Pensieve header: Taking logarithms, for comparisons with penseive://Projects/FullDoPeGDO/.

Startup

(Alt) In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
<< KnotTheory`;
<< "../Profile/Profile.m";
<< "Engine-With-w.m";
<< "Objects-With-w.m";
<< "KT.m";
BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
 Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

» Warning: On Sep 4 2019 I swapped the operations ϵ and η. Some incompatibilities may arise in older notebooks.

(Alt) Out[∘]= Show Profile Monitor

(Alt) In[∘]:=
```
LogE[𝔼_{d_→r_}[L_, Q_, P_]] := Module[{ℰ = L + Q + Log[P], k},
   𝔽_{d→r} @@ Table[CF@SeriesCoefficient[ℰ, {ϵ, 0, k}], {k, 0, $k}]];
```

(Alt) In[∘]:=
```
$k = 1; γ = 1;
```

(Alt) In[∘]:=
```
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

(Alt) In[∘]:= **cm$_{1,2\to3}$**

(Alt) Out[∘]=
$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[a_3\alpha_1+a_3\alpha_2+b_3\beta_1+b_3\beta_2,\ y_3\eta_1+\frac{y_3\eta_2}{\mathcal{A}_1}+\frac{x_3\xi_1}{\mathcal{A}_2}+b_3\eta_2\xi_1+x_3\xi_2,$$
$$1+\left(-\frac{y_3\beta_1\eta_2}{\mathcal{A}_1}-\frac{x_3\beta_2\xi_1}{\mathcal{A}_2}+a_3\eta_2\xi_1-\frac{y_3\eta_2^2\xi_1}{\mathcal{A}_1}-\frac{x_3\eta_2\xi_1^2}{\mathcal{A}_2}-\frac{1}{2}b_3\eta_2^2\xi_1^2\right)\epsilon+$$
$$\left(\frac{y_3\beta_1^2\eta_2}{2\mathcal{A}_1}+\frac{y_3^2\beta_1^2\eta_2^2}{2\mathcal{A}_1^2}+\frac{x_3\beta_2^2\xi_1}{2\mathcal{A}_2}+\frac{x_3y_3\beta_1\beta_2\eta_2\xi_1}{\mathcal{A}_1\mathcal{A}_2}+\frac{y_3\beta_1\eta_2^2\xi_1}{\mathcal{A}_1}-\frac{a_3y_3\beta_1\eta_2^2\xi_1}{\mathcal{A}_1}+\right.$$
$$\frac{y_3^2\beta_1\eta_2^3\xi_1}{\mathcal{A}_1^2}+\frac{x_3^2\beta_2^2\xi_1^2}{2\mathcal{A}_2^2}+\frac{x_3\beta_2\eta_2\xi_1^2}{\mathcal{A}_2}-\frac{a_3x_3\beta_2\eta_2\xi_1^2}{\mathcal{A}_2}-\frac{1}{2}a_3\eta_2^2\xi_1^2+\frac{1}{2}a_3^2\eta_2^2\xi_1^2+$$
$$\frac{x_3y_3\beta_1\eta_2^2\xi_1^2}{\mathcal{A}_1\mathcal{A}_2}+\frac{x_3y_3\beta_2\eta_2^2\xi_1^2}{\mathcal{A}_1\mathcal{A}_2}+\frac{y_3\eta_2^3\xi_1^2}{\mathcal{A}_1}-\frac{a_3y_3\eta_2^3\xi_1^2}{\mathcal{A}_1}+\frac{b_3y_3\beta_1\eta_2^3\xi_1^2}{2\mathcal{A}_1}+\frac{y_3^2\eta_2^4\xi_1^2}{2\mathcal{A}_1^2}+$$
$$\frac{x_3^2\beta_2\eta_2\xi_1^3}{\mathcal{A}_2^2}+\frac{x_3\eta_2^2\xi_1^3}{\mathcal{A}_2}-\frac{a_3x_3\eta_2^2\xi_1^3}{\mathcal{A}_2}+\frac{b_3x_3\beta_2\eta_2^2\xi_1^3}{2\mathcal{A}_2}+\frac{1}{3}b_3\eta_2^3\xi_1^3-\frac{1}{2}a_3b_3\eta_2^3\xi_1^3+$$
$$\left.\frac{x_3y_3\eta_2^3\xi_1^3}{\mathcal{A}_1\mathcal{A}_2}+\frac{b_3y_3\eta_2^4\xi_1^3}{2\mathcal{A}_1}+\frac{x_3^2\eta_2^2\xi_1^4}{2\mathcal{A}_2^2}+\frac{b_3x_3\eta_2^3\xi_1^4}{2\mathcal{A}_2}+\frac{1}{8}b_3^2\eta_2^4\xi_1^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

