Pensieve Header: Solving the two F equations with perturbative hair, where F is written as an exponential.

Ouch! Incorrect F21.

```
$CutoffDegree = 8;
H[h_] := PH[(h /. x[i_] :> z * x[i]) + O[z]^$CutoffDegree];

f1coeffs = DeclareSeries[f1[x[1], x[2]], $CutoffDegree - 2];
f2coeffs = DeclareSeries[f2[x[1], x[2]], $CutoffDegree - 2];
PH[f1]
```

$$\begin{aligned}
&PH\Big[f1[0,0] + (f1[1,0]\,x[1] + f1[0,1]\,x[2])\,z + \\
&\quad\left(\frac{1}{2}f1[2,0]\,x[1]^2 + f1[1,1]\,x[1]\,x[2] + \frac{1}{2}f1[0,2]\,x[2]^2\right)z^2 + \\
&\quad\left(\frac{1}{6}f1[3,0]\,x[1]^3 + \frac{1}{2}f1[2,1]\,x[1]^2\,x[2] + \frac{1}{2}f1[1,2]\,x[1]\,x[2]^2 + \frac{1}{6}f1[0,3]\,x[2]^3\right)z^3 + \\
&\quad\left(\frac{1}{24}f1[4,0]\,x[1]^4 + \frac{1}{6}f1[3,1]\,x[1]^3\,x[2] + \right. \\
&\qquad\left.\frac{1}{4}f1[2,2]\,x[1]^2\,x[2]^2 + \frac{1}{6}f1[1,3]\,x[1]\,x[2]^3 + \frac{1}{24}f1[0,4]\,x[2]^4\right)z^4 + \\
&\quad\left(\frac{1}{120}f1[5,0]\,x[1]^5 + \frac{1}{24}f1[4,1]\,x[1]^4\,x[2] + \frac{1}{12}f1[3,2]\,x[1]^3\,x[2]^2 + \frac{1}{12}f1[2,3]\,x[1]^2\,x[2]^3 + \right. \\
&\qquad\left.\frac{1}{24}f1[1,4]\,x[1]\,x[2]^4 + \frac{1}{120}f1[0,5]\,x[2]^5\right)z^5 + O[z]^6\Big]
\end{aligned}$$

```
F = S[Exp[a1 Ar[2, 1] + a2 Ar[1, 2] + Y[1, 2, 1, PH[f1]] + Y[1, 2, 2, PH[f2]]]];
F21 = S[Exp[a1 Ar[1, 2] + a2 Ar[2, 1] + Y[2, 1, 2, PH[f1]] + Y[2, 1, 1, PH[f2]]]];

Short[lhs = Ar[3, 0] // S[Exp[Ar[1, 3] + Ar[2, 3]]] // F]
```

$$Ar[3,0] + Y[1,2,0,PH[\langle\langle 1\rangle\rangle + \langle\langle 7\rangle\rangle]] + \langle\langle 1\rangle\rangle + Y\Big[2,3,0,PH\Big[1 + \frac{1}{2}x[2]\,z + \frac{1}{6}x[2]^2\,z^2 + \frac{1}{24}x[2]^3\,z^3 + \frac{1}{120}x[2]^4\,z^4 + \frac{1}{720}x[2]^5\,z^5 + \frac{x[2]^6\,z^6}{5040} + \frac{x[2]^7\,z^7}{40\,320} + O[z]^8\Big]\Big]$$

```
Short[rhs = Ar[3, 0] // F // S[sigma[1, 3], sigma[2, 3]]]
```

