Pensieve header: The dequantizators.

Follows code in Projects/SL2Portfolio/SL2PortfolioProgram.nb.

Startup

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
Once[<< "../Profile/Profile.m";];
<< "Engine-Speedy.m";
<< "Objects.m";
$k = 3;
HL[ℰ_] := Style[ℰ, Background → Green];
```

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: June 2018. Original version: July 1994.
```

The Asymmetric Dequantizator

Following pensieve://People/VanDerVeen/Dequant1.pdf.

$$\mathbb{AD}\$f = \gamma\,\frac{\text{Cosh}\left[\hbar\left(a\epsilon+\frac{\gamma\epsilon}{2}-\frac{t}{2}\right)\right]-\text{Cosh}\left[\hbar\sqrt{\left(\frac{t-\gamma\epsilon}{2}\right)^2+\epsilon\omega}\,\right]}{\hbar\, e^{\hbar\left((a+\gamma)\epsilon-t/2\right)}\,\text{Sinh}\left[\frac{\gamma\epsilon\hbar}{2}\right]\left(a^2\epsilon+a\gamma\epsilon-at-\omega\right)};$$

$$\mathbb{AD}\$\omega = \gamma\,\text{CU}[y,x]+\epsilon\,\text{CU}[a,a]-\left(t-\gamma\epsilon\right)\text{CU}[a];$$

```
In[•]:= DeclareMorphism[𝔸𝔻, QU → CU, {a → a_CU, x → CU@x, y :> 𝕊_CU[SS[𝔸𝔻$f], a → a_CU, ω → 𝔸𝔻$ω] ** y_CU}]
```

The Symmetric Dequantizator

Following pensieve://People/VanDerVeen/Dequant1.pdf.

$$\mathbb{SD}\$g = \sqrt{\frac{2\gamma\left(\text{Cosh}\left[\frac{\hbar}{2}\sqrt{t^2+\gamma^2\epsilon^2+4\epsilon\varpi}\,\right]-\text{Cosh}\left[\frac{t-\epsilon\gamma-2\epsilon a}{2/\hbar}\right]\right)}{\text{Sinh}\left[\frac{\gamma\epsilon\hbar}{2}\right]\left(t\left(2a+\gamma\right)-2a\left(a+\gamma\right)\epsilon+2\varpi\right)\hbar}};$$

$$\mathbb{SD}\$f = \text{Simplify}\left[e^{\hbar\left(t/2-\epsilon a\right)}\left(\mathbb{SD}\$g\ /.\ \{a\to -a,\ t\to -t\}\right)\right];$$

$$\mathbb{SD}\$\varpi = \gamma\,\text{CU}[y,x]+\epsilon\,\text{CU}[a,a]-\left(t-\gamma\epsilon\right)\text{CU}[a]-t\gamma\,1_{CU}/2;$$

```
In[•]:= DeclareMorphism[𝕊𝔻, QU → CU, {a → a_CU,
   x :> 𝕊_CU[SS[𝕊𝔻$f], a → a_CU, ϖ → 𝕊𝔻$ϖ] ** x_CU,
   y :> 𝕊_CU[SS[𝕊𝔻$g], a → a_CU, ϖ → 𝕊𝔻$ϖ] ** y_CU }]
```

In[•]:=
```
AD$f = γ (Cosh[ħ (a ϵ + γ ϵ/2 - t/2)] - Cosh[ħ Sqrt[((t - γ ϵ)/2)^2 + ϵ ω]]) / (ħ E^(ħ ((a+γ) ϵ - t/2)) Sinh[γ ϵ ħ/2] (a^2 ϵ + a γ ϵ - a t - ω))
```

Out[•]=
$$\frac{e^{-\left(-\frac{t}{2}+(a+\gamma)\,\epsilon\right)\hbar}\,\gamma\left(\cosh\left[\left(-\frac{t}{2}+a\,\epsilon+\frac{\gamma\,\epsilon}{2}\right)\hbar\right]-\cosh\left[\sqrt{\frac{1}{4}\left(t-\gamma\,\epsilon\right)^2+\epsilon\,\omega}\;\hbar\right]\right)\operatorname{Csch}\left[\frac{\gamma\,\epsilon\,\hbar}{2}\right]}{\left(-a\,t+a^2\,\epsilon+a\,\gamma\,\epsilon-\omega\right)\hbar}$$

