Scatter and Glow - Analytic Testing

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Tails Commute and 4T

```
Der[Ar[1, 2] + Ar[1, 3]][Ar[2, 4]]
```

```
Y[1, 2, 4, AH[-1]]
```

```
Expect[0, Der[Ar[1, 3]][Ar[1, 2]]]
```

```
0
```

```
Expect[0,
 Der[Ar[1, 2] + Ar[1, 3]][Ar[2, 3]]
]
```

```
0
```

```
Der[Ar[1, 2]][Ar[1, 3] + Ar[2, 3]]
```

```
Y[1, 2, 3, AH[-1]]
```

```
Expect[0,
 Der[Ar[1, 2]][Ar[3, 1] + Ar[3, 2]]
]
```

```
0
```

```
Expect[{0, 0},
 Der[Ar[1, 1]]@{Ar[1, 2], Ar[2, 1]}
]
```

```
{0, 0}
```

```
Expect[{0, 0},
 Der[Ar[1, 2]]@{Ar[1, 1], Ar[2, 2]}
]
```

```
{0, 0}
```

Antisymmetry of Der

```
Expect[{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 Table[
  ReducePrimitives[
   Der[Y[1, 2, 3, AH[1]]]@Ar[i, j] + Der[Ar[i, j]]@Y[1, 2, 3, AH[1]]
  ], {i, 4}, {j, 4}
 ]
]
```

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
Expect[{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 Table[
  ReducePrimitives[
   Der[Y[1, 2, 2, AH[1]]]@Ar[i, j] + Der[Ar[i, j]]@Y[1, 2, 2, AH[1]]
  ], {i, 3}, {j, 3}
 ]
]
```

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

- **The braid group on two strands is commutative:**

```
Expect[Ar[1, 2],
 Ar[1, 2] // S[sigma[1, 2]]
]
```

```
Ar[1, 2]
```

- **Reidemeister 2**

```
Expect[SnG[S[], 0],
 SnG[sigma[1, 2], sigbar[1, 2]]
]
```

```
SnG[S[], 0]
```

- **Locallity in Scale (global over local)**

```
S[sigma[3, 1], sigma[3, 2]]
```

$$S\left[\text{Ar}[0,1] \to \text{Ar}[0,1] + Y\left[0,3,1,\text{AH}\left[-\frac{-1+e^{-x[3]}}{x[3]}\right]\right],\right.$$
$$\text{Ar}[0,2] \to \text{Ar}[0,2] + Y\left[0,3,2,\text{AH}\left[-\frac{-1+e^{-x[3]}}{x[3]}\right]\right],$$
$$\text{Ar}[0,3] \to \text{Ar}[0,3] + Y\left[0,3,1,\text{AH}\left[\frac{-1+e^{-x[3]}}{x[3]}\right]\right] + Y\left[0,3,2,\text{AH}\left[\frac{-1+e^{-x[3]}}{x[3]}\right]\right],$$
$$\text{Ar}[1,0] \to \text{Ar}[1,0] + Y\left[1,3,0,\text{AH}\left[-\frac{-1+e^{x[3]}}{x[3]}\right]\right],$$
$$\left.\text{Ar}[2,0] \to \text{Ar}[2,0] + Y\left[2,3,0,\text{AH}\left[-\frac{-1+e^{x[3]}}{x[3]}\right]\right]\right]$$

```
Expect[{Ar[1, 2], Ar[2, 1]},
 {Ar[1, 2], Ar[2, 1]} // S[sigma[3, 1], sigma[3, 2]]
]
```