$$Ar[3,0] + Y[1,2,0,PH[\langle\langle 1\rangle\rangle]] + Y[\langle\langle 1\rangle\rangle] + Y\Big[2,3,0,PH\Big[1 + \frac{1}{2}x[2]\,z + \frac{1}{6}x[2]^2\,z^2 + \frac{1}{24}x[2]^3\,z^3 + \frac{1}{120}x[2]^4\,z^4 + \frac{1}{720}x[2]^5\,z^5 + \frac{x[2]^6\,z^6}{5040} + \frac{x[2]^7\,z^7}{40\,320} + O[z]^8\Big]\Big]$$

```
sol = First[PHSolve[
   Coefficient[lhs, Y[1, 2, 0]] == Coefficient[rhs, Y[1, 2, 0]],
   Join[{a1, a2}, f1coeffs, f2coeffs]
  ]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{f1[5,0] \to \frac{1}{504}\left(a2 - a2^6\right),\right.$$

$$f1[4,1] \to \frac{1}{30\,240}\left(21\,a2 + 42\,a2^2 + 56\,a2^3 + 84\,a2^4 - 90\,a2^5 - 72\,a2^6 + 504\,f2[1,0] - 2016\,a2^3\,f2[1,0] + 5040\,a2\,f2[2,0] - 5040\,f2[3,0] + 10\,080\,a2\,f2[3,0] - 7560\,f2[4,0] - 6048\,f2[5,0]\right),$$

$$f1[3,2] \to \frac{1}{241\,920}\left(-7 + 14\,a2 + 189\,a2^2 + 1680\,a2^3 + 996\,a2^4 - 1464\,a2^5 - 720\,a2^6 + 3360\,f2[1,0] + 30\,240\,a2\,f2[1,0] - 18\,144\,a2^2\,f2[1,0] - 32\,256\,a2^3\,f2[1,0] - 120\,960\,f2[1,0]^2 + 40\,320\,a2\,f2[1,1] - 15\,120\,f2[2,0] + 80\,640\,a2\,f2[2,0] - 60\,480\,f2[2,1] + 120\,960\,a2\,f2[2,1] - 50\,400\,f2[3,0] + 80\,640\,a2\,f2[3,0] - 120\,960\,f2[3,1] - 30\,240\,f2[4,0] - 120\,960\,f2[4,1]\right),$$

$$f1[2,3] \to \frac{1}{483\,840}\left(-35 - 273\,a2 + 588\,a2^2 + 5476\,a2^3 + 2760\,a2^4 - 4512\,a2^5 - 1920\,a2^6 - 5040\,f2[1,0] + 159\,264\,a2\,f2[1,0] - 88\,704\,a2^2\,f2[1,0] - 96\,768\,a2^3\,f2[1,0] - 483\,840\,f2[1,0]^2 - 60\,480\,f2[1,1] + 322\,560\,a2\,f2[1,1] - 120\,960\,f2[1,2] + 241\,920\,a2\,f2[1,2] - 55\,440\,f2[2,0] + 201\,600\,a2\,f2[2,0] - 302\,400\,f2[2,1] + 483\,840\,a2\,f2[2,1] - 362\,880\,f2[2,2] - 60\,480\,f2[3,0] + 80\,640\,a2\,f2[3,0] - 241\,920\,f2[3,1] - 483\,840\,f2[3,2]\right),$$

$$f1[1,4] \to \frac{1}{967\,680}\left(-217 - 560\,a2 + 4036\,a2^2 + 12\,288\,a2^3 + 7152\,a2^4 - 11\,328\,a2^5 - 5760\,a2^6 - 49\,056\,f2[1,0] + 548\,352\,a2\,f2[1,0] - 274\,176\,a2^2\,f2[1,0] - 258\,048\,a2^3\,f2[1,0] - 967\,680\,f2[1,0]^2 - 443\,520\,f2[1,1] + 1\,612\,800\,a2\,f2[1,1] - 1\,209\,600\,f2[1,2] + 1\,935\,360\,a2\,f2[1,2] - 967\,680\,f2[1,3] - 100\,800\,f2[2,0] + 322\,560\,a2\,f2[2,0] - 725\,760\,f2[2,1] + 967\,680\,a2\,f2[2,1] - 1\,451\,520\,f2[2,2] - 1\,935\,360\,f2[2,3]\right),$$

