

Pensieve header: km at \$k=1 with up to 15 inputs.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
<< KnotTheory` ;
<< "../Profile/Profile.m";
(*<<"Archive/Engine-Speedy-190317.m";*) (*174.874 Seconds *)
<< "Engine-Speedy.m"; (*169.953 Seconds*)
<< "Objects.m";
BeginProfile[];
```

Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: June 2018. Original version: July 1994.

```
In[ ]:= $k = 1; Clear[km];
```

```
In[ ]:= km[1] =  $\mathbb{E}_{\{1\} \rightarrow \{1\}}$  [a1 α1 + t τ1, x1 ξ1 + y1 η1, 1];
km[n_Integer] /; n > 1 := km[n] = km[n - 1] // km1, n → 1
```

In[*]:= km[4]

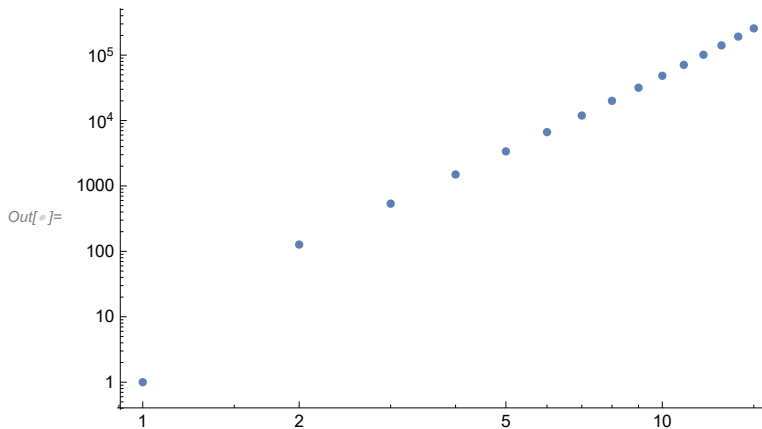
$$\begin{aligned}
\text{Out[*]} = & \mathbb{E}_{\{1,2,3,4\} \rightarrow \{1\}} \left[\mathbf{a}_1 \alpha_1 + \mathbf{a}_1 \alpha_2 + \mathbf{a}_1 \alpha_3 + \mathbf{a}_1 \alpha_4 + \mathbf{t} \tau_1 + \mathbf{t} \tau_2 + \mathbf{t} \tau_3 + \mathbf{t} \tau_4, \right. \\
& \mathbf{y}_1 \eta_1 + \frac{\mathbf{y}_1 \eta_2}{\mathcal{A}_1} + \frac{\mathbf{y}_1 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{y}_1 \eta_4}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \xi_1}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \frac{(1-T) \eta_2 \xi_1}{\hbar} + \frac{(1-T) \eta_3 \xi_1}{\hbar \mathcal{A}_2} + \\
& \frac{(1-T) \eta_4 \xi_1}{\hbar \mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \xi_2}{\mathcal{A}_3 \mathcal{A}_4} + \frac{(1-T) \eta_3 \xi_2}{\hbar} + \frac{(1-T) \eta_4 \xi_2}{\hbar \mathcal{A}_3} + \frac{\mathbf{x}_1 \xi_3}{\mathcal{A}_4} + \frac{(1-T) \eta_4 \xi_3}{\hbar} + \mathbf{x}_1 \xi_4, \\
& 1 + \left(2 T \mathbf{a}_1 \eta_2 \xi_1 + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_2 \xi_1}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_2^2 \xi_1}{2 \mathcal{A}_1} + \frac{2 T \mathbf{a}_1 \eta_3 \xi_1}{\mathcal{A}_2} + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} + \right. \\
& \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_2 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_3^2 \xi_1}{2 \mathcal{A}_1 \mathcal{A}_2^2} + \frac{2 T \mathbf{a}_1 \eta_4 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_4 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3^2 \mathcal{A}_4} + \\
& \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_2 \eta_4 \xi_1}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_3 \eta_4 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_4^2 \xi_1}{2 \mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3^2} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_2 \xi_1^2}{2 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \\
& \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_2^2 \xi_1^2}{4 \hbar} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_3 \xi_1^2}{2 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_2 \eta_3 \xi_1^2}{2 \hbar \mathcal{A}_2} + \\
& \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_3^2 \xi_1^2}{4 \hbar \mathcal{A}_2^2} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_4 \xi_1^2}{2 \mathcal{A}_2^2 \mathcal{A}_3^2 \mathcal{A}_4} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_2 \eta_4 \xi_1^2}{2 \hbar \mathcal{A}_2 \mathcal{A}_3} + \\
& \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_3 \eta_4 \xi_1^2}{2 \hbar \mathcal{A}_2^2 \mathcal{A}_3} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_4^2 \xi_1^2}{4 \hbar \mathcal{A}_2^2 \mathcal{A}_3^2} + 2 T \mathbf{a}_1 \eta_3 \xi_2 + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_3 \xi_2}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \\
& \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_3^2 \xi_2}{2 \mathcal{A}_1 \mathcal{A}_2} + \frac{2 T \mathbf{a}_1 \eta_4 \xi_2}{\mathcal{A}_3} + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_4 \xi_2}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3^2 \mathcal{A}_4} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_3 \eta_4 \xi_2}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_4^2 \xi_2}{2 \mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3^2} + \\
& \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_3 \xi_1 \xi_2}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_3^2 \xi_1 \xi_2}{2 \hbar \mathcal{A}_2} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_4 \xi_1 \xi_2}{\mathcal{A}_2 \mathcal{A}_3^2 \mathcal{A}_4} + \\
& \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_3 \eta_4 \xi_1 \xi_2}{\hbar \mathcal{A}_2 \mathcal{A}_3} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_4^2 \xi_1 \xi_2}{2 \hbar \mathcal{A}_2 \mathcal{A}_3^2} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_3 \xi_2^2}{2 \mathcal{A}_3 \mathcal{A}_4} + \\
& \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_3^2 \xi_2^2}{4 \hbar} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_4 \xi_2^2}{2 \mathcal{A}_3^2 \mathcal{A}_4} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_3 \eta_4 \xi_2^2}{2 \hbar \mathcal{A}_3} + \\
& \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_4^2 \xi_2^2}{4 \hbar \mathcal{A}_3^2} + 2 T \mathbf{a}_1 \eta_4 \xi_3 + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_4 \xi_3}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \frac{(\gamma - 3 T \gamma) \mathbf{y}_1 \eta_4^2 \xi_3}{2 \mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} + \\
& \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_4 \xi_1 \xi_3}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_4^2 \xi_1 \xi_3}{2 \hbar \mathcal{A}_2 \mathcal{A}_3} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_4 \xi_2 \xi_3}{\mathcal{A}_3 \mathcal{A}_4} + \\
& \left. \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_4^2 \xi_2 \xi_3}{2 \hbar \mathcal{A}_3} + \frac{(\gamma - 3 T \gamma) \mathbf{x}_1 \eta_4 \xi_3^2}{2 \mathcal{A}_4} + \frac{(\gamma - 4 T \gamma + 3 T^2 \gamma) \eta_4^2 \xi_3^2}{4 \hbar} \right) \epsilon + \mathbf{O}[\epsilon]^2
\end{aligned}$$