```
{Ar[1, 2], Ar[2, 1]}
```

■ Overcrossings Commute

```
oc1 = SnG[sigma[1, 2], sigma[1, 3]]
```

$$\text{SnG}\Big[\text{S}\Big[\text{Ar}[0,1]\to\text{Ar}[0,1]+\text{Y}\Big[0,1,2,\text{AH}\Big[\frac{-1+e^{-x[1]}}{x[1]}\Big]\Big]+\text{Y}\Big[0,1,3,\text{AH}\Big[\frac{-1+e^{-x[1]}}{x[1]}\Big]\Big],$$

$$\text{Ar}[0,2]\to\text{Ar}[0,2]+\text{Y}\Big[0,1,2,\text{AH}\Big[-\frac{-1+e^{-x[1]}}{x[1]}\Big]\Big],$$

$$\text{Ar}[0,3]\to\text{Ar}[0,3]+\text{Y}\Big[0,1,3,\text{AH}\Big[-\frac{-1+e^{-x[1]}}{x[1]}\Big]\Big],$$

$$\text{Ar}[2,0]\to\text{Ar}[2,0]+\text{Y}\Big[1,2,0,\text{AH}\Big[\frac{-1+e^{x[1]}}{x[1]}\Big]\Big],$$

$$\text{Ar}[3,0]\to\text{Ar}[3,0]+\text{Y}\Big[1,3,0,\text{AH}\Big[\frac{-1+e^{x[1]}}{x[1]}\Big]\Big]\Big],\ \text{Ar}[1,2]+\text{Ar}[1,3]\Big]$$

```
oc2 = SnG[sigma[1, 3], sigma[1, 2]];
Test[oc1 == oc2]
```

```
True
```

■ Reidemeister 3

```
r31 = SnG[sigma[1, 2], sigma[1, 3], sigma[2, 3]]
```

$$\text{SnG}\Big[\text{S}\Big[\text{Ar}[0,1]\to\text{Ar}[0,1]+\text{Y}\Big[0,1,2,\text{AH}\Big[\frac{-1+e^{-x[1]}}{x[1]}\Big]\Big]+\text{Y}\Big[0,1,3,\text{AH}\Big[-\frac{e^{-x[1]}\left(-1+e^{x[1]}\right)}{x[1]}\Big]\Big],$$

$$\text{Ar}[0,2]\to\text{Ar}[0,2]+\text{Y}\Big[0,1,2,\text{AH}\Big[-\frac{-1+e^{-x[1]}}{x[1]}\Big]\Big]+$$

$$\text{Y}\Big[0,1,3,\text{AH}\Big[\frac{\left(-1+e^{-x[1]}\right)\left(-1+e^{-x[2]}\right)}{x[1]}\Big]\Big]+\text{Y}\Big[0,2,3,\text{AH}\Big[\frac{-1+e^{-x[2]}}{x[2]}\Big]\Big],$$

$$\text{Ar}[0,3]\to\text{Ar}[0,3]+\text{Y}\Big[0,1,3,\text{AH}\Big[\frac{e^{-x[1]-x[2]}\left(-1+e^{x[1]}\right)}{x[1]}\Big]\Big]+\text{Y}\Big[0,2,3,\text{AH}\Big[-\frac{-1+e^{-x[2]}}{x[2]}\Big]\Big],$$

$$\text{Ar}[2,0]\to\text{Ar}[2,0]+\text{Y}\Big[1,2,0,\text{AH}\Big[\frac{-1+e^{x[1]}}{x[1]}\Big]\Big],$$

$$\text{Ar}[3,0]\to\text{Ar}[3,0]+\text{Y}\Big[1,2,0,\text{AH}\Big[-\frac{\left(-1+e^{x[1]}\right)\left(-1+e^{x[2]}\right)x[3]}{x[1]\,x[2]}\Big]\Big]+$$

$$\text{Y}\Big[1,3,0,\text{AH}\Big[\frac{e^{x[2]}\left(-1+e^{x[1]}\right)}{x[1]}\Big]\Big]+\text{Y}\Big[2,3,0,\text{AH}\Big[\frac{-1+e^{x[2]}}{x[2]}\Big]\Big]\Big],\ \text{Ar}[1,2]+\text{Ar}[1,3]+\text{Ar}[2,3]\Big]$$

```
r32 = SnG[sigma[2, 3], sigma[1, 3], sigma[1, 2]];
Test[CanonicalForm[r31 == r32]]
```