$$f1[0,5] \to \frac{1}{193\,536}\left(53 + 44\,a2 + 1492\,a2^2 + 2992\,a2^3 - 48\,a2^4 - 960\,a2^5 - 2304\,a2^6 - 18\,144\,f2[1,0] + 153\,216\,a2\,f2[1,0] - 64\,512\,a2^2\,f2[1,0] - 64\,512\,a2^3\,f2[1,0] - 201\,600\,f2[1,1] + 645\,120\,a2\,f2[1,1] - 725\,760\,f2[1,2] + 967\,680\,a2\,f2[1,2] - 967\,680\,f2[1,3] - 967\,680\,f2[1,4]\right),$$

$$f2[0,5] \to \frac{31 - 52\,a2 - 60\,a2^2 + 160\,a2^3 - 240\,a2^4 + 192\,a2^5 - 64\,a2^6}{32\,256},$$

$$f1[4,0] \to \frac{-2 + 7\,a2^2 + 7\,a2^4}{2520},$$

$$f1[3,1] \to \frac{1}{40\,320}\left(-27 - 14\,a2 - 42\,a2^2 + 196\,a2^3 + 126\,a2^4 + 3360\,a2\,f2[1,0] - 5040\,f2[2,0] + 10\,080\,a2\,f2[2,0] - 10\,080\,f2[3,0] - 10\,080\,f2[4,0]\right),$$

$$f1[2,2] \to \frac{1}{725\,760}\left(-425 - 840\,a2 - 2352\,a2^2 + 6720\,a2^3 + 2688\,a2^4 - 30\,240\,f2[1,0] + 161\,280\,a2\,f2[1,0] - 120\,960\,f2[1,1] + 241\,920\,a2\,f2[1,1] - 151\,200\,f2[2,0] + 241\,920\,a2\,f2[2,0] - 362\,880\,f2[2,1] - 120\,960\,f2[3,0] - 483\,840\,f2[3,1]\right),$$

$$f1[1, 3] \to \frac{1}{967\,680}\left(-809 - 2352\,a2 - 2604\,a2^2 + 12\,096\,a2^3 + 4704\,a2^4 - 110\,880\,f2[1, 0] + 403\,200\,a2\,f2[1, 0] - 604\,800\,f2[1, 1] + 967\,680\,a2\,f2[1, 1] - 725\,760\,f2[1, 2] - 181\,440\,f2[2, 0] + 241\,920\,a2\,f2[2, 0] - 725\,760\,f2[2, 1] - 1\,451\,520\,f2[2, 2]\right),$$

$$f1[0, 4] \to \frac{1}{80\,640}\left(-155 - 420\,a2^2 + 1568\,a2^3 + 336\,a2^4 - 16\,800\,f2[1, 0] + 53\,760\,a2\,f2[1, 0] - 120\,960\,f2[1, 1] + 161\,280\,a2\,f2[1, 1] - 241\,920\,f2[1, 2] - 322\,560\,f2[1, 3]\right),$$

$$f2[0, 4] \to \frac{-3 + 168\,a2 - 280\,a2^2 + 224\,a2^3 - 112\,a2^4}{40\,320},$$

$$f1[3, 0] \to \frac{1}{240}\,a2\left(-1 + a2^3\right),$$

$$f1[2, 1] \to \frac{-5\,a2 - 15\,a2^2 + 9\,a2^3 + 12\,a2^4 - 180\,f2[1, 0] + 360\,a2\,f2[1, 0] - 540\,f2[2, 0] - 720\,f2[3, 0]}{2160},$$

$$f1[1, 2] \to \frac{1}{34\,560}\left(5 - 120\,a2 - 348\,a2^2 + 192\,a2^3 + 288\,a2^4 - 7200\,f2[1, 0] + 11\,520\,a2\,f2[1, 0] - 17\,280\,f2[1, 1] - 8640\,f2[2, 0] - 34\,560\,f2[2, 1]\right),$$

$$f1[0, 3] \to \frac{-4 - 16\,a2 - 9\,a2^2 - 12\,a2^3 + 48\,a2^4 - 1080\,f2[1, 0] + 1440\,a2\,f2[1, 0] - 4320\,f2[1, 1] - 8640\,f2[1, 2]}{2880},$$

$$f2[0, 3] \to \frac{-7 + 8\,a2 + 24\,a2^2 - 32\,a2^3 + 16\,a2^4}{3840},$$

$$f1[2, 0] \to \frac{1}{360}\left(1 - 5\,a2^2\right),$$

