Pensieve header: Double Integration.

Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
PP_ := Identity;
<< "Engine-Speedy.m";
<< "Objects.m";
$k = 1;
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, Green, Pink]];
ℏ = γ = 1;
RI := RandomChoice[{-1, 1}] × RandomInteger[{1, 3}];
```

In[∘]:= $\overline{R}_{3,4}$

Out[∘]= $\mathbb{E}_{\{\}\to\{3,4\}}\left[-\hbar\, a_4\, b_3,\ -\frac{\hbar\, x_4\, y_3}{B_3},\ 1\right]$

In[∘]:= $P_{1,2}$

Out[∘]= $\mathbb{E}_{\{1,2\}\to\{\}}\left[\frac{\alpha_2\,\beta_1}{\hbar},\ \frac{\eta_1\,\xi_2}{\hbar},\ 1\right]$

In[∘]:= $\overline{R}_{3,4}$ // $P_{4,3}$

Out[∘]= $\mathbb{E}_{\{\}\to\{\}}[0, 0, 1]$

$\{$
int = $(\mathbb{E}_{\{i\}\to\{1,2\}}[\beta_i\, b_1 + \alpha_i\, a_2,\ \eta_i\, y_1 + \xi_i\, x_2,\ 1] * \overline{R}_{3,4})$ // $bm_{3,1\to1}$ // $am_{4,2\to2}$ // $P_{1,2}$,
$d\Delta_{i\to i,j}$ // int
$\}$ // Column

Out[∘]=
$\mathbb{E}_{\{i\}\to\{\}}\left[\frac{\alpha_i\,\beta_i}{2\hbar},\ \frac{\sqrt{\mathcal{A}_i}\ \eta_i\,\xi_i}{\hbar+\hbar\sqrt{\mathcal{A}_i}},\ \frac{\sqrt{\mathcal{A}_i}}{2+2\sqrt{\mathcal{A}_i}} + O[\epsilon]^1\right]$

$\mathbb{E}_{\{i\}\to\{j\}}\left[a_j\,\alpha_i + b_j\,\beta_i + \frac{\alpha_i\,\beta_i}{2\hbar},\ \frac{y_j\,\eta_i}{\sqrt{\mathcal{A}_i}} + x_j\,\xi_i + \frac{\sqrt{\mathcal{A}_i}\ \eta_i\,\xi_i}{\hbar+\hbar\sqrt{\mathcal{A}_i}},\ \frac{\sqrt{\mathcal{A}_i}}{2+2\sqrt{\mathcal{A}_i}} + O[\epsilon]^1\right]$

In[∘]:= $\mathbb{E}_{\{\}\to\{1\}}[\mu\, a_1\, b_1,\ \nu\, x_1\, y_1,\ 1]$ // $dm_{i,1\to i}$

Out[∘]=
$$\mathbb{E}_{\{i\}\to\{i\}}\Bigg[\mu\, a_i\, b_i + a_i\,\alpha_i + b_i\,\beta_i,\ \frac{\nu\, x_i\, y_i}{\mathcal{A}_i} + y_i\,\eta_i + \frac{\left(\nu - \nu\, B_i + \hbar\, B_i^{\mu/\hbar}\right) x_i\,\xi_i}{\hbar},$$
$$1 + \Bigg(-\frac{\nu\, x_i\, y_i\,\beta_i}{\mathcal{A}_i} + \gamma\,\mu\, B_i^{\mu/\hbar}\, x_i\,\xi_i + a_i\left(\nu\, B_i - \mu\, B_i^{\mu/\hbar}\right) x_i\,\xi_i + \frac{\left(\gamma\,\nu^2 - 3\,\gamma\,\nu^2\, B_i + 2\,\gamma\,\nu\,\hbar\, B_i^{\mu/\hbar}\right) x_i^2\, y_i\,\xi_i}{2\,\mathcal{A}_i} +$$
$$\frac{1}{4\hbar}\left(\gamma\,\nu^2 - 4\,\gamma\,\nu^2\, B_i + 3\,\gamma\,\nu^2\, B_i^2 - 6\,\gamma\,\nu\,\hbar\, B_i^{1+\frac{\mu}{\hbar}} + 2\,\gamma\,\nu\,\hbar\, B_i^{\mu/\hbar} + 4\,\gamma\,\mu\,\hbar\, B_i^{\frac{2\mu}{\hbar}}\right) x_i^2\,\xi_i^2\Bigg)\epsilon + O[\epsilon]^2\Bigg]$$

(171205) (Approx.) On $H^{*cop} \otimes H$ with $R = Id = \rho \otimes r$ (summed), $\int \phi \otimes x := \langle \phi\bar{\rho} \mid xr \rangle$ is an integral. $\frac{1}{2}$**Pf.** $x_1 \int \phi \otimes x_2 = x_1 \langle \phi\bar{\rho} \mid x_2 r \rangle = x_1 r^a r^b \langle \phi\bar{\rho}\bar{\rho}^a \rho^b \mid x_2 r \rangle \sim x_1 r_1 r^b \langle \phi\bar{\rho}\rho^b \mid x_2 r_2 \rangle \sim (xr)_1 r^b \langle \phi\bar{\rho} \mid (xr)_3 \rangle \langle \rho^b \mid (xr)_2 \rangle \sim (xr)_1 (\overline{xr})_2 \langle \phi\bar{\rho} \mid (xr)_3 \rangle = \langle \phi\bar{\rho} \mid xr \rangle = \int \phi \otimes x$. Verify! Attempt in Projects/SL2Portfolio2/DoubleIntegration.nb.

