

Pensieve header: Verifying the $\$PR^4\$$ relation.

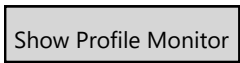
Startup

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
<< "../Profile/Profile.m";
<< "Engine-Speedy.m";
<< "Objects.m";
HL[ε_] := Style[ε, Background → If[TrueQ@ε, Green, Red]];
BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]
```

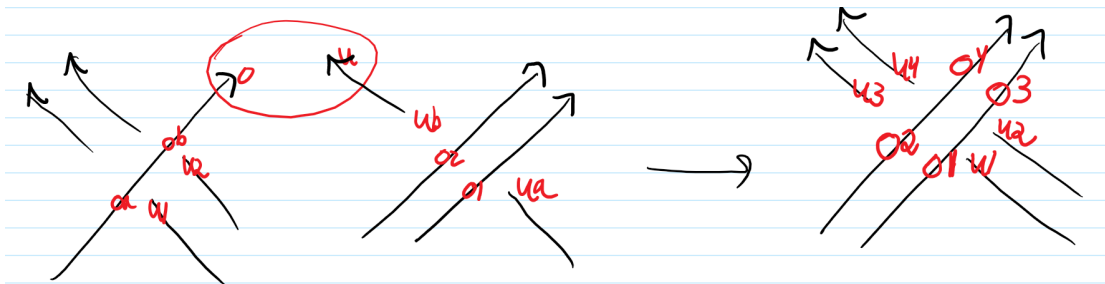
This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: June 2018. Original version: July 1994.

» Warning: On Sep 4 2019 I swapped the operations ϵ and η . Some incompatibilities may arise in older notebooks.