```
True
```

■ Commutators Commute

```
cc11 = SnG[sigma[2, 1], sigma[3, 1], sigbar[2, 1], sigbar[3, 1]]
```

$$\text{SnG}\left[\text{S}\left[\text{Ar}[0,1] \to \text{Ar}[0,1] + \text{Y}\left[0,2,1,\text{AH}\left[-\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[2]}\right]\right] + \text{Y}\left[0,3,1,\text{AH}\left[\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right],\right.\right.$$
$$\text{Ar}[0,2] \to \text{Ar}[0,2] + \text{Y}\left[0,2,1,\text{AH}\left[\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[2]}\right]\right],$$
$$\text{Ar}[0,3] \to \text{Ar}[0,3] + \text{Y}\left[0,3,1,\text{AH}\left[-\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right], \text{Ar}[1,0] \to$$
$$\text{Ar}[1,0] + \text{Y}\left[1,2,0,\text{AH}\left[\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[2]}\right]\right] + \text{Y}\left[1,3,0,\text{AH}\left[-\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right]\Big],$$
$$\left.\text{Y}\left[2,3,1,\text{AH}\left[\frac{-e^{x[2]}x[2] + e^{x[2]+x[3]}x[2] - e^{x[3]}x[3] + e^{x[2]+x[3]}x[3]}{x[2]\,x[3]}\right]\right]\right]$$

```
cc12 = SnG[sigma[4, 1], sigma[5, 1], sigbar[4, 1], sigbar[5, 1]];
Test[(cc11 ** cc12) == (cc12 ** cc11)]
```

```
True
```

```
cc21 = SnG[sigma[2, 1], sigma[3, 1], sigbar[2, 1], sigbar[3, 1]]
```

$$\text{SnG}\left[\text{S}\left[\text{Ar}[0,1] \to \text{Ar}[0,1] + \text{Y}\left[0,2,1,\text{AH}\left[-\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[2]}\right]\right] + \text{Y}\left[0,3,1,\text{AH}\left[\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right],\right.\right.$$
$$\text{Ar}[0,2] \to \text{Ar}[0,2] + \text{Y}\left[0,2,1,\text{AH}\left[\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[2]}\right]\right],$$
$$\text{Ar}[0,3] \to \text{Ar}[0,3] + \text{Y}\left[0,3,1,\text{AH}\left[-\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right], \text{Ar}[1,0] \to$$
$$\text{Ar}[1,0] + \text{Y}\left[1,2,0,\text{AH}\left[\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[2]}\right]\right] + \text{Y}\left[1,3,0,\text{AH}\left[-\frac{\left(-1+e^{x[2]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right]\Big],$$
$$\left.\text{Y}\left[2,3,1,\text{AH}\left[\frac{-e^{x[2]}x[2] + e^{x[2]+x[3]}x[2] - e^{x[3]}x[3] + e^{x[2]+x[3]}x[3]}{x[2]\,x[3]}\right]\right]\right]$$

```
cc22 = SnG[sigma[3, 1], sigma[4, 1], sigbar[3, 1], sigbar[4, 1]];
Test[(cc21 ** cc22) == (cc22 ** cc21)]
```

```
True
```

```
cc31 = SnG[sigma[1, 2], sigma[3, 1], sigbar[1, 2], sigbar[3, 1]]
```

$$\text{SnG}\left[\text{S}\left[\text{Ar}[0,\,1] \to \text{Ar}[0,\,1] + \text{Y}\left[0,\,1,\,2,\,\text{AH}\left[-\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)}{x[1]}\right]\right],\ \text{Ar}[0,\,2] \to \text{Ar}[0,\,2] + \text{Y}\left[0,\,1,\,2,\,\text{AH}\left[\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)}{x[1]}\right]\right] + \text{Y}\left[0,\,3,\,2,\,\text{AH}\left[\frac{\left(-1+e^{x[1]}\right)\left(-1+e^{-x[3]}\right)}{x[3]}\right]\right],\ \text{Ar}[0,\,3] \to \text{Ar}[0,\,3] + \text{Y}\left[0,\,3,\,2,\,\text{AH}\left[\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right],\ \text{Ar}[2,\,0] \to \text{Ar}[2,\,0] + \text{Y}\left[1,\,3,\,0,\,\text{AH}\left[\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)x[2]}{x[1]\,x[3]}\right]\right],\ \text{Y}\left[1,\,3,\,2,\,\text{AH}\left[-\frac{e^{-x[3]}\left(-e^{x[1]}\,x[1] + e^{x[1]+x[3]}\,x[1] - x[3] + e^{x[1]}\,x[3]\right)}{x[1]\,x[3]}\right]\right]\right]$$