In[∘]:=
```
Block[{$k = 1}, Module[
  {RR = R_{3,4}, AM = am_{2,4→2}, BM = bm_{3,1→1}},
  {
   pint = Simplify /@ (
      𝔼_{{}→{1}}[μ a_1 b_1, ν x_1 y_1, 1] // dm_{i,1→i} //
        (𝔼_{{i}→{1,2}}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] RR) // BM // AM // P_{1,2}
     ),
   pΔint = Simplify /@ (
      𝔼_{{}→{1}}[μ a_1 b_1, ν x_1 y_1, 1] // dm_{i,1→i} // dΔ_{i→j,i} //
        (𝔼_{{i}→{1,2}}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] RR) // BM // AM // P_{1,2}
     ),
   HL[pint 𝔼_{{}→{j}}[0, 0, 1] ≡ pΔint]
  }]] // Column
```

$$\mathbb{E}_{\{i\}\to\{\}}\Big[-\frac{\alpha_i \beta_i}{\mu},\ -\frac{\mathcal{A}_i\left(\nu+\hbar\mathcal{A}_i-\nu\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)\eta_i\xi_i}{\nu\hbar},$$
$$\frac{\hbar^2\mathcal{A}_i^{1-\frac{\hbar}{\mu}}}{\mu\nu}-\frac{1}{4(\mu^2\nu^3)}\Big(\hbar\mathcal{A}_i^{1-\frac{2\hbar}{\mu}}\Big(-4\gamma\mu\nu\hbar^3\mathcal{A}_i+2\nu^2\hbar\mathcal{A}_i^{\frac{\hbar}{\mu}}\left(\gamma\hbar(-3\mu+2\hbar)-2(\mu-\hbar)\beta_i\right)+4\gamma\mu\nu\hbar^2\mathcal{A}_i^2\eta_i\xi_i+$$
$$4\gamma\mu\hbar^3\mathcal{A}_i^3\eta_i\xi_i-4\mu\nu^2\mathcal{A}_i^{\frac{\mu+\hbar}{\mu}}(\gamma\hbar-\beta_i)\eta_i\xi_i+4\nu^2\mathcal{A}_i^{1+\frac{2\hbar}{\mu}}\left(\gamma(3\mu-\hbar)\hbar-(\mu+\hbar)\beta_i\right)\eta_i\xi_i-$$
$$4\gamma\mu\nu^2\mathcal{A}_i^{\frac{2(\mu+\hbar)}{\mu}}\eta_i^2\xi_i^2+4\gamma\mu\nu\hbar\mathcal{A}_i^{3+\frac{\hbar}{\mu}}\eta_i^2\xi_i^2+4\gamma\mu\hbar^2\mathcal{A}_i^{4+\frac{\hbar}{\mu}}\eta_i^2\xi_i^2-12\gamma\mu\nu\hbar\mathcal{A}_i^{3+\frac{2\hbar}{\mu}}\eta_i^2\xi_i^2+$$
$$\gamma\nu^2(3\mu+4\hbar)\mathcal{A}_i^{2+\frac{3\hbar}{\mu}}\eta_i^2\xi_i^2+\mu\nu\mathcal{A}_i^{2+\frac{\hbar}{\mu}}\eta_i\xi_i\left(-4\gamma\hbar^2+8\hbar\beta_i+\gamma\nu\eta_i\xi_i\right)\Big)\Big)\epsilon+O[\epsilon]^2\Big]$$

Out[∘]=

$$\mathbb{E}_{\{i\}\to\{j\}}\Big[-\frac{\alpha_i \beta_i}{\mu},\ -\frac{\mathcal{A}_i\left(\nu+\hbar\mathcal{A}_i-\nu\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)\eta_i\xi_i}{\nu\hbar},$$
$$\frac{\hbar^2\mathcal{A}_i^{1-\frac{\hbar}{\mu}}}{\mu\nu}-\frac{1}{4(\mu^2\nu^3)}\Big(\hbar\mathcal{A}_i^{1-\frac{2\hbar}{\mu}}\Big(2\nu^2\hbar\mathcal{A}_i^{\frac{\hbar}{\mu}}\left(\gamma\hbar(-3\mu+2\hbar)-2(\mu-\hbar)\beta_i\right)+4\gamma\mu\nu\hbar^3\mathcal{A}_i(-1+y_j\eta_i)+$$
$$4\gamma\mu\nu\hbar^2\mathcal{A}_i^2\eta_i\xi_i+4\gamma\mu\hbar^3\mathcal{A}_i^3\eta_i\xi_i-4\mu\nu^2\mathcal{A}_i^{\frac{\mu+\hbar}{\mu}}(\gamma\hbar-\beta_i)\eta_i\xi_i+$$