(Alt) In[∘]:= **$k = 2;**

(Alt) In[∘]:= **cm$_{1,2\to3}$ // LogE**

(Alt) Out[∘]=
$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[a_3\alpha_1+a_3\alpha_2+b_3\beta_1+b_3\beta_2+y_3\eta_1+\frac{y_3\eta_2}{\mathcal{A}_1}+\frac{x_3\xi_1}{\mathcal{A}_2}+b_3\eta_2\xi_1+x_3\xi_2,$$
$$-\frac{y_3\beta_1\eta_2}{\mathcal{A}_1}-\frac{x_3\beta_2\xi_1}{\mathcal{A}_2}+a_3\eta_2\xi_1-\frac{y_3\eta_2^2\xi_1}{\mathcal{A}_1}-\frac{x_3\eta_2\xi_1^2}{\mathcal{A}_2}-\frac{1}{2}b_3\eta_2^2\xi_1^2,$$
$$\frac{y_3\beta_1^2\eta_2}{2\mathcal{A}_1}+\frac{x_3\beta_2^2\xi_1}{2\mathcal{A}_2}+\frac{y_3\beta_1\eta_2^2\xi_1}{\mathcal{A}_1}+\frac{x_3\beta_2\eta_2\xi_1^2}{\mathcal{A}_2}-\frac{1}{2}a_3\eta_2^2\xi_1^2+\frac{y_3\eta_2^3\xi_1^2}{\mathcal{A}_1}+\frac{x_3\eta_2^2\xi_1^3}{\mathcal{A}_2}+\frac{1}{3}b_3\eta_2^3\xi_1^3\Big]$$

(Alt) In[∘]:= **(cm$_{1,2\to 1}$ // cm$_{1,3\to 1}$) // LogE**

(Alt) Out[∘]=

$$\mathbb{E}_{\{1,2,3\}\to\{1\}}\Big[a_1\alpha_1+a_1\alpha_2+a_1\alpha_3+b_1\beta_1+b_1\beta_2+b_1\beta_3+$$
$$y_1\eta_1+\frac{y_1\eta_2}{\mathcal{A}_1}+\frac{y_1\eta_3}{\mathcal{A}_1\mathcal{A}_2}+\frac{x_1\xi_1}{\mathcal{A}_2\mathcal{A}_3}+b_1\eta_2\xi_1+\frac{b_1\eta_3\xi_1}{\mathcal{A}_2}+\frac{x_1\xi_2}{\mathcal{A}_3}+b_1\eta_3\xi_2+x_1\xi_3,$$
$$-\frac{y_1\beta_1\eta_2}{\mathcal{A}_1}-\frac{y_1\beta_1\eta_3}{\mathcal{A}_1\mathcal{A}_2}-\frac{y_1\beta_2\eta_3}{\mathcal{A}_1\mathcal{A}_2}-\frac{x_1\beta_2\xi_1}{\mathcal{A}_2\mathcal{A}_3}-\frac{x_1\beta_3\xi_1}{\mathcal{A}_2\mathcal{A}_3}+a_1\eta_2\xi_1-\frac{y_1\eta_2^2\xi_1}{\mathcal{A}_1}+\frac{a_1\eta_3\xi_1}{\mathcal{A}_2}-$$
$$\frac{b_1\beta_2\eta_3\xi_1}{\mathcal{A}_2}-\frac{2y_1\eta_2\eta_3\xi_1}{\mathcal{A}_1\mathcal{A}_2}-\frac{y_1\eta_3^2\xi_1}{\mathcal{A}_1\mathcal{A}_2^2}-\frac{x_1\eta_2\xi_1^2}{\mathcal{A}_2\mathcal{A}_3}-\frac{1}{2}b_1\eta_2^2\xi_1^2-\frac{x_1\eta_3\xi_1^2}{\mathcal{A}_2^2\mathcal{A}_3}-\frac{b_1\eta_2\eta_3\xi_1^2}{\mathcal{A}_2}-$$
$$\frac{b_1\eta_3^2\xi_1^2}{2\mathcal{A}_2^2}-\frac{x_1\beta_3\xi_2}{\mathcal{A}_3}+a_1\eta_3\xi_2-\frac{y_1\eta_3^2\xi_2}{\mathcal{A}_1\mathcal{A}_2}-\frac{2x_1\eta_3\xi_1\xi_2}{\mathcal{A}_2\mathcal{A}_3}-\frac{b_1\eta_3^2\xi_1\xi_2}{\mathcal{A}_2}-\frac{x_1\eta_3\xi_2^2}{\mathcal{A}_3}-\frac{1}{2}b_1\eta_3^2\xi_2^2,$$
