Pensieve header: Searching for a Seifert Formula; continues Seifert.nb at pensieve://People/VanDerVeen/.

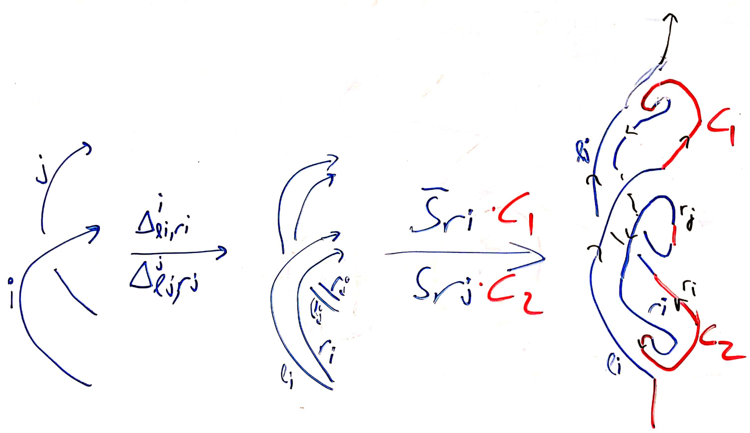
Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
PP_ := Identity;
<< "Engine-Speedy.m";
<< "Objects.m";
$k = 1;
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, Green, Pink]];
ħ = γ = 1;
```

» Warning: On Sep 4 2019 I swapped the operations ϵ and η. Some incompatibilities may arise in older notebooks.

BS (BanderSnatch)

(See https://en.wikipedia.org/wiki/Bandersnatch)



In[∘]:=
```
Define[BS_{i,j→k} =
  C₁ C₂ dΔ_{i→li,ri} dΔ_{j→lj,rj} // d̄S̄_{ri} // dS_{rj} // dm_{li,1→k} // dm_{k,rj→k} // dm_{k,ri→k} // dm_{k,2→k} // dm_{k,lj→k}]
```

In[∘]:=
```
Define[tBS_{i,j→k} = (t2b_i t2b_j) // BS_{i,j→k} // b2t_k]
```

In[∘]:=
```
BS_{1,2→3} // Short
```

Out[∘]//Short=

$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[0,\ y_3\ (1 - \ll 1 \gg)\ \eta_1 + \ll 7 \gg,\ B_3 + \Big(\ll 56 \gg + \ll 1 \gg + \frac{1}{4}\left(-3\, B_3\, \mathcal{A}_2^2 + \ll 8 \gg + B_3^3\, \mathcal{A}_1^2\, \mathcal{A}_2^2\right) \eta_2^2\, \xi_2^2\Big)\ \epsilon + O[\epsilon]^2\Big]$$

In[∘]:=
```
FreeQ[BS_{1,2→3}, β]
```

Out[∘]= False

In[∘]:= **Simplify@tBS$_{1,2\to3}$**

Out[∘]=
$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[0,\ -\frac{y_3\,\mathcal{A}_2\,\eta_2}{\mathcal{A}_1}+y_3\left(-\left(-1+\mathcal{A}_2\right)\eta_1+\mathcal{A}_2\,\eta_2\right)+x_3\left(\mathcal{A}_1\left(\left(1-\frac{1}{\mathcal{A}_2}\right)\xi_1-\xi_2\right)+\xi_2\right)+$$
$$\left(-1+T_3\right)\left(\mathcal{A}_2\,\eta_2\,(\xi_1-\xi_2)+\mathcal{A}_1\,(\eta_1-\eta_2)\left(\left(-1+\mathcal{A}_2\right)\xi_1-\mathcal{A}_2\,\xi_2\right)\right),$$
$$T_3+\frac{1}{4}T_3\left(\frac{2\,y_3^2\,\mathcal{A}_2\left(\mathcal{A}_1^2\left(-1+\mathcal{A}_2\right)\eta_1^2-\mathcal{A}_2\,\eta_2^2+\mathcal{A}_1\,\mathcal{A}_2\,\eta_2^2\right)}{\mathcal{A}_1^2}+\frac{2\,x_3^2\,\mathcal{A}_1\left(-\mathcal{A}_2^2\,\xi_2^2+\mathcal{A}_1\left(\left(-1+\mathcal{A}_2\right)\xi_1^2+\mathcal{A}_2^2\,\xi_2^2\right)\right)}{\mathcal{A}_2^2}-\right.$$
