / \\
& \left(2 (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 \right. \\
& \left. \left. \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \right) \right] \right] + \\
& Y \left[0, 2, 1, \text{AH} \left[- \left(e^{-\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} \left(-1 + e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} \right) \right) \right. \right. \right. \\
& \left. \left. \left. \left. (-1 + 4 f1 x[1]) \left(-x[1]^2 - e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} x[1]^2 + \right. \right. \right. \right. \\
& 2 x[1] x[2] + 2 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} x[1] x[2] - \\
& 8 f1 x[1]^2 x[2] - 8 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f1 x[1]^2 x[2] + \\
& 8 f2 x[1]^2 x[2] + 8 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f2 x[1]^2 x[2] - \\
& x[2]^2 - e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} x[2]^2 + 8 f1 x[1] x[2]^2 + \\
& 8 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f1 x[1] x[2]^2 - 8 f2 x[1] x[2]^2 - \\
& 8 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f2 x[1] x[2]^2 - 16 f1^2 x[1]^2 x[2]^2 - \\
& 16 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f1^2 x[1]^2 x[2]^2 + 32 f1 f2 x[1]^2 x[2]^2 + \\
& 32 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f1 f2 x[1]^2 x[2]^2 - 16 f2^2 x[1]^2 x[2]^2 - \\
& 16 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f2^2 x[1]^2 x[2]^2 + x[1] \\
& \left. \left. \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} \right. \right. \right. \right. \\
& x[1] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + x[2] \\
& \left. \left. \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} \right. \right. \right. \right. \\
& x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - \\
& 4 f1 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + \\
& 4 e^{\frac{1}{4} \sqrt{(x[1]-x[2]+4 f1 x[1] x[2]-4 f2 x[1] x[2])^2}} f1 x[1] x[2] \\
& \left. \left. \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - \right. \right. \right. \right. \\
& 4 f2 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} +
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \frac{x[1] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 + x[2]} + \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}}}{x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - 4 f1 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + 4 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} f1 x[1] x[2]} \right. \\
& \left. \frac{\sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - 4 f2 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + 4 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} f2 x[1] x[2]}{\sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}}}{\left. \left(2 (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \right) \right]} + \\
& Y \left[0, 2, 1, \text{AH} \left[\left(e^{-\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \left(-1 + e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \right) \right) \right. \right. \\
& \left. \left. (-1 + 4 f1 x[1]) \left(-x[1]^2 - e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} x[1]^2 + \right. \right. \right. \\
& \left. \left. 2 x[1] x[2] + 2 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} x[1] x[2] - \right. \right. \\
& \left. \left. 8 f1 x[1]^2 x[2] - 8 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f1 x[1]^2 x[2] + \right. \right. \\
& \left. \left. 8 f2 x[1]^2 x[2] + 8 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f2 x[1]^2 x[2] - \right. \right. \\
& \left. \left. x[2]^2 - e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} x[2]^2 + 8 f1 x[1] x[2]^2 + \right. \right. \\
& \left. \left. 8 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f1 x[1] x[2]^2 - 8 f2 x[1] x[2]^2 - \right. \right. \\
& \left. \left. 8 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f2 x[1] x[2]^2 - 16 f1^2 x[1]^2 x[2]^2 - \right. \right. \\
& \left. \left. 16 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f1^2 x[1]^2 x[2]^2 + 32 f1 f2 x[1]^2 x[2]^2 + \right. \right. \\
& \left. \left. 32 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f1 f2 x[1]^2 x[2]^2 - 16 f2^2 x[1]^2 x[2]^2 - \right. \right. \\
& \left. \left. 16 e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} f2^2 x[1]^2 x[2]^2 + x[1] \right. \right. \\