```
In[ ]:= $k = 1; (*ħ=γ=1;*)
```



$$\text{In[]:= rhs} = (R_{o1,u1} R_{o3,u2} R_{o2,u3} R_{o4,u4}) // (bm_{o1,o3 \to o1} bm_{o2,o4 \to o2} am_{u1,u3 \to u1} am_{u2,u4 \to u2})$$

$$\text{Out[]:= } \mathbb{E}_{\{ \} \to \{o1,o2,u1,u2\}} \left[\hbar a_{u1} b_{o1} + \hbar a_{u2} b_{o1} + \hbar a_{u1} b_{o2} + \hbar a_{u2} b_{o2}, \hbar B_{o2} x_{u1} y_{o1} + \hbar B_{o2} x_{u2} y_{o1} + \hbar x_{u1} y_{o2} + \hbar x_{u2} y_{o2}, \right. \\ \left. 1 + \left(-\hbar^2 a_{u1} B_{o2} x_{u2} y_{o1} - \frac{1}{4} \gamma \hbar^3 B_{o2}^2 x_{u1}^2 y_{o1}^2 - \frac{1}{4} \gamma \hbar^3 B_{o2}^2 x_{u2}^2 y_{o1}^2 - \right. \right. \\ \left. \left. \hbar^2 a_{u1} x_{u2} y_{o2} + \gamma \hbar^3 B_{o2} x_{u1} x_{u2} y_{o1} y_{o2} - \frac{1}{4} \gamma \hbar^3 x_{u1}^2 y_{o2}^2 - \frac{1}{4} \gamma \hbar^3 x_{u2}^2 y_{o2}^2 \right) \epsilon + O[\epsilon]^2 \right]$$

$$\text{In[]:= lhs} = (R_{oa,u1} R_{ob,u2} // bm_{oa,ob \to o}) (R_{o1,ua} R_{o2,ub} // am_{ua,ub \to u}) // P_{o,u};$$

$$\text{In[]:= lhs} \equiv \text{rhs}$$

Out[]:= True

(The above was tested to $\$k=4$).

In[]:= **am**_{1,2→3}

$$\text{Out[]} = \mathbb{E}_{\{1,2\} \rightarrow \{3\}} \left[\mathbf{a}_3 \alpha_1 + \mathbf{a}_3 \alpha_2, \frac{x_3 \xi_1}{\mathcal{A}_2} + x_3 \xi_2, \mathbf{1} \right]$$

In[]:= **bm**_{1,2→3}

$$\text{Out[]} = \mathbb{E}_{\{1,2\} \rightarrow \{3\}} \left[\mathbf{b}_3 \beta_1 + \mathbf{b}_3 \beta_2, \mathbf{y}_3 \eta_1 + \mathbf{y}_3 \eta_2, \mathbf{1} - \mathbf{y}_3 \beta_1 \eta_2 \in + \mathbf{0} [\in]^2 \right]$$

In[]:= **? *Values**

System`

	DefaultValues	DynamicModule` eValues	NValues	SingularValues	SynthesizeMisc` singValues	Values
Out[]:=	DownValues	FormatValues	OwnValues	SubValues	UpValues	

In[]:= **Cases** [DownValues [Subscript], (a_ -> _) /; ! FreeQ[a, R]] // Column

$$\text{HoldPattern} [R_{\{i_, j_\}, 1}] \Rightarrow \mathbb{E}_{\{\} \rightarrow \{i, j\}} \left[\hbar a_j b_i, \hbar x_j y_i, \mathbf{1} - \frac{1}{4} (\gamma \hbar^3 x_j^2 y_i^2) \in + \mathbf{0} [\in]^2 \right]$$

HoldPattern [R_{ii\$1111, jj\$1111}, \$k_Integer] ->

$$\text{PP}_{\text{Boot}} \left[\text{Block} \left[\{i, j, k\}, R_{\{i_, j_\}, \$k} = \mathbb{E}_{\{\} \rightarrow \{i, j\}} \left[\hbar a_j b_i + \sum_{k=1}^{k+1} \frac{(1 - e^{\gamma \in h})^k (\hbar y_i x_j)^k}{k (1 - e^{k \gamma \in h})} \right]; \right. \right. \\ \left. \left. R_{\{ii$1111, jj$1111\}, \$k} \right] \right]$$

HoldPattern [R_{i_, j_}] -> R_{i, j}, \$k

Out[]:= HoldPattern [R_{sis\$1111}] -> R_{sis\$1111}

HoldPattern [R_{ii\$1112, jj\$1112}, \$k_Integer] ->

$$\text{PP}_{\text{Boot}} \left[\text{Block} \left[\{i, j, k\}, \bar{R}_{\{i_, j_\}, \$k} = \text{CF} \left[\mathbb{E}_{\{\} \rightarrow \{i, j\}} \left[-\hbar a_j b_i, -\frac{\hbar x_j y_i}{B_i}, \mathbf{1} + \text{If} [\$k == 0, 0, \right. \right. \right. \\ \left. \left. \left. \bar{R}_{\{i, j\}, \$k-1} [3] - (\text{bm}_{i, 3 \rightarrow i} \text{am}_{j, 4 \rightarrow j}) \left[(\text{bm}_{i, 1 \rightarrow i} \text{am}_{j, 2 \rightarrow j}) \left[\bar{R}_{\{i, j\}, 0} R_{1, 2} \bar{R}_{\{3, 4\}, \$k-1} [3] \right] \right] \right] \right] \right]; \right. \right. \\ \left. \left. \bar{R}_{\{ii$1112, jj$1112\}, \$k} \right] \right]$$