$$f1[1, 1] \to \frac{11 - 50\,a2^2 - 720\,f2[1, 0] - 1440\,f2[2, 0]}{2880},$$

$$f1[0, 2] \to \frac{9 - 20\,a2 - 10\,a2^2 - 720\,f2[1, 0] - 2880\,f2[1, 1]}{1440},$$

$$f2[0, 2] \to \frac{1 - 20\,a2 + 20\,a2^2}{1440},$$

$$f1[0, 1] \to \frac{1}{96}\left(1 + 4\,a2 - 8\,a2^2 - 96\,f2[1, 0]\right),$$

$$f1[1, 0] \to \frac{1}{24}\left(a2 - a2^2\right),$$

$$f2[0, 1] \to \frac{1}{96}\left(1 - 4\,a2^2\right),$$

$$f1[0,\,0] \to \frac{1}{12}\,(-1+3\,a2),\quad f2[0,\,0] \to \frac{1}{24}\,(-1+6\,a2),\quad a1 \to \frac{1}{2}\,(-1+2\,a2)\Big\}$$

PH[f1] /. sol

$$\begin{aligned}
&PH\Big[\frac{1}{12}\,(-1+3\,a2)+\Big(\frac{1}{24}\left(a2-a2^2\right)x[1]+\frac{1}{96}\left(1+4\,a2-8\,a2^2-96\,f2[1,\,0]\right)x[2]\Big)\,z+\\
&\Big(\frac{1}{720}\left(1-5\,a2^2\right)x[1]^2+\frac{\left(11-50\,a2^2-720\,f2[1,\,0]-1440\,f2[2,\,0]\right)x[1]\,x[2]}{2880}+\\
&\quad\frac{\left(9-20\,a2-10\,a2^2-720\,f2[1,\,0]-2880\,f2[1,\,1]\right)x[2]^2}{2880}\Big)\,z^2+\\
&\Big(\frac{a2\left(-1+a2^3\right)x[1]^3}{1440}+\frac{1}{4320}\left(-5\,a2-15\,a2^2+9\,a2^3+12\,a2^4-180\,f2[1,\,0]+\right.\\
&\quad\left.360\,a2\,f2[1,\,0]-540\,f2[2,\,0]-720\,f2[3,\,0]\right)x[1]^2\,x[2]+\frac{1}{69\,120}\\
&\quad\left(5-120\,a2-348\,a2^2+192\,a2^3+288\,a2^4-7200\,f2[1,\,0]+11\,520\,a2\,f2[1,\,0]-17\,280\,f2[1,\,1]-\right.\\
&\quad\left.8640\,f2[2,\,0]-34\,560\,f2[2,\,1]\right)x[1]\,x[2]^2+\frac{1}{17\,280}\left(-4-16\,a2-9\,a2^2-12\,a2^3+\right.\\
&\quad\left.48\,a2^4-1080\,f2[1,\,0]+1440\,a2\,f2[1,\,0]-4320\,f2[1,\,1]-8640\,f2[1,\,2]\right)x[2]^3\Big)\,z^3+\\
&\Big(\frac{\left(-2+7\,a2^2+7\,a2^4\right)x[1]^4}{60\,480}+\frac{1}{241\,920}\left(-27-14\,a2-42\,a2^2+196\,a2^3+126\,a2^4+3360\,a2\,f2[1,\,0]-\right.\\
&\quad\left.5040\,f2[2,\,0]+10\,080\,a2\,f2[2,\,0]-10\,080\,f2[3,\,0]-10\,080\,f2[4,\,0]\right)x[1]^3\,x[2]+\\
&\quad\frac{1}{2\,903\,040}\left(-425-840\,a2-2352\,a2^2+6720\,a2^3+2688\,a2^4-30\,240\,f2[1,\,0]+\right.\\
&\quad\left.161\,280\,a2\,f2[1,\,0]-120\,960\,f2[1,\,1]+241\,920\,a2\,f2[1,\,1]-151\,200\,f2[2,\,0]+\right.\\
&\quad\left.241\,920\,a2\,f2[2,\,0]-362\,880\,f2[2,\,1]-120\,960\,f2[3,\,0]-483\,840\,f2[3,\,1]\right)x[1]^2\,x[2]^2+\\
&\quad\frac{1}{5\,806\,080}\left(-809-2352\,a2-2604\,a2^2+12\,096\,a2^3+4704\,a2^4-110\,880\,f2[1,\,0]+\right.\\
&\quad\left.403\,200\,a2\,f2[1,\,0]-604\,800\,f2[1,\,1]+967\,680\,a2\,f2[1,\,1]-725\,760\,f2[1,\,2]-\right.\\