& \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \right. \right. \\
& \left. \left. x[1] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} + x[2] \right. \right. \\
& \left. \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} - e^{\frac{1}{4}} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \frac{x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 - 4 f1 x[1] x[2]} \right. \right. \right. \right. \right. \\
& \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 + 4 e^{\frac{1}{4} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} \\
& f1 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 - 4 f2 x[1] x[2]} \\
& \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 + 4 e^{\frac{1}{4} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} \\
& f2 x[1] x[2] \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} \Big) \Big) \Big) \Big) / \\
& \left(2 (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 \right. \\
& \left. \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2} \right) \Big) \Big) + \\
& Y \left[0, 2, 2, \text{AH} \left[\left(e^{-\frac{1}{4} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} \left(-1 + e^{\frac{1}{4} \sqrt{(x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2}} \right)^2 \right. \right. \right. \\
& \left. \left. \left. x[1] (-1 + 4 f1 x[1]) (-1 + 4 f2 x[2]) \right) \right] \right] / \\
& \left. \left. \left. \left. \left. (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])^2 \right) \right] \right] \right], \\
& \text{Ar}[1, 0] \rightarrow \text{Ar}[1, 0] + Y \left[1, 2, \right. \\
& 0, \\
& \text{AH} \left[\right. \\
& \left. \frac{e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]} \right) (-1 + 4 f1 x[1])}{x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]} \right] \right], \text{Ar} \left[\right. \\
& 2, 0] \rightarrow \text{Ar}[2, \\
& 0] + \\
& Y \left[\right. \\
& 1, \\
& 2, \\
& 0, \\
& \text{AH} \left[\right. \\
& \left. \frac{e^{-\frac{x[2]}{4} - f2 x[1] x[2]} \left(-e^{\frac{x[1]}{4} + f1 x[1] x[2]} + e^{\frac{x[2]}{4} + f2 x[1] x[2]} \right) (-1 + 4 f2 x[2])}{-x[1] + x[2] - 4 f1 x[1] x[2] + 4 f2 x[1] x[2]} \right] \right] \Big) \Big) \Big) \Big) \Big) \\
& \text{Short} \left[\right. \\
& \text{F21} = \text{S}[\text{Exp}[-1/4 \text{Ar}[1, 2] + 1/4 \text{Ar}[2, 1] + Y[2, 1, 2, \text{AH}[f1]] + Y[2, 1, 1, \text{AH}[f2]]]] \\
&] \\
& \text{S}[\ll 1 \gg] \\
& \text{Short}[\text{lhs1} = \text{Ar}[3, 0] // \text{S}[\text{Exp}[\text{Ar}[1, 3] + \text{Ar}[2, 3]]] // \text{F}] \\
& \text{Ar}[3, 0] + Y \left[1, 2, 0, \text{AH} \left[-\frac{e^{-\ll 1 \gg - \ll 1 \gg} (\ll 1 \gg) x[3]}{x[1] x[2] (\ll 1 \gg) (\ll 1 \gg)} \right] \right] + Y[\ll 1 \gg] + Y \left[2, 3, 0, \text{AH} \left[\frac{-1 + e^{x[2]}}{x[2]} \right] \right]
\end{aligned}$$

```

Short[rhs1 = Ar[3, 0] // F // S[sigma[1, 3], sigma[2, 3]]

Ar[3, 0] + Y[1, 2, 0, AH[- ( -1 + e^x <<1>> <<1>> ) <<1>> x[3] ] ] +
Y[1, <<2>>, AH[ ( <<1>> / <<1>> ) ] ] + Y[2, 3, 0, AH[ ( -1 + e^x[2] / x[2] ) ] ]

lhs2 = Ar[1, 0] // F21 // S[sigma[1, 2]] // S[Exp[1 / 2 Ar[1, 1]]] // S[Exp[1 / 2 Ar[2, 2]]]

Ar[1, 0] + Y[1, 2, 0, AH[- ( e^(3x[1]/4 - f2x[1]x[2]) ( -e^(x[2]/4 + f1x[1]x[2]) + e^(x[1]/4 + f2x[1]x[2]) ) (-1 + 4 f2 x[1]) ) /
(-x[1] + x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2]) ] ]

rhs2 = Ar[1, 0] // S[Exp[Expand[1 / 2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] // F

Ar[1, 0] +
Y[1, 2, 0, AH[ ( e^(-x[2]/4 - f2x[1]x[2]) ( 2 e^(x[1]/4 + f1x[1]x[2]) x[1] - e^(3x[1]/4 + x[2]/2 + f1x[1]x[2]) x[1] - e^(x[2]/4 + f2x[1]x[2]) x[1] -
4 e^(x[1]/4 + f1x[1]x[2]) f1 x[1]^2 + 4 e^(x[2]/4 + f2x[1]x[2]) f1 x[1]^2 + e^(3x[1]/4 + x[2]/2 + f1x[1]x[2]) x[2] -
e^(x[2]/4 + f2x[1]x[2]) x[2] - 4 e^(3x[1]/4 + x[2]/2 + f1x[1]x[2]) f1 x[1] x[2] + 4 e^(x[2]/4 + f2x[1]x[2]) f1 x[1] x[2] -
4 e^(x[1]/4 + f1x[1]x[2]) f2 x[1] x[2] + 4 e^(3x[1]/4 + x[2]/2 + f1x[1]x[2]) f2 x[1] x[2] ) ) /
((x[1] + x[2]) (x[1] - x[2] + 4 f1 x[1] x[2] - 4 f2 x[1] x[2])) ] ]

```

The following was aborted after a day of computation:

```

sol = Solve[
{
Coefficient[lhs1, Y[1, 2, 0]] == Coefficient[rhs1, Y[1, 2, 0]],
Coefficient[lhs2, Y[1, 2, 0]] == Coefficient[rhs2, Y[1, 2, 0]]
},
{f1, f2}
]
$Aborted

```

■ To do

- Print lhs1 and rhs1 in full.
- See if matters can be simplified by picking specific coefficients of or values for x[3].