HoldPattern [R_{i_, j_}] -> R_{i, j}, \$k

HoldPattern [R_{sis\$1112}] -> R_{sis\$1112}

In[]:= **Cases** [DownValues [Subscript], (_ [a_] -> _) /; a === R] // Column

$$\text{HoldPattern} [R_{\{i_, j_\}, 1}] \Rightarrow \mathbb{E}_{\{\} \rightarrow \{i, j\}} \left[\hbar a_j b_i, \hbar x_j y_i, \mathbf{1} - \frac{1}{4} (\gamma \hbar^3 x_j^2 y_i^2) \in + \mathbf{0} [\in]^2 \right]$$

HoldPattern [R_{ii\$1111, jj\$1111}, \$k_Integer] ->

$$\text{PP}_{\text{Boot}} \left[\text{Block} \left[\{i, j, k\}, R_{\{i_, j_\}, \$k} = \mathbb{E}_{\{\} \rightarrow \{i, j\}} \left[\hbar a_j b_i + \sum_{k=1}^{k+1} \frac{(1 - e^{\gamma \in h})^k (\hbar y_i x_j)^k}{k (1 - e^{k \gamma \in h})} \right]; \right. \right. \\ \left. \left. R_{\{ii$1111, jj$1111\}, \$k} \right] \right]$$

HoldPattern [R_{i_, j_}] -> R_{i, j}, \$k

HoldPattern [R_{sis\$1111}] -> R_{sis\$1111}

In[*]:= Cases[DownValues[Subscript], _[_[_List,Integer],_] // Column

$$\text{HoldPattern}[\text{am}_{\{i,j \rightarrow k\},1}] \Rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[\mathbf{a}_k \alpha_i + \mathbf{a}_k \alpha_j, \frac{\mathbf{x}_k \xi_i}{\mathcal{A}_j} + \mathbf{x}_k \xi_j, \mathbf{1} \right]$$

$$\text{HoldPattern}[\text{bm}_{\{i,j \rightarrow k\},1}] \Rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[\mathbf{b}_k \beta_i + \mathbf{b}_k \beta_j, \mathbf{y}_k \eta_i + \mathbf{y}_k \eta_j, \mathbf{1} - \mathbf{y}_k \beta_i \eta_j \in + \mathcal{O}[\epsilon]^2 \right]$$

Out[*]:= $\text{HoldPattern}[\text{R}_{\{i\$,j\$,1\}}] \Rightarrow \mathbb{E}_{\{\} \rightarrow \{i\$,j\}}$ $\left[\hbar \mathbf{a}_j \$, \hbar \mathbf{x}_j \$, \mathbf{y}_i \$, \mathbf{1} - \frac{1}{4} \left(\gamma \hbar^3 \mathbf{x}_{j\$}^2 \mathbf{y}_{i\$}^2 \right) \in + \mathcal{O}[\epsilon]^2 \right]$

$$\text{HoldPattern}[\text{P}_{\{i\$,j\$,0\}}] \Rightarrow \mathbb{E}_{\{i\$,j\} \rightarrow \{\}} \left[\frac{\alpha_{i\$} \beta_{i\$}}{\hbar}, \frac{\eta_{i\$} \xi_{j\$}}{\hbar}, \mathbf{1} \right]$$

$$\text{HoldPattern}[\text{P}_{\{i\$,j\$,1\}}] \Rightarrow \mathbb{E}_{\{i\$,j\} \rightarrow \{\}} \left[\frac{\alpha_{i\$} \beta_{i\$}}{\hbar}, \frac{\eta_{i\$} \xi_{j\$}}{\hbar}, \mathbf{1} + \frac{\gamma \eta_{i\$}^2 \xi_{j\$}^2 \epsilon}{4 \hbar} + \mathcal{O}[\epsilon]^2 \right]$$

In[*]:= DownValues[Subscript] =

Cases[DownValues[Subscript], Except[[(a[_] => _) /; a === R] | (_[_[_List,Integer],_)]]]

Out[*]:= {HoldPattern[PPProfile`private`label_] => Function[Profile`private`expr, Profile[Profile`private`label, Profile`private`expr], {HoldAll}], HoldPattern[E[L_, Q_, P_]\$_k_] => E[L, Q, Series[Normal[P], {epsilon, 0, \$k}]], HoldPattern[E_dr_[L_, Q_, P_]\$_k_] => E_dr_@@E[L, Q, P]\$_k_, HoldPattern[sm_{i,j->k}_] => E_{i,j} -> {k} [b_k (beta_i + beta_j) + t_k (tau_i + tau_j) + a_k (alpha_i + alpha_j) + y_k (eta_i + eta_j) + x_k (xi_i + xi_j)], HoldPattern[sDelta_{i->j,k}_] => E_{i} -> {j,k} [beta_i (b_j + b_k) + tau_i (t_j + t_k) + alpha_i (a_j + a_k) + eta_i (y_j + y_k) + xi_i (x_j + x_k)], HoldPattern[sS_{i_}] => E_{(i)->(i)} [-beta_i b_i - tau_i t_i - alpha_i a_i - eta_i y_i - xi_i x_i], HoldPattern[sEta_{i_}] => E_{(i)->(i)} [0], HoldPattern[sE_{i_}] => E_{(i)->(i)} [0], HoldPattern[sSigma_{i->j_}] => E_{(i)->(j)} [beta_i b_j + tau_i t_j + alpha_i a_j + eta_i y_j + xi_i x_j], HoldPattern[sY_{i->j,k,l,m}_] => E_{(i)->(j,k,l,m)} [beta_i b_k + tau_i t_k + alpha_i a_l + eta_i y_j + xi_i x_m], HoldPattern[cm_{ii\$1101_,jj\$1101->kk\$1101_},\$_k_Integer] => PPBoot[Block[{i, j, k}, cm_{i,j->k}_,\$k = E_{i,j} -> {k} [C^Lambda]; cm_{ii\$1101, jj\$1101->kk\$1101}_,\$k]], HoldPattern[cm_{i,j->k}_] => cm_{i,j->k}_,\$k, HoldPattern[cm_{sis\$1101_}] => cm_{sis\$1101_}, HoldPattern[cSigma_{ii\$1102->jj\$1102_},\$_k_Integer] => PPBoot[Block[{i, j, k}, cSigma_{i->j_},\$k = sSigma_{i,j} / . tau_i -> 0; cSigma_{ii\$1102->jj\$1102_},\$k]], HoldPattern[cSigma_{i->j_}] => cSigma_{i->j_},\$k, HoldPattern[cSigma_{sis\$1102_}] => cSigma_{sis\$1102_}, HoldPattern[cE_{ii\$1103_},\$_k_Integer] => PPBoot[Block[{i, j, k}, cE_{i_},\$k = sE_{i}; cE_{ii\$1103_},\$k]], HoldPattern[cE_{i_}] => cE_{i_},\$k, HoldPattern[cE_{sis\$1103_}] => cE_{sis\$1103_}, HoldPattern[cEta_{ii\$1104_},\$_k_Integer] => PPBoot[Block[{i, j, k}, cEta_{i_},\$k = sEta_{i}; cEta_{ii\$1104_},\$k]], HoldPattern[cEta_{i_}] => cEta_{i_},\$k, HoldPattern[cEta_{sis\$1104_}] => cEta_{sis\$1104_}, HoldPattern[cDelta_{ii\$1105->jj\$1105, kk\$1105_},\$_k_Integer] => PPBoot[Block[{i, j, k}, cDelta_{i->j,k}_,\$k = sDelta_{i->j,k}; cDelta_{ii\$1105->jj\$1105, kk\$1105_},\$k]], HoldPattern[cDelta_{i->j,k}_] => cDelta_{i->j,k}_,\$k, HoldPattern[cDelta_{sis\$1105_}] => cDelta_{sis\$1105_}, HoldPattern[cS_{ii\$1106_},\$_k_Integer] => PPBoot[Block[{i, j, k}, cS_{i_},\$k = cm_{i,1->i} [cm_{i,2->i} [cm_{4,3->i} [SY_{i->1,2,3,4} [sS_{i}]]]]]; cS_{ii\$1106_},\$k]], HoldPattern[cS_{i_}] => cS_{i_},\$k, HoldPattern[cS_{sis\$1106_}] => cS_{sis\$1106_}, HoldPattern[aSigma_{ii\$1107->jj\$1107_},\$_k_Integer] => PPBoot[Block[{i, j, k}, aSigma_{i->j_},\$k = E_{(i)->(j)} [a_j alpha_i + x_j xi_i]; aSigma_{ii\$1107->jj\$1107_},\$k]], HoldPattern[aSigma_{i->j_}] => aSigma_{i->j_},\$k, HoldPattern[aSigma_{sis\$1107_}] => aSigma_{sis\$1107_}, HoldPattern[bSigma_{ii\$1108->jj\$1108_},\$_k_Integer] => PPBoot[Block[{i, j, k}, bSigma_{i->j_},\$k = E_{(i)->(j)} [b_j beta_i + y_j eta_i]; bSigma_{ii\$1108->jj\$1108_},\$k]], HoldPattern[bSigma_{i->j_}] => bSigma_{i->j_},\$k, HoldPattern[bSigma_{sis\$1108_}] => bSigma_{sis\$1108_}, HoldPattern[am_{ii\$1109_,jj\$1109->kk\$1109_},\$_k_Integer] => PPBoot[Block[{i, j, k}, am_{i,j->k}_,\$k = E_{i,j} -> {k} [(alpha_i + alpha_j) a_k + (xi_i / A_j + xi_j) x_k]; am_{ii\$1109, jj\$1109->kk\$1109_},\$k]], HoldPattern[am_{i,j->k}_] => am_{i,j->k}_,\$k, HoldPattern[am_{sis\$1109_}] => am_{sis\$1109_}, HoldPattern[bm_{ii\$1110_,jj\$1110->kk\$1110_},\$_k_Integer] =>


