```
Table[
 Print[i, j, " ", ReducePrimitives[
   Der[Y[1, 2, 2, H[1]]]@Ar[i, j] + Der[Ar[i, j]]@Y[1, 2, 2, H[1]]
  ]], {i, 3}, {j, 3}
]
```

```
11 0

12 0

13 0

21 0

22 0

23 0

31 0

32 0

33 0
```

```
{{Null, Null, Null}, {Null, Null, Null}, {Null, Null, Null}}
```

```
{i, j} = {2, 1}
```

```
{2, 1}
```

```
Der[Y[1, 2, 2, 1]]@Ar[i, j]
```

```
Y[1, 2, 1, x[2]] + Y[1, 2, 2, x[2]]
```

```
Der[Y[1, 2, 2, 1]]
```

```
Der[Ar[2, 0] → Y[1, 2, 0, x[2]], Ar[0, 1] → Y[1, 0, 2, x[2]], Ar[0, 2] → Y[1, 0, 2, -x[2]]]
```

```
Table[
 Print[i, j, ReducePrimitives[
   Der[Y[1, 2, 3, H[1]]]@Ar[i, j] + Der[Ar[i, j]]@Y[1, 2, 3, H[1]]
  ]], {i, 4}, {j, 4}
]
```

110

120

130

140

210

220

230

240

310

320

330

340

410

420

430

440

```
{{Null, Null, Null, Null}, {Null, Null, Null, Null},
 {Null, Null, Null, Null}, {Null, Null, Null, Null}}
```

{i, j} = {1, 3}

```
{1, 3}
```

Der[Y[1, 2, 3, H[1]]]@Ar[i, j]

```
Y[1, 2, 3, AH[-x[1]]]
```

Der[Ar[i, j]]@Y[1, 2, 3, H[1]]

```
Y[1, 2, 3, AH[x[1]]]
```

Der[Y[1, 2, 3, H[1]]]

```
Der[Ar[0, 1] → Y[0, 1, 3, AH[-x[2]]], Ar[0, 2] → Y[0, 2, 3, AH[x[1]]],
 Ar[0, 3] → Y[0, 1, 3, AH[x[2]]] + Y[0, 2, 3, AH[-x[1]]], Ar[3, 0] → Y[1, 2, 0, AH[x[3]]]]
```

Der[Y[1, 2, 3, H[1]]]@Ar[1, 3]

```
Y[1, 2, 3, AH[-x[1]]]
```

CanonicalForm[S[sigma[1, 2]]]

$$S\left[Ar[0,1] \to Ar[0,1] + Y\left[0,1,2,AH\left[-\frac{e^{-x[1]}\left(-1+e^{x[1]}\right)}{x[1]}\right]\right],\right.$$
$$Ar[0,2] \to Ar[0,2] + Y\left[0,1,2,AH\left[\frac{e^{-x[1]}\left(-1+e^{x[1]}\right)}{x[1]}\right]\right],$$
$$\left.Ar[2,0] \to Ar[2,0] + Y\left[1,2,0,AH\left[\frac{-1+e^{x[1]}}{x[1]}\right]\right]\right]$$

S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, H[bch]]]]

$$S\left[Ar[0,1] \to AH\left[\frac{e^{-x[1]-x[2]}\left(e^{x[1]+x[2]}\,Ar[0,1]\,x[1] + Y\left[0,1,3,AH\left[1-e^{x[1]}\right]\right]\right)}{x[1]}\right],\right.$$
$$Ar[0,2] \to AH\left[\frac{e^{-x[2]}\left(e^{x[2]}\,Ar[0,2]\,x[2] + Y\left[0,2,3,AH\left[1-e^{x[2]}\right]\right]\right)}{x[2]}\right],$$
$$Ar[0,3] \to AH\left[\frac{1}{x[1]\,x[2]}\,e^{-x[1]-x[2]}\left(e^{x[1]+x[2]}\,Ar[0,3]\,x[1]\,x[2] + \right.\right.$$
$$\left.\left.Y\left[0,1,3,AH\left[\left(-1+e^{x[1]}\right)x[2]\right]\right] + Y\left[0,2,3,AH\left[e^{x[1]}\left(-1+e^{x[2]}\right)x[1]\right]\right]\right)\right],$$
$$Ar[3,0] \to AH\left[\frac{1}{x[1]\,x[2]}\left(Ar[3,0]\,x[1]\,x[2] + Y\left[1,2,0,AH\left[-e^{x[1]}\left(-1+e^{x[2]}\right)x[3]\right]\right] + \right.\right.$$
$$\left.\left.\left.Y\left[1,3,0,AH\left[\left(-1+e^{x[1]+x[2]}\right)x[2]\right]\right]\right)\right]\right]$$

S[sigma[1, 3], sigma[2, 3]]

$$S\left[Ar[0,1] \to Ar[0,1] + Y\left[0,1,3,AH\left[-\frac{e^{-x[1]-x[2]}\left(-1+e^{x[1]}\right)}{x[1]}\right]\right],\right.$$
$$Ar[0,2] \to Ar[0,2] + Y\left[0,2,3,AH\left[-\frac{e^{-x[2]}\left(-1+e^{x[2]}\right)}{x[2]}\right]\right],$$
$$Ar[0,3] \to Ar[0,3] + Y\left[0,1,3,AH\left[\frac{e^{-x[1]-x[2]}\left(-1+e^{x[1]}\right)}{x[1]}\right]\right] + Y\left[0,2,3,AH\left[\frac{e^{-x[2]}\left(-1+e^{x[2]}\right)}{x[2]}\right]\right],$$
$$Ar[3,0] \to Ar[3,0] + Y\left[1,2,0,AH\left[-\frac{\left(-1+e^{x[1]}\right)\left(-1+e^{x[2]}\right)x[3]}{x[1]\,x[2]}\right]\right] +$$
$$\left.Y\left[1,3,0,AH\left[\frac{e^{x[2]}\left(-1+e^{x[1]}\right)}{x[1]}\right]\right] + Y\left[2,3,0,AH\left[\frac{-1+e^{x[2]}}{x[2]}\right]\right]\right]$$

Factor[

$$\left(\frac{x[1]}{\left(-1+e^{x[1]+x[2]}\right)^2(x[1]+x[2])} - \frac{2\,e^{x[1]+x[2]}\,x[1]}{\left(-1+e^{x[1]+x[2]}\right)^2(x[1]+x[2])} + \frac{e^{2x[1]+2x[2]}\,x[1]}{\left(-1+e^{x[1]+x[2]}\right)^2(x[1]+x[2])} + \right.$$
$$\left.\frac{x[2]}{\left(-1+e^{x[1]+x[2]}\right)^2(x[1]+x[2])} - \frac{2\,e^{x[1]+x[2]}\,x[2]}{\left(-1+e^{x[1]+x[2]}\right)^2(x[1]+x[2])} + \frac{e^{2x[1]+2x[2]}\,x[2]}{\left(-1+e^{x[1]+x[2]}\right)^2(x[1]+x[2])}\right)\Bigg]$$

1