$$8\,a_3\left(1+T_3\left(\mathcal{A}_2\,\eta_2\,(\xi_1-\xi_2)+\mathcal{A}_1\,(\eta_1-\eta_2)\left(\left(-1+\mathcal{A}_2\right)\xi_1-\mathcal{A}_2\,\xi_2\right)\right)\right)+$$
$$\frac{1}{\mathcal{A}_2}2\,x_3\left(-\left(-1+T_3\right)\mathcal{A}_2^2\,\eta_2\,\xi_2^2+\mathcal{A}_1\,\mathcal{A}_2\left(\left(3+3\,T_3\left(-1+\mathcal{A}_2\right)-\mathcal{A}_2\right)\eta_2\,\xi_1^2-4\,\xi_1\right.\right.$$
$$\left(-1+\left(1+T_3\left(-1+\mathcal{A}_2\right)\right)\eta_2\,\xi_2\right)+\xi_2\left(-4+\mathcal{A}_2\left(-\left(-1+T_3\right)\eta_1+2\left(-1+2\,T_3\right)\eta_2\right)\xi_2\right)\Big)+$$
$$\mathcal{A}_1^2\left(\eta_1\left(\left(-1+\mathcal{A}_2\right)\left(-3+\mathcal{A}_2+T_3\left(1+\mathcal{A}_2\right)\right)\xi_1^2-4\left(-1+\mathcal{A}_2\right)\mathcal{A}_2\,\xi_1\,\xi_2+\left(1+T_3\right)\mathcal{A}_2^2\,\xi_2^2\right)-\eta_2\right.$$
$$\left.\left.\left(\left(-1+\mathcal{A}_2\right)\left(-1-\mathcal{A}_2+T_3\left(-1+3\,\mathcal{A}_2\right)\right)\xi_1^2-4\,T_3\left(-1+\mathcal{A}_2\right)\mathcal{A}_2\,\xi_1\,\xi_2+\left(-1+3\,T_3\right)\mathcal{A}_2^2\,\xi_2^2\right)\right)\right)-$$
$$\frac{1}{\mathcal{A}_1}2\,y_3\left(\mathcal{A}_1\left(-4\,x_3\,\eta_2\,\xi_1+\mathcal{A}_2\left(\eta_1\left(4-4\left(-1+T_3\right)\eta_2\,\xi_1\right)+\eta_2\left(-4+2\,x_3\,\xi_1+3\left(-1+T_3\right)\eta_2\,\xi_1\right)\right)+\right.\right.$$
$$\left.4\,\mathcal{A}_2^2\,(\eta_1-\eta_2)\,\eta_2\,(T_3\,\xi_1-\xi_2)\right)+\mathcal{A}_2^2\,\eta_2^2\left(\left(1+T_3\right)\xi_1+\left(-3+T_3\right)\xi_2\right)+$$
$$\mathcal{A}_1^2\left(\eta_1^2\left(\left(-1+\mathcal{A}_2\right)\left(1-\mathcal{A}_2+T_3\left(-1+3\,\mathcal{A}_2\right)\right)\xi_1-\mathcal{A}_2\left(1+T_3\left(-1+\mathcal{A}_2\right)+\mathcal{A}_2\right)\xi_2\right)+\right.$$
$$2\,\mathcal{A}_2\,\eta_1\left(-2\,T_3\left(-1+\mathcal{A}_2\right)\eta_2\,\xi_1+\left(x_3+2\,\mathcal{A}_2\,\eta_2\right)\xi_2\right)+$$
$$\left.\left.\eta_2\left(-2\,x_3\left(-1+\mathcal{A}_2\right)\xi_1+\mathcal{A}_2\,\eta_2\left(\left(-1+3\,T_3\right)\left(-1+\mathcal{A}_2\right)\xi_1-\left(1+T_3\right)\mathcal{A}_2\,\xi_2\right)\right)\right)\right)+$$