```
cc32 = SnG[sigma[1, 4], sigma[5, 1], sigbar[1, 4], sigbar[5, 1]];
Test[(cc31 ** cc32) == (cc32 ** cc31)]
```

```
True
```

- **This last one we expect to fail:**

```
cc41 = SnG[sigma[1, 2], sigma[3, 1], sigbar[1, 2], sigbar[3, 1]]
```

$$\text{SnG}\left[\text{S}\left[\text{Ar}[0,\,1] \to \text{Ar}[0,\,1] + \text{Y}\left[0,\,1,\,2,\,\text{AH}\left[-\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)}{x[1]}\right]\right],\ \text{Ar}[0,\,2] \to \text{Ar}[0,\,2] + \text{Y}\left[0,\,1,\,2,\,\text{AH}\left[\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)}{x[1]}\right]\right] + \text{Y}\left[0,\,3,\,2,\,\text{AH}\left[\frac{\left(-1+e^{x[1]}\right)\left(-1+e^{-x[3]}\right)}{x[3]}\right]\right],\ \text{Ar}[0,\,3] \to \text{Ar}[0,\,3] + \text{Y}\left[0,\,3,\,2,\,\text{AH}\left[\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)}{x[3]}\right]\right],\ \text{Ar}[2,\,0] \to \text{Ar}[2,\,0] + \text{Y}\left[1,\,3,\,0,\,\text{AH}\left[\frac{e^{-x[3]}\left(-1+e^{x[1]}\right)\left(-1+e^{x[3]}\right)x[2]}{x[1]\,x[3]}\right]\right],\ \text{Y}\left[1,\,3,\,2,\,\text{AH}\left[-\frac{e^{-x[3]}\left(-e^{x[1]}\,x[1] + e^{x[1]+x[3]}\,x[1] - x[3] + e^{x[1]}\,x[3]\right)}{x[1]\,x[3]}\right]\right]\right]$$

```
cc42 = SnG[sigma[4, 1], sigma[5, 1], sigbar[4, 1], sigbar[5, 1]];
Expect[False,
 (cc41 ** cc42) === (cc42 ** cc41)
]
```

```
False
```

■ Commutators Commutators are Central (along strand 1)