$$4\nu^2\mathcal{A}_i^{1+\frac{2\hbar}{\mu}}\left(\gamma(3\mu-\hbar)\hbar-(\mu+\hbar)\beta_i\right)\eta_i\xi_i-4\gamma\mu\nu^2\mathcal{A}_i^{\frac{2(\mu+\hbar)}{\mu}}\eta_i^2\xi_i^2+4\gamma\mu\nu\hbar\mathcal{A}_i^{3+\frac{\hbar}{\mu}}\eta_i^2\xi_i^2+$$
$$4\gamma\mu\hbar^2\mathcal{A}_i^{4+\frac{\hbar}{\mu}}\eta_i^2\xi_i^2-12\gamma\mu\nu\hbar\mathcal{A}_i^{3+\frac{2\hbar}{\mu}}\eta_i^2\xi_i^2+\gamma\nu^2(3\mu+4\hbar)\mathcal{A}_i^{2+\frac{3\hbar}{\mu}}\eta_i^2\xi_i^2+4\gamma\mu\nu\hbar^2x_j$$
$$\left(\nu\hbar y_j+\mathcal{A}_i\left(\nu+\hbar\mathcal{A}_i-\nu\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)\xi_i\right)+\mu\nu\mathcal{A}_i^{2+\frac{\hbar}{\mu}}\eta_i\xi_i\left(-4\gamma\hbar^2+8\hbar\beta_i+\gamma\nu\eta_i\xi_i\right)\Big)\Big)\epsilon+O[\epsilon]^2\Big]$$

$$\frac{1}{\mu\nu^2}\mathcal{A}_i^{-\frac{2\hbar}{\mu}}\left(\gamma\epsilon\nu\hbar^4x_jy_j\mathcal{A}_i+\gamma\epsilon\hbar^4y_j\mathcal{A}_i^2\eta_i+\gamma\epsilon\nu\hbar^3x_j\mathcal{A}_i^2\xi_i+\gamma\epsilon\hbar^4x_j\mathcal{A}_i^3\xi_i-\gamma\epsilon\nu\hbar^3x_j\mathcal{A}_i^{2+\frac{\hbar}{\mu}}\xi_i\right)==0$$

```
In[•]:= Block[{$k = 1}, Table[
    pint = Simplify /@ (
        𝔼{}→{1}[μ a1 b1, ν x1 y1, 1] // dmi,1→i //
          (𝔼{i}→{1,2}[βi b1 + αi a2, ηi y1 + ξi x2, 1] RR) // BM // AM // P1,2
      );
    pΔint = Simplify /@ (
        𝔼{}→{1}[μ a1 b1, ν x1 y1, 1] // dmi,1→i // dΔi→j,i //
          (𝔼{i}→{1,2}[βi b1 + αi a2, ηi y1 + ξi x2, 1] RR) // BM // AM // P1,2
      );
    Echo@HL@TrueQ[pint 𝔼{}→{j}[0, 0, 1] ≡ pΔint],
    {RR, {(*R3,4,*)R3,4 // bS3 // bS3}},
    {AM, {am2,4→2, am4,2→2}}, {BM, {(*bm1,3→1,*)bm3,1→1}}
  ]]
```

QZip4 fail at {L,Q,P} =

$$\Bigg\{-\frac{\alpha_i\,\beta_i}{\mu},\ \frac{x_{n\$11979[2]}\,y_{n\$11979[1]}\left(\hbar\,\mathcal{A}_i+\nu\,\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)}{\mathcal{A}_i}+y_{n\$11979[1]}\,\eta_i+\frac{x_{n\$11979[2]}\left(\nu\,\mathcal{A}_i^{\frac{\hbar}{\mu}}-\nu\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+\hbar\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{1+\frac{\mu}{\hbar}}\right)\xi_i}{\hbar}+\frac{\eta_{n\$11979[1]}\,\xi_{n\$11979[2]}}{\hbar},\ -\frac{\hbar}{\mu}+\Bigg(\frac{1}{\mu^2\,\mathcal{A}_i}x_{n\$11979[2]}\,y_{n\$11979[1]}\left(-\gamma\,\mu\,\hbar^3\,\mathcal{A}_i-\gamma\,\mu\,\nu\,\hbar^2\,\mathcal{A}_i^{\frac{\hbar}{\mu}}+\gamma\,\nu\,\hbar^3\,\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)+\frac{1}{4\,\mu^2\,\mathcal{A}_i^2}x^2_{n\$11979[2]}\,y^2_{n\$11979[1]}\left(\gamma\,\mu\,\hbar^4\,\mathcal{A}_i^2-4\,\gamma\,\mu\,\nu^2\,\hbar^2\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+4\,\gamma\,\nu^2\,\hbar^3\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}\right)+\frac{x_{n\$11979[2]}\,y_{n\$11979[1]}\left(\mu\,\nu\,\hbar\,\mathcal{A}_i^{\frac{\hbar}{\mu}}-\nu\,\hbar^2\,\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)\beta_i}{\mu^2\,\mathcal{A}_i}+\frac{1}{\mu^2\,\mathcal{A}_i}x_{n\$11979[2]}\,y^2_{n\$11979[1]}\left(-\gamma\,\mu\,\nu\,\hbar^2\,\mathcal{A}_i^{\frac{\hbar}{\mu}}+\gamma\,\nu\,\hbar^3\,\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)\eta_i-\frac{\hbar^2\,y_{n\$11979[1]}\,\beta_i\,\eta_i}{\mu^2}+\frac{x_{n\$11979[2]}\left(-2\,\gamma\,\nu\,\hbar^2\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+\gamma\,\mu\,\hbar^2\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{1+\frac{\mu}{\hbar}}\right)\xi_i}{\mu^2}+\frac{1}{2\,\mu^2\,\mathcal{A}_i}x^2_{n\$11979[2]}\,y_{n\$11979[1]}\left(-3\,\gamma\,\mu\,\nu^2\,\hbar\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+2\,\gamma\,\nu^2\,\hbar^2\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+5\,\gamma\,\mu\,\nu^2\,\hbar\,\mathcal{A}_i^{\frac{3\hbar}{\mu}}-6\,\gamma\,\nu^2\,\hbar^2\,\mathcal{A}_i^{\frac{3\hbar}{\mu}}+2\,\gamma\,\nu\,\hbar^3\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{\mu/\hbar}\right)\xi_i+\frac{x_{n\$11979[2]}\left(\nu\,\hbar\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}-\mu\,\hbar\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{1+\frac{\mu}{\hbar}}\right)\beta_i\,\xi_i}{\mu^2}+\frac{1}{\mu^2}x_{n\$11979[2]}\,y_{n\$11979[1]}\left(-\gamma\,\mu\,\nu\,\hbar\,\mathcal{A}_i^{\frac{\hbar}{\mu}}+\gamma\,\nu\,\hbar^2\,\mathcal{A}_i^{\frac{\hbar}{\mu}}+\gamma\,\mu\,\nu\,\hbar\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}-2\,\gamma\,\nu\,\hbar^2\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+\gamma\,\hbar^3\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{1+\frac{\mu}{\hbar}}\right)\eta_i\,\xi_i+\frac{1}{4\,\mu^2}x^2_{n\$11979[2]}\left(-\gamma\,\mu\,\nu^2\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}+4\,\gamma\,\mu\,\nu^2\,\mathcal{A}_i^{\frac{3\hbar}{\mu}}-4\,\gamma\,\nu^2\,\hbar\,\mathcal{A}_i^{\frac{3\hbar}{\mu}}-3\,\gamma\,\mu\,\nu^2\,\mathcal{A}_i^{\frac{4\hbar}{\mu}}+8\,\gamma\,\nu^2\,\hbar\,\mathcal{A}_i^{\frac{4\hbar}{\mu}}+2\,\gamma\,\mu\,\nu\,\hbar\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{\mu/\hbar