$$\left(-1+T_3\right)\left(\mathcal{A}_2\,\eta_2\left(4\,\xi_2+\mathcal{A}_2\,\eta_2\left(\left(1+T_3\right)\xi_1^2+2\left(-3+T_3\right)\xi_1\,\xi_2-\left(-3+T_3\right)\xi_2^2\right)\right)+\mathcal{A}_1^2\right.$$
$$\left(\eta_1^2\left(\left(-1+\mathcal{A}_2\right)\left(-3+\mathcal{A}_2+T_3\left(1+\mathcal{A}_2\right)\right)\xi_1^2+2\left(-3+T_3\right)\left(-1+\mathcal{A}_2\right)\mathcal{A}_2\,\xi_1\,\xi_2-\left(-3+T_3\right)\mathcal{A}_2^2\,\xi_2^2\right)-\right.$$
$$2\,\eta_1\,\eta_2\left(\left(-1+\mathcal{A}_2\right)\left(-3+\mathcal{A}_2+T_3\left(1+\mathcal{A}_2\right)\right)\xi_1^2+2\left(-3+T_3\right)\left(-1+\mathcal{A}_2\right)\mathcal{A}_2\,\xi_1\,\xi_2-\right.$$
$$\left.\left(-3+T_3\right)\mathcal{A}_2^2\,\xi_2^2\right)+\eta_2^2\left(\left(-1+\mathcal{A}_2\right)\left(-1-\mathcal{A}_2+T_3\left(-1+3\,\mathcal{A}_2\right)\right)\xi_1^2-\right.$$
$$\left.\left.2\left(1+T_3\right)\left(-1+\mathcal{A}_2\right)\mathcal{A}_2\,\xi_1\,\xi_2+\left(1+T_3\right)\mathcal{A}_2^2\,\xi_2^2\right)\right)+2\,\mathcal{A}_1$$
$$\left(\eta_1\left(\mathcal{A}_2\left(1+T_3\left(-1+\mathcal{A}_2\right)+\mathcal{A}_2\right)\eta_2\,\xi_1^2-\left(-3+T_3\right)\mathcal{A}_2^2\,\eta_2\,\xi_2^2+2\,\xi_1\left(1+\mathcal{A}_2+\left(-3+T_3\right)\mathcal{A}_2^2\,\eta_2\,\xi_2\right)\right)+\right.$$
$$\mathcal{A}_2\,\eta_2\left(-2\left(1+T_3\left(-1+\mathcal{A}_2\right)\right)\eta_2\,\xi_1^2-2\,\xi_2\left(-1+\mathcal{A}_2\,\eta_2\,\xi_2\right)+\right.$$
$$\left.\left.\left.\left.\left.\xi_1\left(-4+\left(1-T_3+4\,\mathcal{A}_2\right)\eta_2\,\xi_2\right)\right)\right)\right)\right)\right)\epsilon+O[\epsilon]^2\Big]$$

In[∘]:= **Simplify@(tBS$_{1,2\to3}$ /. $\mathcal{A}$_ → 1)**

Out[∘]=
$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[0,\ \left(-1+T_3\right)(\eta_2\,\xi_1-\eta_1\,\xi_2),$$
$$T_3+\frac{1}{4}T_3\left(-8\,a_3-8\,y_3\,\eta_1+8\,y_3\,\eta_2+8\,x_3\,\xi_1+8\left(-1+T_3\right)\eta_1\,\xi_1-8\left(-1+T_3\right)\eta_2\,\xi_1-8\,a_3\,T_3\,\eta_2\,\xi_1+\right.$$