```
In[ ]:= Table[LeafCount /@ List@@km[n], {n, 15}] // MatrixForm
```

Out[]:= MatrixForm=

13	15	1
25	54	127
37	123	536
49	227	1497
61	371	3374
73	560	6649
85	799	11937
97	1093	20001
109	1447	31767
121	1866	48339
133	2355	71014
145	2919	101297
157	3563	140916
169	4292	191837
181	5111	256279

```
In[ ]:= ListLogLogPlot[Table[LeafCount[km[n][[3]]], {n, 15}]]
```



```
In[ ]:= N@Table[Log@LeafCount[km[n][[3]]], {n, 2, 15}]
           Log[n]
```

Out[]:= {6.98868, 5.72007, 5.27393, 5.04763, 4.91261, 4.82417,
4.76259, 4.71785, 4.6843, 4.65852, 4.63833, 4.62228, 4.60937, 4.59889}

```
In[ ]:= PrintProfile[]
```

Out[]:= ProfileRoot is root. Profiled time: 169.953
 (14) 0.345/ 168.530 above B
 (1) 0/ 1.422 above Boot
 Together: called 74368 times, time in 69.844/75.404
 (74368) 69.844/ 75.404 under CCF
 (74368) 5.560/ 5.560 above Exp
 CCF: called 74368 times, time in 46.78/122.184
 (74368) 46.780/ 122.180 under CF
 (74368) 69.844/ 75.404 above Together
 CF: called 2493 times, time in 33.738/155.922
 (28) 0/ 0.016 under EEQ
 (11) 0/ 0 under Boot
 (120) 5.563/ 23.983 under LZip
 (1332) 20.626/ 95.274 under QZip

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( 1002) 7.549/ 36.649 under QZip4
( 74368) 46.780/ 122.180 above CCF
QZip: called 30 times, time in 8.297/143.39
( 30) 8.297/ 143.390 under B
( 1332) 20.626/ 95.274 above CF
( 30) 0.390/ 38.366 above QZip4
( 30) 0.609/ 1.453 above Zip
Exp: called 74368 times, time in 5.56/5.56
( 74368) 5.560/ 5.560 under Together
Zip: called 423 times, time in 3.345/7.879
( 30) 0.359/ 1.438 under LZip
( 30) 0.609/ 1.453 under QZip
( 60) 0.548/ 1.327 under QZip4
( 303) 1.829/ 3.661 under Zip
( 423) 0.873/ 0.873 above Collect
( 303) 1.829/ 3.661 above Zip
Collect: called 423 times, time in 0.873/0.873
( 423) 0.873/ 0.873 under Zip
LZip: called 30 times, time in 0.718/26.202
( 30) 0.718/ 26.202 under B
( 28) 0.047/ 0.063 above EEQ
( 120) 5.563/ 23.983 above CF
( 30) 0.359/ 1.438 above Zip
QZip4: called 30 times, time in 0.39/38.366
( 30) 0.390/ 38.366 under QZip
( 1002) 7.549/ 36.649 above CF
( 60) 0.548/ 1.327 above Zip
B: called 30 times, time in 0.345/169.937
( 16) 0/ 1.406 under Boot
( 14) 0.345/ 168.530 under ProfileRoot
( 30) 0.718/ 26.202 above LZip
( 30) 8.297/ 143.390 above QZip
EEQ: called 28 times, time in 0.047/0.063
( 28) 0.047/ 0.063 under LZip
( 28) 0/ 0.016 above CF
Boot: called 16 times, time in 0.016/3.406
( 15) 0.016/ 1.984 under Boot
( 1) 0/ 1.422 under ProfileRoot
( 16) 0/ 1.406 above B
( 15) 0.016/ 1.984 above Boot
( 11) 0/ 0 above CF

```