# Scatter and Glow

## Project goals

- Verify R2 and R3.
- Recover the Alexander polynomial of all knots.
- Recover the multi-variable Alexander polynomial of all links.
- The scatter and glow of an arbitrary exponential.
- Find an explicit BCH formula.
- Solve R4 for F at the scatter level.
- Verify the pentagon.
- Solve for F at the glow level.
- Check the Hexagons.
- Solve the \theta-R-F equation.
- Verify the Hexagons.
- Recover the Lieberum formulas.

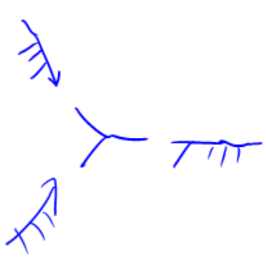
## Conventions

Ar[i, j] is an arrow going from i to j.
Y[i,j,k] := [Ar[i,k], Ar[j,k]] = Ar[i,k]Ar[j,k] - Ar[j,k]Ar[i,k] =: Ad[Ar[i,k]][Ar[j,k]] = -[Ar[i,j], Ar[j,k]]
x[l]Y[i,j,k] := [Ar[l,k], Y[i,j,k]]

hY[i,j,k] → Y[i,j,k,h]

scatter[s][prims] :=
prims /. {Ar[..] →
Y[i,j,k] → Y[Ar[i]/.s, Ar[j]/.s, Ar[k]/.s]

/. Distribute & glue



## Program

```
S[sigma[i_, j_]] := S[{
    Ar[0, j] → Ar[0, j] - (Exp[-x[i]] - 1) / x[i] * Y[0, i, j],
    Ar[0, i] → Ar[0, i] + (Exp[-x[i]] - 1) / x[i] * Y[0, i, j],
    Ar[j, 0] → Ar[j, 0] - (Exp[-x[i]] - 1) / x[i] * Y[i, j, 0]
   }];

ReducePrimitives[prims_] := prims;

SlideLeg[i_, S[SRules_List]][prims_] := ReducePrimitives[prims /. {
     Ar[i, j_] ⧴ (Ar[i, 0] /. SRules /. 0 → j),
     Ar[j_, i] ⧴ (Ar[0, i] /. SRules /. 0 → j),
     Y[i, j_, i] ⧴ ImClueless,
     Y[j_, i, i] ⧴ ImClueless,
     Y[i, i, j_] ⧴ Trouble,
     Y[i, j_, k_] ⧴ (Ar[i, 0] /. SRules /. {
         Ar[i, 0] ⧴ Y[i, j, k],
         Y[l_, m_, 0] ⧴ -x[j] Y[l, m, k]
        }),
     Y[j_, i, k_] ⧴ (Ar[i, 0] /. SRules /. {
         Ar[i, 0] ⧴ Y[j, i, k],
         Y[l_, m_, 0] ⧴ x[j] Y[l, m, k]
        }),
     Y[j_, k_, i] ⧴ (Ar[0, i] /. SRules /. {
         Ar[0, i] ⧴ Y[j, k, i],
         Y[0, l_, m_] ⧴ -x[l] Y[j, k, m],
         Y[l_, 0, m_] ⧴ x[l] Y[j, k, m]
        })
    }];
Scatter[s_S][prims_] := prims /. {
   Ar[i_, j_] ⧴ (Ar[i, j] // SlideLeg[i, s] // SlideLeg[j, s]),
   Y[i_, j_, k_] ⧴ (Y[i, j, k] // SlideLeg[i, s] // SlideLeg[j, s] // SlideLeg[k, s])
  }

Scatter[S[sigma[1, 2]]][Ar[1, 2]]
```

$$\mathrm{Ar}[1,2]-\frac{\left(-1+e^{-x[1]}\right)Y[1,1,2]}{x[1]}$$

```
{Ar[1, 4], Ar[2, 4], Ar[3, 4]} // Scatter[S[sigma[1, 2]]] // Scatter[S[sigma[1, 3]]] //
  Scatter[S[sigma[2, 3]]] // Expand
```