&\quad\left.181\,440\,f2[2,\,0]+241\,920\,a2\,f2[2,\,0]-725\,760\,f2[2,\,1]-1\,451\,520\,f2[2,\,2]\right)x[1]\,x[2]^3+
\end{aligned}$$

$$\frac{1}{1\,935\,360}\left(-155-420\,a2^2+1568\,a2^3+336\,a2^4-16\,800\,f2[1,0]+53\,760\,a2\,f2[1,0]-120\,960\,f2[1,1]+161\,280\,a2\,f2[1,1]-241\,920\,f2[1,2]-322\,560\,f2[1,3]\right)x[2]^4\Bigg)z^4+$$

$$\Bigg(\frac{\left(a2-a2^6\right)x[1]^5}{60\,480}+\frac{1}{725\,760}\left(21\,a2+42\,a2^2+56\,a2^3+84\,a2^4-90\,a2^5-72\,a2^6+504\,f2[1,0]-2016\,a2^3\,f2[1,0]+5040\,a2\,f2[2,0]-5040\,f2[3,0]+10\,080\,a2\,f2[3,0]-7560\,f2[4,0]-6048\,f2[5,0]\right)x[1]^4\,x[2]+\frac{1}{2\,903\,040}\left(-7+14\,a2+189\,a2^2+1680\,a2^3+996\,a2^4-1464\,a2^5-720\,a2^6+3360\,f2[1,0]+30\,240\,a2\,f2[1,0]-18\,144\,a2^2\,f2[1,0]-32\,256\,a2^3\,f2[1,0]-120\,960\,f2[1,0]^2+40\,320\,a2\,f2[1,1]-15\,120\,f2[2,0]+80\,640\,a2\,f2[2,0]-60\,480\,f2[2,1]+120\,960\,a2\,f2[2,1]-50\,400\,f2[3,0]+80\,640\,a2\,f2[3,0]-120\,960\,f2[3,1]-30\,240\,f2[4,0]-120\,960\,f2[4,1]\right)x[1]^3\,x[2]^2+\frac{1}{5\,806\,080}\left(-35-273\,a2+588\,a2^2+5476\,a2^3+2760\,a2^4-4512\,a2^5-1920\,a2^6-5040\,f2[1,0]+159\,264\,a2\,f2[1,0]-88\,704\,a2^2\,f2[1,0]-96\,768\,a2^3\,f2[1,0]-483\,840\,f2[1,0]^2-60\,480\,f2[1,1]+322\,560\,a2\,f2[1,1]-120\,960\,f2[1,2]+241\,920\,a2\,f2[1,2]-55\,440\,f2[2,0]+201\,600\,a2\,f2[2,0]-302\,400\,f2[2,1]+483\,840\,a2\,f2[2,1]-362\,880\,f2[2,2]-60\,480\,f2[3,0]+80\,640\,a2\,f2[3,0]-241\,920\,f2[3,1]-483\,840\,f2[3,2]\right)x[1]^2\,x[2]^3+\frac{1}{23\,224\,320}\left(-217-560\,a2+4036\,a2^2+12\,288\,a2^3+7152\,a2^4-11\,328\,a2^5-5760\,a2^6-49\,056\,f2[1,0]+548\,352\,a2\,f2[1,0]-274\,176\,a2^2\,f2[1,0]-258\,048\,a2^3\,f2[1,0]-967\,680\,f2[1,0]^2-443\,520\,f2[1,1]+1\,612\,800\,a2\,f2[1,1]-1\,209\,600\,f2[1,2]+1\,935\,360\,a2\,f2[1,2]-967\,680\,f2[1,3]-100\,800\,f2[2,0]+322\,560\,a2\,f2[2,0]-725\,760\,f2[2,1]+967\,680\,a2\,f2[2,1]-1\,451\,520\,f2[2,2]-1\,935\,360\,f2[2,3]\right)x[1]\,x[2]^4+\frac{1}{23\,224\,320}\left(53+44\,a2+1492\,a2^2+2992\,a2^3-48\,a2^4-960\,a2^5-2304\,a2^6-18\,144\,f2[1,0]+153\,216\,a2\,f2[1,0]-64\,512\,a2^2\,f2[1,0]-64\,512\,a2^3\,f2[1,0]-201\,600\,f2[1,1]+645\,120\,a2\,f2[1,1]-725\,760\,f2[1,2]+967\,680\,a2\,f2[1,2]-967\,680\,f2[1,3]-967\,680\,f2[1,4]\right)x[2]^5\Bigg)z^5+O[z]^6\Big]$$