$$\frac{y_1\beta_1^2\eta_2}{2\mathcal{A}_1}+\frac{y_1\beta_1^2\eta_3}{2\mathcal{A}_1\mathcal{A}_2}+\frac{y_1\beta_1\beta_2\eta_3}{\mathcal{A}_1\mathcal{A}_2}+\frac{y_1\beta_2^2\eta_3}{2\mathcal{A}_1\mathcal{A}_2}+\frac{x_1\beta_2^2\xi_1}{2\mathcal{A}_2\mathcal{A}_3}+\frac{x_1\beta_2\beta_3\xi_1}{\mathcal{A}_2\mathcal{A}_3}+\frac{x_1\beta_3^2\xi_1}{2\mathcal{A}_2\mathcal{A}_3}+\frac{y_1\beta_1\eta_2^2\xi_1}{\mathcal{A}_1}-$$
$$\frac{a_1\beta_2\eta_3\xi_1}{\mathcal{A}_2}+\frac{b_1\beta_2^2\eta_3\xi_1}{2\mathcal{A}_2}+\frac{2y_1\beta_1\eta_2\eta_3\xi_1}{\mathcal{A}_1\mathcal{A}_2}+\frac{2y_1\beta_2\eta_2\eta_3\xi_1}{\mathcal{A}_1\mathcal{A}_2}+\frac{y_1\beta_1\eta_3^2\xi_1}{\mathcal{A}_1\mathcal{A}_2^2}+\frac{2y_1\beta_2\eta_3^2\xi_1}{\mathcal{A}_1\mathcal{A}_2^2}+$$
$$\frac{x_1\beta_2\eta_2\xi_1^2}{\mathcal{A}_2\mathcal{A}_3}+\frac{x_1\beta_3\eta_2\xi_1^2}{\mathcal{A}_2\mathcal{A}_3}-\frac{1}{2}a_1\eta_2^2\xi_1^2+\frac{y_1\eta_2^3\xi_1^2}{\mathcal{A}_1}+\frac{2x_1\beta_2\eta_3\xi_1^2}{\mathcal{A}_2^2\mathcal{A}_3}+\frac{x_1\beta_3\eta_3\xi_1^2}{\mathcal{A}_2^2\mathcal{A}_3}-\frac{a_1\eta_2\eta_3\xi_1^2}{\mathcal{A}_2}+$$
$$\frac{b_1\beta_2\eta_2\eta_3\xi_1^2}{\mathcal{A}_2}+\frac{3y_1\eta_2^2\eta_3\xi_1^2}{\mathcal{A}_1\mathcal{A}_2}-\frac{a_1\eta_3^2\xi_1^2}{2\mathcal{A}_2^2}+\frac{b_1\beta_2\eta_3^2\xi_1^2}{\mathcal{A}_2^2}+\frac{3y_1\eta_2\eta_3^2\xi_1^2}{\mathcal{A}_1\mathcal{A}_2^2}+\frac{y_1\eta_3^3\xi_1^2}{\mathcal{A}_1\mathcal{A}_2^3}+\frac{x_1\eta_2^2\xi_1^3}{\mathcal{A}_2\mathcal{A}_3}+$$
$$\frac{1}{3}b_1\eta_2^3\xi_1^3+\frac{2x_1\eta_2\eta_3\xi_1^3}{\mathcal{A}_2^2\mathcal{A}_3}+\frac{b_1\eta_2^2\eta_3\xi_1^3}{\mathcal{A}_2}+\frac{x_1\eta_3^2\xi_1^3}{\mathcal{A}_2^3\mathcal{A}_3}+\frac{b_1\eta_2\eta_3^2\xi_1^3}{\mathcal{A}_2^2}+\frac{b_1\eta_3^3\xi_1^3}{3\mathcal{A}_2^3}+\frac{x_1\beta_3^2\xi_2}{2\mathcal{A}_3}+$$
$$\frac{y_1\beta_1\eta_3^2\xi_2}{\mathcal{A}_1\mathcal{A}_2}+\frac{y_1\beta_2\eta_3^2\xi_2}{\mathcal{A}_1\mathcal{A}_2}+\frac{2x_1\beta_2\eta_3\xi_1\xi_2}{\mathcal{A}_2\mathcal{A}_3}+\frac{2x_1\beta_3\eta_3\xi_1\xi_2}{\mathcal{A}_2\mathcal{A}_3}-\frac{a_1\eta_3^2\xi_1\xi_2}{\mathcal{A}_2}+\frac{b_1\beta_2\eta_3^2\xi_1\xi_2}{\mathcal{A}_2}+$$
$$\frac{2y_1\eta_2\eta_3^2\xi_1\xi_2}{\mathcal{A}_1\mathcal{A}_2}+\frac{2y_1\eta_3^3\xi_1\xi_2}{\mathcal{A}_1\mathcal{A}_2^2}+\frac{2x_1\eta_2\eta_3\xi_1^2\xi_2}{\mathcal{A}_2\mathcal{A}_3}+\frac{3x_1\eta_3^2\xi_1^2\xi_2}{\mathcal{A}_2^2\mathcal{A}_3}+\frac{b_1\eta_2\eta_3^2\xi_1^2\xi_2}{\mathcal{A}_2}+\frac{b_1\eta_3^3\xi_1^2\xi_2}{\mathcal{A}_2^2}+$$
$$\frac{x_1\beta_3\eta_3\xi_2^2}{\mathcal{A}_3}-\frac{1}{2}a_1\eta_3^2\xi_2^2+\frac{y_1\eta_3^3\xi_2^2}{\mathcal{A}_1\mathcal{A}_2}+\frac{3x_1\eta_3^2\xi_1\xi_2^2}{\mathcal{A}_2\mathcal{A}_3}+\frac{b_1\eta_3^3\xi_1\xi_2^2}{\mathcal{A}_2}+\frac{x_1\eta_3^2\xi_2^3}{\mathcal{A}_3}+\frac{1}{3}b_1\eta_3^3\xi_2^3\Big]$$