```
(ccc = SnG[
    sigma[1, 2], sigma[3, 1], sigbar[1, 2], sigbar[3, 1],
    sigma[4, 1], sigma[5, 1], sigbar[4, 1], sigbar[5, 1],
    sigma[3, 1], sigma[1, 2], sigbar[3, 1], sigbar[1, 2],
    sigma[5, 1], sigma[4, 1], sigbar[5, 1], sigbar[4, 1]
  ]) // Last
```

$$Y\Big[1, 4, 2,$$

$$AH\Big[\frac{1}{x[1]\,x[4]}\,e^{-x[3]}\,\Big(-e^{x[1]}\,x[1]+e^{x[1]+x[3]}\,x[1]+e^{x[1]+x[4]}\,x[1]-e^{x[1]+x[3]+x[4]}\,x[1]+e^{x[1]+x[5]}\,x[1]-$$
$$e^{x[1]+x[3]+x[5]}\,x[1]-e^{x[1]+x[4]+x[5]}\,x[1]+e^{x[1]+x[3]+x[4]+x[5]}\,x[1]-x[3]+e^{x[1]}\,x[3]+$$
$$e^{x[4]}\,x[3]-e^{x[1]+x[4]}\,x[3]+e^{x[5]}\,x[3]-e^{x[1]+x[5]}\,x[3]-e^{x[4]+x[5]}\,x[3]+$$
$$e^{x[1]+x[4]+x[5]}\,x[3]-e^{x[4]}\,x[4]+e^{x[1]+x[4]}\,x[4]+e^{x[3]+x[4]}\,x[4]-e^{x[1]+x[3]+x[4]}\,x[4]+$$
$$e^{x[4]+x[5]}\,x[4]-e^{x[1]+x[4]+x[5]}\,x[4]-e^{x[3]+x[4]+x[5]}\,x[4]+e^{x[1]+x[3]+x[4]+x[5]}\,x[4]-$$
$$e^{x[5]}\,x[5]+e^{x[1]+x[5]}\,x[5]+e^{x[3]+x[5]}\,x[5]-e^{x[1]+x[3]+x[5]}\,x[5]+e^{x[4]+x[5]}\,x[5]-$$
$$e^{x[1]+x[4]+x[5]}\,x[5]-e^{x[3]+x[4]+x[5]}\,x[5]+e^{x[1]+x[3]+x[4]+x[5]}\,x[5]\Big)\Big]+$$

$$Y\Big[1, 5, 2, AH\Big[-\frac{1}{x[1]\,x[5]}\,e^{-x[3]}\,\Big(-e^{x[1]}\,x[1]+e^{x[1]+x[3]}\,x[1]+e^{x[1]+x[4]}\,x[1]-e^{x[1]+x[3]+x[4]}\,x[1]+$$
$$e^{x[1]+x[5]}\,x[1]-e^{x[1]+x[3]+x[5]}\,x[1]-e^{x[1]+x[4]+x[5]}\,x[1]+e^{x[1]+x[3]+x[4]+x[5]}\,x[1]-$$
$$x[3]+e^{x[1]}\,x[3]+e^{x[4]}\,x[3]-e^{x[1]+x[4]}\,x[3]+e^{x[5]}\,x[3]-e^{x[1]+x[5]}\,x[3]-$$
$$e^{x[4]+x[5]}\,x[3]+e^{x[1]+x[4]+x[5]}\,x[3]-e^{x[4]}\,x[4]+e^{x[1]+x[4]}\,x[4]+e^{x[3]+x[4]}\,x[4]-$$
$$e^{x[1]+x[3]+x[4]}\,x[4]+e^{x[4]+x[5]}\,x[4]-e^{x[1]+x[4]+x[5]}\,x[4]-e^{x[3]+x[4]+x[5]}\,x[4]+$$
$$e^{x[1]+x[3]+x[4]+x[5]}\,x[4]-e^{x[5]}\,x[5]+e^{x[1]+x[5]}\,x[5]+e^{x[3]+x[5]}\,x[5]-e^{x[1]+x[3]+x[5]}\,x[5]+$$
$$e^{x[4]+x[5]}\,x[5]-e^{x[1]+x[4]+x[5]}\,x[5]-e^{x[3]+x[4]+x[5]}\,x[5]+e^{x[1]+x[3]+x[4]+x[5]}\,x[5]\Big)\Big]\Big]$$

```
Test[ccc ** SnG[sigma[6, 1]] == SnG[sigma[6, 1]] ** ccc]
```

```
True
```

```
Test[ccc ** SnG[sigma[1, 6]] == SnG[sigma[1, 6]] ** ccc]
```

```
True
```

■ Scattering by Exponentials

```
S[Exp[Ar[1, 2]]]
```

$$S\Big[Ar[0, 1] \to Ar[0, 1] + Y\Big[0, 1, 2, AH\Big[-\frac{e^{-x[1]}\,\left(-1+e^{x[1]}\right)}{x[1]}\Big]\Big],$$

$$Ar[0, 2] \to Ar[0, 2] + Y\Big[0, 1, 2, AH\Big[\frac{e^{-x[1]}\,\left(-1+e^{x[1]}\right)}{x[1]}\Big]\Big],$$

$$Ar[2, 0] \to Ar[2, 0] + Y\Big[1, 2, 0, AH\Big[\frac{-1+e^{x[1]}}{x[1]}\Big]\Big]\Big]$$