```

HoldPattern[d $\Delta_{\text{sis}\$1126\_}$ ] := d $\Delta_{\{\text{sis}\$1126\}}$ , HoldPattern[C $_{\{\text{ii}\$1127\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, C $_{\{i\_}, \$k = \mathbb{E}_{\{\} \rightarrow \{i\}}[\theta, \theta, \sqrt{B_i} e^{\frac{1}{2}(-h) \in a_i}]_{\$k}$ ;
    C $_{\{\text{ii}\$1127}, \$k}$ ]], HoldPattern[C $_{\{i\_}$ ] := C $_{\{i\}, \$k}$ , HoldPattern[C $_{\text{sis}\$1127\_}$ ] := C $_{\{\text{sis}\$1127\}}$ ,
HoldPattern[C $_{\{\text{ii}\$1128\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, C $_{\{i\_}, \$k = \mathbb{E}_{\{\} \rightarrow \{i\}}[\theta, \theta, \frac{1}{\sqrt{B_i}} e^{\frac{1}{2}h \in a_i}]_{\$k}$ ;
    C $_{\{\text{ii}\$1128}, \$k}$ ]], HoldPattern[C $_{\{i\_}$ ] := C $_{\{i\}, \$k}$ , HoldPattern[C $_{\text{sis}\$1128\_}$ ] := C $_{\{\text{sis}\$1128\}}$ ,
HoldPattern[Kink $_{\{\text{ii}\$1129\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, Kink $_{\{i\_}, \$k = \text{dm}_{1,3 \rightarrow i}[\text{dm}_{1,2 \rightarrow 1}[\text{R}_{1,3} \text{C}_2]]$ ;
    Kink $_{\{\text{ii}\$1129}, \$k}$ ]], HoldPattern[Kink $_{\{i\_}$ ] := Kink $_{\{i\}, \$k}$ ,
HoldPattern[Kink $_{\text{sis}\$1129\_}$ ] := Kink $_{\{\text{sis}\$1129\}}$ , HoldPattern[Kink $_{\{\text{ii}\$1130\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, Kink $_{\{i\_}, \$k = \text{dm}_{1,3 \rightarrow i}[\text{dm}_{1,2 \rightarrow 1}[\bar{\text{R}}_{1,3} \text{C}_2]]$ ;
    Kink $_{\{\text{ii}\$1130}, \$k}$ ]], HoldPattern[Kink $_{\{i\_}$ ] := Kink $_{\{i\}, \$k}$ ,
HoldPattern[Kink $_{\text{sis}\$1130\_}$ ] := Kink $_{\{\text{sis}\$1130\}}$ , HoldPattern[b2t $_{\{\text{ii}\$1131\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, b2t $_{\{i\_}, \$k = \mathbb{E}_{\{i\} \rightarrow \{i\}}[\alpha_i a_i + \frac{\beta_i (\in a_i - t_i)}{\gamma} + \xi_i x_i + \eta_i y_i]$ ;
    b2t $_{\{\text{ii}\$1131}, \$k}$ ]], HoldPattern[b2t $_{\{i\_}$ ] := b2t $_{\{i\}, \$k}$ ,
HoldPattern[b2t $_{\text{sis}\$1131\_}$ ] := b2t $_{\{\text{sis}\$1131\}}$ , HoldPattern[t2b $_{\{\text{ii}\$1132\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, t2b $_{\{i\_}, \$k = \mathbb{E}_{\{i\} \rightarrow \{i\}}[\alpha_i a_i + \tau_i (\in a_i - \gamma b_i) + \xi_i x_i + \eta_i y_i]$ ;
    t2b $_{\{\text{ii}\$1132}, \$k}$ ]], HoldPattern[t2b $_{\{i\_}$ ] := t2b $_{\{i\}, \$k}$ ,
HoldPattern[t2b $_{\text{sis}\$1132\_}$ ] := t2b $_{\{\text{sis}\$1132\}}$ , HoldPattern[kR $_{\{\text{ii}\$1133_, \text{jj}\$1133\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, kR $_{\{i_, j\_}, \$k = (b2t_i b2t_j / . t_{i|j} \rightarrow t)[R_{i,j}]$ ;
    kR $_{\{\text{ii}\$1133, \text{jj}\$1133}, \$k}$ ]], HoldPattern[kR $_{\{i_, j\_}$ ] := kR $_{\{i,j\}, \$k}$ ,
HoldPattern[kR $_{\text{sis}\$1133\_}$ ] := kR $_{\{\text{sis}\$1133\}}$ , HoldPattern[kR $_{\{\text{ii}\$1134_, \text{jj}\$1134\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, kR $_{\{i_, j\_}, \$k = (b2t_i b2t_j / . \{t_{i|j} \rightarrow t, T_{i|j} \rightarrow T\})[\bar{R}_{i,j}]$ ;