(Alt) In[∘]:= **$k = 1; R$_{i,j}$ // LogE**

(Alt) Out[∘]=

$$\mathbb{E}_{\{\}\to\{i,j\}}\Big[\hbar\, a_j b_i+\hbar\, x_j y_i,\ -\frac{1}{4}\hbar^3 x_j^2 y_i^2\Big]$$

(Alt) In[∘]:= **$k = 3; $\overline{\mathrm{R}}_{i,j}$ // LogE**

(Alt) Out[∘]= $\mathbb{F}_{\{\}\to\{i,j\}}\Big[-\hbar\, a_j\, b_i - \frac{\hbar\, x_j\, y_i}{B_i},\ -\frac{\hbar^2 a_j x_j y_i}{B_i} - \frac{3\hbar^3 x_j^2 y_i^2}{4 B_i^2},$
$-\frac{\hbar^3 a_j^2 x_j y_i}{2 B_i} + \frac{\hbar^4 x_j^2 y_i^2}{2 B_i^2} - \frac{3\hbar^4 a_j x_j^2 y_i^2}{2 B_i^2} - \frac{10\hbar^5 x_j^3 y_i^3}{9 B_i^3},$
$-\frac{\hbar^4 a_j^3 x_j y_i}{6 B_i} - \frac{3\hbar^5 x_j^2 y_i^2}{16 B_i^2} + \frac{\hbar^5 a_j x_j^2 y_i^2}{B_i^2} - \frac{3\hbar^5 a_j^2 x_j^2 y_i^2}{2 B_i^2} + \frac{2\hbar^6 x_j^3 y_i^3}{B_i^3} - \frac{10\hbar^6 a_j x_j^3 y_i^3}{3 B_i^3} - \frac{35\hbar^7 x_j^4 y_i^4}{16 B_i^4}\Big]$

(Alt) In[∘]:= **$k = 0; LogE[t1 = $\mathbb{E}_{\{\}\to\{1,2,3,4\}}\left[\hbar a_2 b_1 - \hbar a_4 b_3,\ \hbar x_2 y_1 - \frac{\hbar x_4 y_3}{B_3},\ 1\right]$]**

(Alt) Out[∘]= $\mathbb{F}_{\{\}\to\{1,2,3,4\}}\left[\hbar a_2 b_1 - \hbar a_4 b_3 + \hbar x_2 y_1 - \frac{\hbar x_4 y_3}{B_3}\right]$

(Alt) In[∘]:= **LogE[t2 = $\mathbb{E}_{\{1,2,3,4\}\to\{1,2\}}\left[a_2(\alpha_2+\alpha_4) + b_1\beta_1 + b_1\beta_3,\ y_1\eta_1 + y_1\eta_3 + x_2\xi_2 + \frac{x_2\xi_4}{\mathcal{A}_2},\ 1\right]$]**

(Alt) Out[∘]= $\mathbb{F}_{\{1,2,3,4\}\to\{1,2\}}\left[a_2\alpha_2 + a_2\alpha_4 + b_1\beta_1 + b_1\beta_3 + y_1\eta_1 + y_1\eta_3 + x_2\xi_2 + \frac{x_2\xi_4}{\mathcal{A}_2}\right]$

(Alt) In[∘]:= **t1 // t2**

(Alt) Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[0, 0, 1]$

(Alt) In[∘]:= **$k = 3; $\mathrm{P}_{i,j}$ // LogE**

(Alt) Out[∘]= $\mathbb{F}_{\{i,j\}\to\{\}}\left[\frac{\alpha_j\beta_i}{\hbar} + \frac{\eta_i\xi_j}{\hbar},\ \frac{\eta_i^2\xi_j^2}{4\hbar},\ \frac{1}{8}\eta_i^2\xi_j^2 + \frac{5\eta_i^3\xi_j^3}{36\hbar},\ \frac{1}{24}\hbar\,\eta_i^2\xi_j^2 + \frac{1}{6}\eta_i^3\xi_j^3 + \frac{5\eta_i^4\xi_j^4}{48\hbar}\right]$

(Alt) In[∘]:= **$k = 3; aS_i // LogE**

(Alt) Out[∘]= $\mathbb{F}_{\{i\}\to\{i\}}\Big[-a_i\alpha_i - x_i\mathcal{A}_i\xi_i,\ -\hbar a_i x_i\mathcal{A}_i\xi_i - \frac{1}{2}\hbar x_i^2\mathcal{A}_i^2\xi_i^2,$
$-\frac{1}{2}\hbar^2 a_i^2 x_i\mathcal{A}_i\xi_i + \frac{1}{4}\hbar^2 x_i^2\mathcal{A}_i^2\xi_i^2 - \hbar^2 a_i x_i^2\mathcal{A}_i^2\xi_i^2 - \frac{1}{2}\hbar^2 x_i^3\mathcal{A}_i^3\xi_i^3,\ -\frac{1}{6}\hbar^3 a_i^3 x_i\mathcal{A}_i\xi_i -$