```
Test[CanonicalForm[
  S[Exp[Ar[1, 2]]] == S[sigma[1, 2]]
 ]]
```

```
True
```

```
Test[CanonicalForm[
  S[Exp[-Ar[1, 2]]] == S[sigbar[1, 2]]
 ]]
```

```
True
```

■ The BCH Formula

```
S[sigma[1, 3], sigma[2, 3]] // Short
```

$$S\left[Ar[0, 1] \to Ar[0, 1] + Y\left[0, 1, 3, AH\left[-\frac{e^{\ll 1\gg}\ \ll 1\gg}{x[1]}\right]\right],\ \ll 2\gg,\ Ar[3, 0] \to \ll 1\gg\right]$$

```
S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, bc]]] // Short
```

$$S\left[Ar[0, 1] \to Ar[0, 1] + Y\left[0, 1, 3, AH\left[\frac{\ll 1\gg}{x[1] + \ll 1\gg}\right]\right],\ \ll 1\gg,\ \ll 1\gg,\ Ar[3, 0] \to \ll 1\gg\right]$$

```
bch = bc /. First[HSolve[
    Coefficient[
      Ar[0, 1] // S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, bc]]],
      Y[0, 1, 3]
     ] == Coefficient[
      Ar[0, 1] // S[sigma[1, 3], sigma[2, 3]],
      Y[0, 1, 3]
     ],
    bc
   ]]
```

$$\frac{-e^{x[1]}\,x[1] + e^{x[1]+x[2]}\,x[1] + x[2] - e^{x[1]}\,x[2]}{\left(-1 + e^{x[1]+x[2]}\right)\,x[1]\,x[2]}$$

```
Test[CanonicalForm[
  S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, AH[bch]]]] == S[sigma[1, 3], sigma[2, 3]]
 ]]
```

```
True
```

■ Compare with Kurlin

$$\text{Test}\left[\text{Simplify}\left[(\text{bch} /. \{x[1] \to x,\ x[2] \to y\}) == \frac{1}{y}\left(1 - \frac{e^x - 1}{x}\,\frac{x + y}{e^{x+y} - 1}\right)\right]\right]$$

```
True
```

Testing Code

```
SetAttributes[{Test, Expect}, {HoldAll}];
Test[expr_] := If[TrueQ[Check[expr, False]], True,
   If[Head[$FailLog] =!= List, $FailLog = {}];
   AppendTo[$FailLog,
    "On " <> ToString[Date[]] <> " failed in " <> ToString[HoldForm[expr]]];
   Print[Last[$FailLog]]
  ];
Expect[val_, expr_] := If[TrueQ[Test[val == expr]], val];

SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\ScatterAndGlow"]
<< ScatterAndGlow.m
```

```
C:\drorbn\AcademicPensieve\Projects\ScatterAndGlow
```

Test Test

```
Test[0 == 1]
```

```
On {2009, 1, 14, 11, 51, 8.0802000} failed in 0 == 1
```

Failed Tests

```
$FailLog
```

```
{On {2009, 1, 14, 11, 45, 20.6852000} failed in 0 == 1,
 On {2009, 1, 14, 11, 45, 25.7602000} failed in {{0, 0, 0, 0}, {0, 0, 0, 0}, {0,
   0, 0, 0}, {0, 0, 0, 0}} == Table[ReducePrimitives[Der[ToPH[3, Y[1, 2, 3,
   AH[1]]]][Ar[i, j]] + Der[Ar[i, j]][Y[1, 2, 3, ToPH[3, PH[1]]]]], {i, 4}, {j, 4}],
 On {2009, 1, 14, 11, 45, 27.6082000} failed in 0 == 1,
 On {2009, 1, 14, 11, 51, 0.9332000} failed in 0 == 1,
 On {2009, 1, 14, 11, 51, 8.0802000} failed in 0 == 1}
```