```
CanonicalForm[S[Exp[Ar[1, 3] + Ar[2, 3]]] ** F == F ** S[sigma[1, 3], sigma[2, 3]] /. sol]
```

```
True
```

```
S[Exp[1 / 2 Ar[1, 1]]]
```

```
S[]
```

```
Cases[
 lhs2 = F21 ** S[sigma[1, 2]] ** S[Exp[1 / 2 Ar[1, 1]]] ** S[Exp[1 / 2 Ar[2, 2]]],
 Y[ijk__, h_] :> Y[ijk],
 Infinity
]
```

```
{Y[0, 1, 1], Y[0, 1, 2], Y[0, 2, 1], Y[0, 2, 2],
 Y[0, 1, 1], Y[0, 1, 2], Y[0, 2, 1], Y[0, 2, 2], Y[1, 2, 0], Y[1, 2, 0]}
```

```
Cases[
 rhs2 = S[Exp[Expand[1 / 2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] ** F,
 Y[ijk__, h_] :> Y[ijk],
 Infinity
]
```

```
{Y[0, 1, 1], Y[0, 1, 2], Y[0, 2, 1], Y[0, 2, 2],
 Y[0, 1, 1], Y[0, 1, 2], Y[0, 2, 1], Y[0, 2, 2], Y[1, 2, 0], Y[1, 2, 0]}
```