$$\Big\{\mathrm{Ar}[1,4],\ \mathrm{Ar}[2,4]+\frac{Y[1,2,4]}{x[1]}-\frac{e^{-x[1]}\,Y[1,2,4]}{x[1]},\ \mathrm{Ar}[3,4]+\frac{Y[1,3,4]}{x[1]}-\frac{e^{-x[1]}\,Y[1,3,4]}{x[1]}+\frac{2\,Y[2,3,4]}{x[2]}-\frac{e^{-x[1]}\,Y[2,3,4]}{x[2]}+\frac{e^{-x[1]-x[2]}\,Y[2,3,4]}{x[2]}-\frac{2\,e^{-x[2]}\,Y[2,3,4]}{x[2]}\Big\}$$

```
{Ar[1, 4], Ar[2, 4], Ar[3, 4]} // Scatter[S[sigma[2, 3]]] // Scatter[S[sigma[1, 3]]] //
  Scatter[S[sigma[1, 2]]] // Expand
```

$$\Big\{Ar[1,4],\ Ar[2,4] + \frac{Y[1,2,4]}{x[1]} - \frac{e^{-x[1]}\,Y[1,2,4]}{x[1]},$$
$$Ar[3,4] - \frac{x[3]\,Y[1,2,4]}{x[1]\,x[2]} + \frac{e^{-x[1]}\,x[3]\,Y[1,2,4]}{x[1]\,x[2]} - \frac{e^{-x[1]-x[2]}\,x[3]\,Y[1,2,4]}{x[1]\,x[2]} +$$
$$\frac{e^{-x[2]}\,x[3]\,Y[1,2,4]}{x[1]\,x[2]} + \frac{2\,Y[1,3,4]}{x[1]} - \frac{2\,e^{-x[1]}\,Y[1,3,4]}{x[1]} +$$
$$\frac{e^{-x[1]-x[2]}\,Y[1,3,4]}{x[1]} - \frac{e^{-x[2]}\,Y[1,3,4]}{x[1]} + \frac{Y[2,3,4]}{x[2]} - \frac{e^{-x[2]}\,Y[2,3,4]}{x[2]}\Big\}$$

```
Ar[3, 4] // Scatter[S[sigma[2, 3]]]
```

$$Ar[3,4] - \frac{\left(-1 + e^{-x[2]}\right)Y[2,3,4]}{x[2]}$$

$$\left(Ar[3,4] - \frac{\left(-1 + e^{-x[2]}\right)Y[2,3,4]}{x[2]}\right) \texttt{ // Scatter[S[sigma[1, 3]]] // Expand}$$

$$Ar[3,4] + \frac{2\,Y[1,3,4]}{x[1]} - \frac{2\,e^{-x[1]}\,Y[1,3,4]}{x[1]} +$$
$$\frac{e^{-x[1]-x[2]}\,Y[1,3,4]}{x[1]} - \frac{e^{-x[2]}\,Y[1,3,4]}{x[1]} + \frac{Y[2,3,4]}{x[2]} - \frac{e^{-x[2]}\,Y[2,3,4]}{x[2]}$$

```
Ar[3, 4] // Scatter[S[sigma[1, 3]]] // Expand
```

$$Ar[3,4] + \frac{Y[1,3,4]}{x[1]} - \frac{e^{-x[1]}\,Y[1,3,4]}{x[1]}$$

```
Y[2, 3, 4] // Scatter[S[sigma[1, 3]]] // Expand
```

$$\frac{x[2]\,Y[1,3,4]}{x[1]} - \frac{e^{-x[1]}\,x[2]\,Y[1,3,4]}{x[1]} + Y[2,3,4]$$

## Mathematica Experiments

```
{1, 2, 3} /. i_Integer ⧴ i^2
```

{1, 4, 9}

```
{1, 2, 3} /. 2 → i^2
```

$\{1, i^2, 3\}$

```
f[x]
```

f[x]

```
x // f
```

f[x]

```
f@x
f[x]
```