$\frac{1}{12}\hbar^3 x_i^2\mathcal{A}_i^2\xi_i^2 + \frac{1}{2}\hbar^3 a_i x_i^2\mathcal{A}_i^2\xi_i^2 - \hbar^3 a_i^2 x_i^2\mathcal{A}_i^2\xi_i^2 + \frac{2}{3}\hbar^3 x_i^3\mathcal{A}_i^3\xi_i^3 - \frac{3}{2}\hbar^3 a_i x_i^3\mathcal{A}_i^3\xi_i^3 - \frac{2}{3}\hbar^3 x_i^4\mathcal{A}_i^4\xi_i^4\Big]$

(Alt) In[∘]:= **$k = 3; $\overline{\mathrm{aS}}_i$ // LogE**

(Alt) Out[∘]= $\mathbb{F}_{\{i\}\to\{i\}}\Big[-a_i\alpha_i - x_i\mathcal{A}_i\xi_i,\ \hbar x_i\mathcal{A}_i\xi_i - \hbar a_i x_i\mathcal{A}_i\xi_i - \frac{1}{2}\hbar x_i^2\mathcal{A}_i^2\xi_i^2,$
$-\frac{1}{2}\hbar^2 x_i\mathcal{A}_i\xi_i + \hbar^2 a_i x_i\mathcal{A}_i\xi_i - \frac{1}{2}\hbar^2 a_i^2 x_i\mathcal{A}_i\xi_i + \frac{5}{4}\hbar^2 x_i^2\mathcal{A}_i^2\xi_i^2 - \hbar^2 a_i x_i^2\mathcal{A}_i^2\xi_i^2 - \frac{1}{2}\hbar^2 x_i^3\mathcal{A}_i^3\xi_i^3,$
$\frac{1}{6}\hbar^3 x_i\mathcal{A}_i\xi_i - \frac{1}{2}\hbar^3 a_i x_i\mathcal{A}_i\xi_i + \frac{1}{2}\hbar^3 a_i^2 x_i\mathcal{A}_i\xi_i - \frac{1}{6}\hbar^3 a_i^3 x_i\mathcal{A}_i\xi_i - \frac{19}{12}\hbar^3 x_i^2\mathcal{A}_i^2\xi_i^2 +$
$\frac{5}{2}\hbar^3 a_i x_i^2\mathcal{A}_i^2\xi_i^2 - \hbar^3 a_i^2 x_i^2\mathcal{A}_i^2\xi_i^2 + \frac{13}{6}\hbar^3 x_i^3\mathcal{A}_i^3\xi_i^3 - \frac{3}{2}\hbar^3 a_i x_i^3\mathcal{A}_i^3\xi_i^3 - \frac{2}{3}\hbar^3 x_i^4\mathcal{A}_i^4\xi_i^4\Big]$

(Alt) In[∘]:= **\$k = 1; dS₁ // LogE**

(Alt) Out[∘]=
$$\mathbb{F}_{\{1\}\to\{1\}}\Big[-a_1\alpha_1-b_1\beta_1-\frac{y_1\mathcal{A}_1\eta_1}{B_1}-x_1\mathcal{A}_1\xi_1+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\eta_1\xi_1}{\hbar B_1},$$
$$\frac{\hbar y_1\mathcal{A}_1\eta_1}{B_1}-\frac{y_1\mathcal{A}_1\beta_1\eta_1}{B_1}-\frac{\hbar y_1^2\mathcal{A}_1^2\eta_1^2}{2B_1^2}-\hbar a_1x_1\mathcal{A}_1\xi_1-x_1\mathcal{A}_1\beta_1\xi_1+\frac{a_1\mathcal{A}_1\eta_1\xi_1}{B_1}-$$
$$\frac{\hbar x_1y_1\mathcal{A}_1^2\eta_1\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\mathcal{A}_1)\eta_1\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar B_1}+\frac{y_1\left(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2\right)\eta_1^2\xi_1}{2B_1^2}-$$
$$\frac{1}{2}\hbar x_1^2\mathcal{A}_1^2\xi_1^2+\frac{x_1\left(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2\right)\eta_1\xi_1^2}{2B_1}+\frac{\left(-3\mathcal{A}_1^2+4B_1\mathcal{A}_1^2-B_1^2\mathcal{A}_1^2\right)\eta_1^2\xi_1^2}{4\hbar B_1^2}\Big]$$

(Alt) In[∘]:= **\$k = 1; $\overline{\mathrm{dS}}_1$ // LogE**

(Alt) Out[∘]=