$$4\,x_3\,y_3\,\eta_2\,\xi_1-8\,y_3\,\eta_1\,\eta_2\,\xi_1+4\,y_3\,\eta_2^2\,\xi_1+4\,x_3\,\eta_2\,\xi_1^2+4\left(-1+T_3\right)\eta_1\,\eta_2\,\xi_1^2+\left(3-4\,T_3+T_3^2\right)\eta_2^2\,\xi_1^2-$$
$$8\,x_3\,\xi_2+8\,a_3\,T_3\,\eta_1\,\xi_2-4\,x_3\,y_3\,\eta_1\,\xi_2+4\,y_3\,\eta_1^2\,\xi_2+8\left(-1+T_3\right)\eta_2\,\xi_2-8\,x_3\,\eta_2\,\xi_1\,\xi_2+$$
$$\left.4\left(3-4\,T_3+T_3^2\right)\eta_1\,\eta_2\,\xi_1\,\xi_2+4\left(-1+T_3\right)\eta_2^2\,\xi_1\,\xi_2+4\,x_3\,\eta_1\,\xi_2^2-\left(3-4\,T_3+T_3^2\right)\eta_1^2\,\xi_2^2\right)\epsilon+O[\epsilon]^2\Big]$$

In[∘]:= **Simplify@(tBS$_{1,2\to3}$ /. {$\mathcal{A}$_ → 1, (y | a | x)_ → 0})**

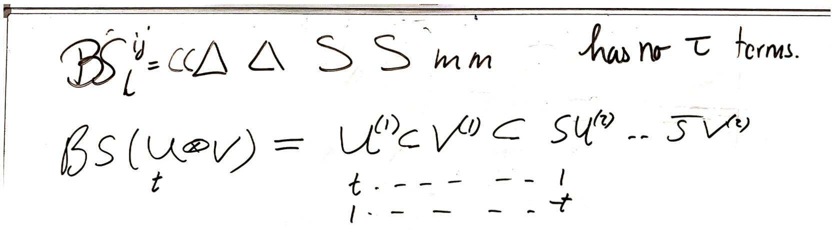
Out[∘]=
$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[0,\ \left(-1+T_3\right)(\eta_2\,\xi_1-\eta_1\,\xi_2),$$
$$T_3-\frac{1}{4}\left(\left(-1+T_3\right)T_3\left(\left(-3+T_3\right)\eta_1^2\,\xi_2^2-4\,\eta_1\,\xi_1\left(2+\eta_2\left(\xi_1+\left(-3+T_3\right)\xi_2\right)\right)-\right.\right.$$
$$\left.\left.\eta_2\left(\left(-3+T_3\right)\eta_2\,\xi_1^2+8\,\xi_2+4\,\xi_1\left(-2+\eta_2\,\xi_2\right)\right)\right)\right)\epsilon+O[\epsilon]^2\Big]$$

In[∘]:= **FreeQ[tBS$_{1,2\to3}$, τ]**

Out[∘]= True

Theorem. tBS is τ-free.

Proof. Follows immediately from the fact that tBS is a "commutator" and $(1\otimes S)\,\Delta(t) = t\otimes 1 - 1\otimes t$, and that t is central:



In[∘]:= $\text{Define}\left[tR_{i,j} = R_{i,j} \,//\, b2t_i \,//\, b2t_j,\ \overline{tR}_{i,j} = \overline{R}_{i,j} \,//\, b2t_i \,//\, b2t_j\right]$

In[∘]:= $tR_{1,2}$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}\left[-a_2\, t_1,\ x_2\, y_1,\ 1 + \left(a_1\, a_2 - \frac{1}{4}\, x_2^2\, y_1^2\right)\epsilon + O[\epsilon]^2\right]$

In[∘]:= $\overline{tR}_{1,2}$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}\left[a_2\, t_1,\ -\frac{x_2\, y_1}{T_1},\ 1 + \left(-a_1\, a_2 - \frac{a_1\, x_2\, y_1}{T_1} - \frac{a_2\, x_2\, y_1}{T_1} - \frac{3\, x_2^2\, y_1^2}{4\, T_1^2}\right)\epsilon + O[\epsilon]^2\right]$

In[∘]:= C_1

Out[∘]= $\mathbb{E}_{\{\}\to\{1\}}\left[0,\ 0,\ \sqrt{B_1} - \frac{1}{2}\left(a_1\sqrt{B_1}\right)\epsilon + O[\epsilon]^2\right]$