```
sol2 = First[PHSolve[
   {
    Coefficient[lhs, Y[1, 2, 0]] == Coefficient[rhs, Y[1, 2, 0]],
    Coefficient[Ar[1, 0] // lhs2, Y[1, 2, 0]] == Coefficient[Ar[1, 0] // rhs2, Y[1, 2, 0]]
   },
   Join[{a1, a2}, f1coeffs, f2coeffs]
  ]]
```

$$\left\{f1[5,0] \to \frac{341}{688\,128}, f1[4,1] \to \frac{1073}{6\,193\,152}, f1[3,2] \to \frac{4807}{30\,965\,760}, f1[2,3] \to \frac{4807}{30\,965\,760},\right.$$
$$f1[1,4] \to \frac{1073}{6\,193\,152}, f1[0,5] \to \frac{341}{688\,128}, f2[5,0] \to \frac{341}{688\,128}, f2[4,1] \to \frac{1073}{6\,193\,152},$$
$$f2[3,2] \to \frac{4807}{30\,965\,760}, f2[2,3] \to \frac{4807}{30\,965\,760}, f2[1,4] \to \frac{1073}{6\,193\,152}, f2[0,5] \to \frac{341}{688\,128},$$
$$f1[4,0] \to -\frac{131}{215\,040}, f1[3,1] \to -\frac{1}{2880}, f1[2,2] \to -\frac{1573}{5\,806\,080}, f1[1,3] \to -\frac{1}{2880},$$
$$f1[0,4] \to -\frac{131}{215\,040}, f2[4,0] \to \frac{131}{215\,040}, f2[3,1] \to \frac{1}{2880}, f2[2,2] \to \frac{1573}{5\,806\,080},$$
$$f2[1,3] \to \frac{1}{2880}, f2[0,4] \to \frac{131}{215\,040}, f1[3,0] \to -\frac{21}{20\,480}, f1[2,1] \to -\frac{239}{552\,960},$$
$$f1[1,2] \to -\frac{239}{552\,960}, f1[0,3] \to -\frac{21}{20\,480}, f2[3,0] \to -\frac{21}{20\,480}, f2[2,1] \to -\frac{239}{552\,960},$$
$$f2[1,2] \to -\frac{239}{552\,960}, f2[0,3] \to -\frac{21}{20\,480}, f1[2,0] \to \frac{11}{5760}, f1[1,1] \to \frac{1}{576}, f1[0,2] \to \frac{11}{5760},$$
$$f2[2,0] \to -\frac{11}{5760}, f2[1,1] \to -\frac{1}{576}, f2[0,2] \to -\frac{11}{5760}, f1[1,0] \to \frac{1}{128}, f1[0,1] \to \frac{1}{128},$$
$$\left.f2[1,0] \to \frac{1}{128}, f2[0,1] \to \frac{1}{128}, f1[0,0] \to -\frac{1}{48}, f2[0,0] \to \frac{1}{48}, a1 \to -\frac{1}{4}, a2 \to \frac{1}{4}\right\}$$

```
CanonicalForm[
 F21 ** S[sigma[1, 2]] ** S[Exp[1 / 2 Ar[1, 1]]] ** S[Exp[1 / 2 Ar[2, 2]]] ==
   S[Exp[Expand[1 / 2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] ** F /. sol2
]
```

```
True
```

```
Simplify[
 {
   Coefficient[lhs, Y[1, 2, 0]] == Coefficient[rhs, Y[1, 2, 0]],
   Coefficient[Ar[1, 0] // lhs2, Y[1, 2, 0]] == Coefficient[Ar[1, 0] // rhs2, Y[1, 2, 0]]
  } /. {a1 → -1/2 + a2}
]
```

A very large output was generated. Here is a sample of it:

$$\Big\{-\frac{1}{24}\left(\left(\left(-2+6\,a2-24\,f1[0,0]\right)x[1]+\left(-1+6\,a2-24\,f2[0,0]\right)x[2]\right)x[3]\right)z^2-\frac{1}{48}\left(\left(\left(-2+6\,a2+4\,a2^2-24\,f1[0,0]-24\,a2\,f1[0,0]-48\,f1[1,0]\right)x[1]^2+\left(-1+8\,a2^2-48\,f1[0,1]-36\,f2[0,0]-8\,a2\left(-1+3\,f1[0,0]+3\,f2[0,0]\right)-48\,f2[1,0]\right)x[1]\,x[2]+2\left(a2+2\,a2^2-12\,a2\,f2[0,0]-6\left(f2[0,0]+4\,f2[0,1]\right)\right)x[2]^2\right)x[3]\right)z^3-\frac{(\ll 1\gg)\,z^4}{1920}-\frac{((\ll 1\gg)\ll 1\gg)\ll 1\gg}{11520}+\left(\frac{1}{720}\left(x[1]+x[2]\right)\left(x[1]^2+x[1]\,x[2]+x[2]^2\right)^2x[3]-\frac{(\ll 1\gg)\,x[3]}{322560}\right)z^6+\frac{\left(\ll 13\gg+32\left(4\,x[1]^6+14\,x[1]^5\,x[2]+\ll 4\gg+4\,x[2]^6\right)\right)x[3]\,z^7}{645120}+O[z]^8==0,\ \ll 1\gg==0\Big\}$$

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