$$\mathbb{F}_{\{1\}\to\{1\}}\Big[-a_1\alpha_1-b_1\beta_1-\frac{y_1\mathcal{A}_1\eta_1}{B_1}-x_1\mathcal{A}_1\xi_1+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\eta_1\xi_1}{\hbar B_1},$$
$$-\frac{y_1\mathcal{A}_1\beta_1\eta_1}{B_1}-\frac{\hbar y_1^2\mathcal{A}_1^2\eta_1^2}{2B_1^2}+\hbar x_1\mathcal{A}_1\xi_1-\hbar a_1x_1\mathcal{A}_1\xi_1-x_1\mathcal{A}_1\beta_1\xi_1+\frac{a_1\mathcal{A}_1\eta_1\xi_1}{B_1}-$$
$$\frac{\hbar x_1y_1\mathcal{A}_1^2\eta_1\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\mathcal{A}_1)\eta_1\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar B_1}+\frac{y_1\left(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2\right)\eta_1^2\xi_1}{2B_1^2}-$$
$$\frac{1}{2}\hbar x_1^2\mathcal{A}_1^2\xi_1^2+\frac{x_1\left(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2\right)\eta_1\xi_1^2}{2B_1}+\frac{\left(-3\mathcal{A}_1^2+4B_1\mathcal{A}_1^2-B_1^2\mathcal{A}_1^2\right)\eta_1^2\xi_1^2}{4\hbar B_1^2}\Big]$$

(Alt) In[∘]:= **$\mathrm{dm}_{1,2\to3}$**

(Alt) Out[∘]=
$$\mathbb{E}_{\{1,2\}\to\{3\}}\Big[a_3\alpha_1+a_3\alpha_2+b_3\beta_1+b_3\beta_2,\ y_3\eta_1+\frac{y_3\eta_2}{\mathcal{A}_1}+\frac{x_3\xi_1}{\mathcal{A}_2}+\frac{(1-B_3)\eta_2\xi_1}{\hbar}+x_3\xi_2,$$
$$1+\Bigg(-\frac{y_3\beta_1\eta_2}{\mathcal{A}_1}-\frac{x_3\beta_2\xi_1}{\mathcal{A}_2}+a_3B_3\eta_2\xi_1+\frac{\hbar x_3y_3\eta_2\xi_1}{\mathcal{A}_1\mathcal{A}_2}+$$
$$\frac{(1-3B_3)y_3\eta_2^2\xi_1}{2\mathcal{A}_1}+\frac{(1-3B_3)x_3\eta_2\xi_1^2}{2\mathcal{A}_2}+\frac{\left(1-4B_3+3B_3^2\right)\eta_2^2\xi_1^2}{4\hbar}\Bigg)\epsilon+O[\epsilon]^2\Big]$$

(Alt) In[∘]:= **$\mathrm{dm}_{1,2\to3}$ // LogE**

(Alt) Out[∘]=
$$\mathbb{F}_{\{1,2\}\to\{3\}}\Big[a_3\alpha_1+a_3\alpha_2+b_3\beta_1+b_3\beta_2+y_3\eta_1+\frac{y_3\eta_2}{\mathcal{A}_1}+\frac{x_3\xi_1}{\mathcal{A}_2}+\frac{(1-B_3)\eta_2\xi_1}{\hbar}+x_3\xi_2,\ -\frac{y_3\beta_1\eta_2}{\mathcal{A}_1}-$$
$$\frac{x_3\beta_2\xi_1}{\mathcal{A}_2}+a_3B_3\eta_2\xi_1+\frac{\hbar x_3y_3\eta_2\xi_1}{\mathcal{A}_1\mathcal{A}_2}+\frac{(1-3B_3)y_3\eta_2^2\xi_1}{2\mathcal{A}_1}+\frac{(1-3B_3)x_3\eta_2\xi_1^2}{2\mathcal{A}_2}+\frac{\left(1-4B_3+3B_3^2\right)\eta_2^2\xi_1^2}{4\hbar}\Big]$$

(Alt) In[∘]:= **$\overline{\mathrm{dS}}_1$ // dS_1**

(Alt) Out[∘]=
$$\mathbb{E}_{\{1\}\to\{1\}}\left[a_1\alpha_1+b_1\beta_1,\ y_1\eta_1+x_1\xi_1,\ 1+O[\epsilon]^2\right]$$

(Alt) In[∘]:=
```
Timing[Block[{$k = 3},
  HL /@ {(dΔ1→1,2 // dΔ2→2,3) ≡ (dΔ1→1,3 // dΔ1→1,2), (dm1,2→1 // dm1,3→1) ≡ (dm2,3→2 // dm1,2→1)}]
]
```

(Alt) Out[∘]= {26.1406, {True, True}}

(Alt) In[∘]:=
```
Timing@Block[{$k = 1}, Z[Knot[8, 17]] // LogE]
```

KnotTheory: Loading precomputed data in PD4Knots`.