    kR $_{\{\text{ii}\$1134, \text{jj}\$1134}, \$k}$ ]], HoldPattern[kR $_{\{i_, j\_}$ ] := kR $_{\{i,j\}, \$k}$ ,
HoldPattern[kR $_{\text{sis}\$1134\_}$ ] := kR $_{\{\text{sis}\$1134\}}$ , HoldPattern[km $_{\{\text{ii}\$1135_, \text{jj}\$1135\_ \rightarrow \text{kk}\$1135\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, km $_{\{i_, j\_ \rightarrow k\_}, \$k = (b2t_k / . \{t_k \rightarrow t, T_k \rightarrow T, \tau_{i|j} \rightarrow \theta\})[\text{dm}_{i,j \rightarrow k}[t2b_i t2b_j]]$ ;
    km $_{\{\text{ii}\$1135, \text{jj}\$1135 \rightarrow \text{kk}\$1135}, \$k}$ ]], HoldPattern[km $_{\{i_, j\_ \rightarrow k\_}$ ] := km $_{\{i,j \rightarrow k\}, \$k}$ ,
HoldPattern[km $_{\text{sis}\$1135\_}$ ] := km $_{\{\text{sis}\$1135\}}$ ,
HoldPattern[kC $_{\{\text{ii}\$1136\_}, \$k\_Integer}$ ] := PP $_{\text{Boot}}$ [Block[{i, j, k}, kC $_{\{i\_}, \$k = (b2t_i / . T_i \rightarrow T)[C_i]$ ;
    kC $_{\{\text{ii}\$1136}, \$k}$ ]], HoldPattern[kC $_{\{i\_}$ ] := kC $_{\{i\}, \$k}$ , HoldPattern[kC $_{\text{sis}\$1136\_}$ ] := kC $_{\{\text{sis}\$1136\}}$ ,
HoldPattern[kC $_{\{\text{ii}\$1137\_}, \$k\_Integer}$ ] := PP $_{\text{Boot}}$ [Block[{i, j, k}, kC $_{\{i\_}, \$k = (b2t_i / . T_i \rightarrow T)[\bar{C}_i]$ ;
    kC $_{\{\text{ii}\$1137}, \$k}$ ]], HoldPattern[kC $_{\{i\_}$ ] := kC $_{\{i\}, \$k}$ ,
HoldPattern[kC $_{\text{sis}\$1137\_}$ ] := kC $_{\{\text{sis}\$1137\}}$ , HoldPattern[kKink $_{\{\text{ii}\$1138\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, kKink $_{\{i\_}, \$k = (b2t_i / . \{t_i \rightarrow t, T_i \rightarrow T\})[Kink_i]$ ;
    kKink $_{\{\text{ii}\$1138}, \$k}$ ]], HoldPattern[kKink $_{\{i\_}$ ] := kKink $_{\{i\}, \$k}$ ,
HoldPattern[kKink $_{\text{sis}\$1138\_}$ ] := kKink $_{\{\text{sis}\$1138\}}$ , HoldPattern[kKink $_{\{\text{ii}\$1139\_}, \$k\_Integer}$ ] :=
  PP $_{\text{Boot}}$ [Block[{i, j, k}, kKink $_{\{i\_}, \$k = (b2t_i / . \{t_i \rightarrow t, T_i \rightarrow T\})[Kink_i]$ ;
    kKink $_{\{\text{ii}\$1139}, \$k}$ ]], HoldPattern[kKink $_{\{i\_}$ ] := kKink $_{\{i\}, \$k}$ ,
HoldPattern[kKink $_{\text{sis}\$1139\_}$ ] := kKink $_{\{\text{sis}\$1139\}}$ 

```

In[]:= Cases[DownValues[Subscript], (_[a__] := _) /; a === R] // Column

Out[]:=

```
In[ ]:= Cases [DownValues [Subscript], _[_[_List,_Integer], _]] // Column
```

```
Out[ ]:=
```

```
In[ ]:= Define [Ri,j = i + j]
```

```
In[ ]:= Cases [DownValues [Subscript], (_[a_>>_] /; a === R] // Column
```

```
HoldPattern [R{ii$4547_,jj$4547_},$k_Integer] := PPBoot [Block[{i, j, k}, R{i_,j_},$k = i + j;
Out[ ]:= R{ii$4547, jj$4547},$k]
```

```
HoldPattern [R{i_,j_}] := R{i,j},$k
```

```
HoldPattern [Rsis$4547_}] := R{sis$4547}
```

```
In[ ]:= Cases [DownValues [Subscript], _[_[_List,_Integer], _]] // Column
```

```
Out[ ]:=
```