}-6\,\gamma\,\mu\,\nu\,\hbar\,\mathcal{A}_i^{\frac{3\hbar}{\mu}}\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{\mu/\hbar}-4\,\gamma\,\nu\,\hbar^2\,\mathcal{A}_i^{\frac{3\hbar}{\mu}}\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{\mu/\hbar}+4\,\gamma\,\mu\,\hbar^2\,\mathcal{A}_i^{\frac{2\hbar}{\mu}}\left(\mathcal{A}_i^{\frac{\hbar}{\mu}}\right)^{\frac{2\mu}{\hbar}}\right)\xi_i^2-\frac{\gamma\,\eta^2_{n\$11979[1]}\,\xi^2_{n\$11979[2]}}{4\,\mu}\Bigg)\epsilon+O[\epsilon]^2\Bigg\}$$

"

True

"

False

Out[ ]= {{{True}, {False}}}

In[ ]:= $(\overline{R}_{3,4}$ // $bS_3) \equiv (R_{3,4}$ // bS_3 // $bS_3)$

Out[ ]= True

```
In[◦]:= Block[{$k = 1}, Table[
          pint = Simplify /@ (
              𝔼_{}→{1}[μ a_1 b_1, ν x_1 y_1, 1] // dm_{i,1→i} //
                 (𝔼_{i}→{1,2}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] RR) // BM // AM // P_{1,2}
            );
          pΔint = Simplify /@ (
              𝔼_{}→{1}[μ a_1 b_1, ν x_1 y_1, 1] // dm_{i,1→i} // dΔ_{i→i,j} //
                 (𝔼_{i}→{1,2}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] RR) // BM // AM // P_{1,2}
            );
          Echo@HL@TrueQ[pint 𝔼_{}→{j}[0, 0, 1] ≡ pΔint],
          {RR, {R_{3,4}, R_{3,4} // bS_3 // bS_3, R_{3,4} // \bar{bS}_3 // \bar{bS}_3, R_{3,4} // aS_4 // aS_4, R_{3,4} // \bar{aS}_4 // \bar{aS}_4}},
          {AM, {am_{2,4→2}, am_{4,2→2}}}, {BM, {bm_{1,3→1}, bm_{3,1→1}}}
        ]]
```

```
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```

```
Out[◦]= {{{False, False}, {False, False}},
         {{False, False}, {False, False}}, {{False, False}, {False, False}},
         {{False, False}, {False, False}}, {{False, False}, {False, False}}}
```

In[∘]:=
```
Block[{$k = 1}, inp = Simplify /@ (𝔼_{}→{1}[RI a_1 b_1, RI x_1 y_1, 1] // dm_{i,1→i});
  Table[
   Echo@HL@TrueQ[
     𝔼_{}→{j}[0, 0, 1] (inp // (𝔼_{i}→{1,2}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] RR) // BM // AM // P_{1,2} ) ≡
      (inp // ΔΔ // (𝔼_{i}→{1,2}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] RR) // BM // AM // P_{1,2})],
   {ΔΔ, {dΔ_{i→i,j}, dΔ_{i→j,i}}}, {AM, {dm_{2,4→2}, dm_{4,2→2}}}, {BM, {dm_{1,3→1}, dm_{3,1→1}}},
   {RR, {R_{3,4}, R_{3,4} // dS_3 // dS_3, R_{3,4} // dS_4 // dS_4}}
  ]] // MatrixForm
```

» False

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Out[∘]//MatrixForm=

False	False	False	False	False	True
False	False	False	False	False	False
False	False	False	False	False	False
False	False	True	False	False	False