(Alt) Out[∘]=

$$\left\{47.5938, \mathbb{F}_{\{\}\to\{0\}}\left[\mathrm{Log}\left[-\frac{T^3}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}\right], \frac{-3\hbar+8T\hbar-8T^2\hbar+8T^4\hbar-8T^5\hbar+3T^6\hbar}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}+\frac{a\left(-6\hbar+16T\hbar-16T^2\hbar+16T^4\hbar-16T^5\hbar+6T^6\hbar\right)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}+\frac{x\,y\left(-6\hbar^2+10T\hbar^2-6T^2\hbar^2-6T^3\hbar^2+10T^4\hbar^2-6T^5\hbar^2\right)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}\right]\right\}$$

(Alt) In[∘]:= **Timing@Block[{$k = 2}, Z[Knot[8, 17]] // LogE]**

(Alt) Out[∘]=

$$\left\{2267.69, \mathbb{F}_{\{\}\to\{0\}}\left[\text{Log}\left[-\frac{T^3}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}\right],\right.\right.$$

$$\frac{-3\hbar+8T\hbar-8T^2\hbar+8T^4\hbar-8T^5\hbar+3T^6\hbar}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}+\frac{a\left(-6\hbar+16T\hbar-16T^2\hbar+16T^4\hbar-16T^5\hbar+6T^6\hbar\right)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}+$$

$$\frac{x\,y\left(-6\hbar^2+10T\hbar^2-6T^2\hbar^2-6T^3\hbar^2+10T^4\hbar^2-6T^5\hbar^2\right)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6},$$

$$\left(a\left(8T\hbar^2-64T^2\hbar^2+262T^3\hbar^2-608T^4\hbar^2+952T^5\hbar^2-1096T^6\hbar^2+952T^7\hbar^2-608T^8\hbar^2+262T^9\hbar^2-64T^{10}\hbar^2+8T^{11}\hbar^2\right)\right)\Big/$$

$$\left(1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}\right)+$$

$$\left(a^2\left(8T\hbar^2-64T^2\hbar^2+262T^3\hbar^2-608T^4\hbar^2+952T^5\hbar^2-1096T^6\hbar^2+952T^7\hbar^2-608T^8\hbar^2+262T^9\hbar^2-64T^{10}\hbar^2+8T^{11}\hbar^2\right)\right)\Big/$$

$$\left(1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}\right)+$$

$$\left(4T\hbar^2-50T^2\hbar^2+307T^3\hbar^2-1160T^4\hbar^2+3062T^5\hbar^2-6127T^6\hbar^2+9760T^7\hbar^2-12754T^8\hbar^2+13916T^9\hbar^2-12754T^{10}\hbar^2+9760T^{11}\hbar^2-6127T^{12}\hbar^2+3062T^{13}\hbar^2-1160T^{14}\hbar^2+307T^{15}\hbar^2-50T^{16}\hbar^2+4T^{17}\hbar^2\right)\Big/$$

$$\left(2-24T+144T^2-578T^3+1728T^4-4056T^5+7708T^6-12072T^7+15744T^8-17194T^9+15744T^{10}-12072T^{11}+7708T^{12}-4056T^{13}+1728T^{14}-578T^{15}+144T^{16}-24T^{17}+2T^{18}\right)+$$

$$\left(a\,x\,y\left(28T\hbar^3-168T^2\hbar^3+544T^3\hbar^3-1000T^4\hbar^3+1248T^5\hbar^3-1096T^6\hbar^3+656T^7\hbar^3-216T^8\hbar^3-20T^9\hbar^3+40T^{10}\hbar^3-12T^{11}\hbar^3\right)\right)\Big/$$

$$\left(1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}\right)+$$

$$\left(x\,y\left(-18\hbar^3+78T\hbar^3-146T^2\hbar^3+110T^3\hbar^3+78T^4\hbar^3-274T^5\hbar^3+274T^6\hbar^3-78T^7\hbar^3-110T^8\hbar^3+146T^9\hbar^3-78T^{10}\hbar^3+18T^{11}\hbar^3\right)\right)\Big/$$

$$\left(1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}\right)+$$

$$\left.\left.\frac{x^2y^2\left(3\hbar^4-37T^2\hbar^4+153T^3\hbar^4-261T^4\hbar^4+325T^5\hbar^4-261T^6\hbar^4+153T^7\hbar^4-37T^8\hbar^4+3T^{10}\hbar^4\right)}{1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}}\right]\right\}$$

(Alt) In[∘]:=
```
Timing@Block[{$k = 3}, Z[Knot[3, 1]] // LogE]
```

KnotTheory: Loading precomputed data in PD4Knots`.

(Alt) Out[∘]=

$$\Big\{1380.94, \mathbb{F}_{\{\}\to\{0\}}\Big[\mathrm{Log}\Big[\frac{T}{1-T+T^2}\Big], \frac{a\,(-2\hbar+2T^2\hbar)}{1-T+T^2} + \frac{-2\hbar+3T\hbar-2T^2\hbar+T^3\hbar}{1-2T+3T^2-2T^3+T^4} + \frac{x\,y\,(-2\hbar^2-2T\hbar^2)}{1-T+T^2},$$

$$\frac{a^2\,(2T\hbar^2-8T^2\hbar^2+2T^3\hbar^2)}{1-2T+3T^2-2T^3+T^4} + \frac{a\,(2T\hbar^2-14T^2\hbar^2+12T^3\hbar^2-6T^4\hbar^2+2T^5\hbar^2)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} +$$

$$\frac{T\hbar^2-11T^2\hbar^2+16T^3\hbar^2-12T^4\hbar^2+8T^5\hbar^2-3T^6\hbar^2+T^7\hbar^2}{2-8T+20T^2-32T^3+38T^4-32T^5+20T^6-8T^7+2T^8} + \frac{a\,x\,y\,(8T\hbar^3-8T^2\hbar^3-4T^3\hbar^3)}{1-2T+3T^2-2T^3+T^4} +$$

$$\frac{x\,y\,(-2\hbar^3-2T^2\hbar^3-6T^3\hbar^3+2T^5\hbar^3)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} + \frac{x^2\,y^2\,(\hbar^4+5T\hbar^4+T^2\hbar^4)}{1-2T+3T^2-2T^3+T^4},$$

$$\frac{a^3\,(-4T\hbar^3+28T^2\hbar^3-28T^4\hbar^3+4T^5\hbar^3)}{3-9T+18T^2-21T^3+18T^4-9T^5+3T^6} +$$

$$\frac{a^2\,(-2T\hbar^3+24T^2\hbar^3-12T^3\hbar^3-32T^4\hbar^3+20T^5\hbar^3-8T^6\hbar^3+2T^7\hbar^3)}{1-4T+10T^2-16T^3+19T^4-16T^5+10T^6-4T^7+T^8} +$$

$$\frac{a\,(-T\hbar^3+19T^2\hbar^3-19T^3\hbar^3-34T^4\hbar^3+40T^5\hbar^3-22T^6\hbar^3+11T^7\hbar^3-3T^8\hbar^3+T^9\hbar^3)}{1-5T+15T^2-30T^3+45T^4-51T^5+45T^6-30T^7+15T^8-5T^9+T^{10}} +$$

$$(-T\hbar^3+29T^2\hbar^3-43T^3\hbar^3-71T^4\hbar^3+131T^5\hbar^3-84T^6\hbar^3+53T^7\hbar^3-23T^8\hbar^3+11T^9\hbar^3-3T^{10}\hbar^3+T^{11}\hbar^3)\,/\,(6-36T+126T^2-300T^3+540T^4-756T^5+846T^6-756T^7+540T^8-300T^9+126T^{10}-36T^{11}+6T^{12}) + \frac{a^2\,x\,y\,(-8T\hbar^4+8T^2\hbar^4+36T^3\hbar^4-20T^4\hbar^4-4T^5\hbar^4)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} +$$

$$\frac{a\,x\,y\,(12T\hbar^4-16T^2\hbar^4+40T^3\hbar^4-16T^4\hbar^4-56T^5\hbar^4+8T^6\hbar^4+4T^7\hbar^4)}{1-4T+10T^2-16T^3+19T^4-16T^5+10T^6-4T^7+T^8} +$$

$$\frac{x\,y\,(-4\hbar^4+3T\hbar^4-6T^2\hbar^4-9T^3\hbar^4-15T^4\hbar^4-63T^5\hbar^4-9T^6\hbar^4+42T^7\hbar^4+3T^8\hbar^4-4T^9\hbar^4)}{3-15T+45T^2-90T^3+135T^4-153T^5+135T^6-90T^7+45T^8-15T^9+3T^{10}} +$$

$$\frac{a\,x^2\,y^2\,(-14T\hbar^5-6T^2\hbar^5+30T^3\hbar^5+4T^4\hbar^5)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} +$$

$$\frac{x^2\,y^2\,(2\hbar^5+23T\hbar^5-10T^2\hbar^5+11T^3\hbar^5+42T^4\hbar^5-29T^5\hbar^5-8T^6\hbar^5)}{1-4T+10T^2-16T^3+19T^4-16T^5+10T^6-4T^7+T^8} +$$

$$\frac{x^3\,y^3\,(-2\hbar^6-24T\hbar^6-24T^2\hbar^6-2T^3\hbar^6)}{3-9T+18T^2-21T^3+18T^4-9T^5+3T^6}\Big]\Big\}$$

(Alt) In[∘]:=
```
Timing@Block[{$k = 3}, Z[Knot[8, 17]] // LogE]
```