In[•]:=
```
Series[AD$f, {ħ, 0, 4}] // Simplify
```

Out[•]=
$$1+\frac{1}{2}\left(t-2\,(a+\gamma)\,\epsilon\right)\hbar+\frac{1}{12}\left(2\,t^2-7\,t\,(a+\gamma)\,\epsilon+\epsilon\left(7\,a^2\,\epsilon+13\,a\,\gamma\,\epsilon+6\,\gamma^2\,\epsilon+\omega\right)\right)\hbar^2+$$
$$\frac{1}{24}\left(t-2\,(a+\gamma)\,\epsilon\right)\left(t^2-3\,t\,(a+\gamma)\,\epsilon+\epsilon\left(3\,a^2\,\epsilon+5\,a\,\gamma\,\epsilon+2\,\gamma^2\,\epsilon+\omega\right)\right)\hbar^3+$$
$$\frac{1}{720}\left(6\,t^4-39\,t^3\,(a+\gamma)\,\epsilon+t^2\,\epsilon\left(101\,a^2\,\epsilon+192\,a\,\gamma\,\epsilon+91\,\gamma^2\,\epsilon+9\,\omega\right)-\right.$$
$$t\,\epsilon^2\left(124\,a^3\,\epsilon+337\,a^2\,\gamma\,\epsilon+302\,a\,\gamma^2\,\epsilon+89\,\gamma^3\,\epsilon+32\,a\,\omega+33\,\gamma\,\omega\right)+\epsilon^2\left(62\,a^4\,\epsilon^2+214\,a^3\,\gamma\,\epsilon^2+\right.$$
$$\left.\left.30\,\gamma^4\,\epsilon^2+29\,\gamma^2\,\epsilon\,\omega+2\,\omega^2+a^2\,\epsilon\left(271\,\gamma^2\,\epsilon+32\,\omega\right)+a\,\gamma\,\epsilon\left(149\,\gamma^2\,\epsilon+62\,\omega\right)\right)\right)\hbar^4+O[\hbar]^5$$

In[•]:=
```
(Series[AD$f, {ħ, 0, 4}] /. ϵ -> 0) // Simplify
```

Out[•]=
$$1+\frac{t\,\hbar}{2}+\frac{t^2\,\hbar^2}{6}+\frac{t^3\,\hbar^3}{24}+\frac{t^4\,\hbar^4}{120}+O[\hbar]^5$$

In[•]:=
```
Collect[Series[AD$f, {ħ, 0, 4}] // Normal, ϵ]
```

Out[•]=
$$1+\frac{t\,\hbar}{2}+\frac{t^2\,\hbar^2}{6}+\frac{t^3\,\hbar^3}{24}+\frac{t^4\,\hbar^4}{120}+\epsilon^4\left(\frac{31\,a^4\,\hbar^4}{360}+\frac{107}{360}\,a^3\,\gamma\,\hbar^4+\frac{271}{720}\,a^2\,\gamma^2\,\hbar^4+\frac{149}{720}\,a\,\gamma^3\,\hbar^4+\frac{\gamma^4\,\hbar^4}{24}\right)+$$
$$\epsilon\left(-a\,\hbar-\gamma\,\hbar-\frac{7}{12}\,a\,t\,\hbar^2-\frac{7}{12}\,t\,\gamma\,\hbar^2+\frac{\omega\,\hbar^2}{12}-\frac{5}{24}\,a\,t^2\,\hbar^3-\right.$$
$$\left.\frac{5}{24}\,t^2\,\gamma\,\hbar^3+\frac{1}{24}\,t\,\omega\,\hbar^3-\frac{13}{240}\,a\,t^3\,\hbar^4-\frac{13}{240}\,t^3\,\gamma\,\hbar^4+\frac{1}{80}\,t^2\,\omega\,\hbar^4\right)+$$
$$\epsilon^3\left(-\frac{1}{4}\,a^3\,\hbar^3-\frac{2}{3}\,a^2\,\gamma\,\hbar^3-\frac{7}{12}\,a\,\gamma^2\,\hbar^3-\frac{\gamma^3\,\hbar^3}{6}-\frac{31}{180}\,a^3\,t\,\hbar^4-\frac{337}{720}\,a^2\,t\,\gamma\,\hbar^4-\right.$$
$$\left.\frac{151}{360}\,a\,t\,\gamma^2\,\hbar^4-\frac{89}{720}\,t\,\gamma^3\,\hbar^4+\frac{2}{45}\,a^2\,\omega\,\hbar^4+\frac{31}{360}\,a\,\gamma\,\omega\,\hbar^4+\frac{29}{720}\,\gamma^2\,\omega\,\hbar^4\right)+$$
$$\epsilon^2\left(\frac{7\,a^2\,\hbar^2}{12}+\frac{13}{12}\,a\,\gamma\,\hbar^2+\frac{\gamma^2\,\hbar^2}{2}+\frac{3}{8}\,a^2\,t\,\hbar^3+\frac{17}{24}\,a\,t\,\gamma\,\hbar^3+\frac{1}{3}\,t\,\gamma^2\,\hbar^3-\frac{1}{12}\,a\,\omega\,\hbar^3-\frac{1}{12}\,\gamma\,\omega\,\hbar^3+\right.$$
$$\left.\frac{101}{720}\,a^2\,t^2\,\hbar^4+\frac{4}{15}\,a\,t^2\,\gamma\,\hbar^4+\frac{91}{720}\,t^2\,\gamma^2\,\hbar^4-\frac{2}{45}\,a\,t\,\omega\,\hbar^4-\frac{11}{240}\,t\,\gamma\,\omega\,\hbar^4+\frac{\omega^2\,\hbar^4}{360}\right)$$

In[•]:=
```
Series[Normal@Series[AD$f, {ħ, 0, 5}], {ϵ, 0, 2}]
```

Out[•]=
$$\frac{1}{720}\left(720+360\,t\,\hbar+120\,t^2\,\hbar^2+30\,t^3\,\hbar^3+6\,t^4\,\hbar^4+t^5\,\hbar^5\right)+$$
$$\left(-(a+\gamma)\,\hbar+\frac{1}{12}\left(-7\,a\,t-7\,t\,\gamma+\omega\right)\hbar^2+\frac{1}{24}\left(-5\,a\,t^2-5\,t^2\,\gamma+t\,\omega\right)\hbar^3+\right.$$
$$\left.\frac{1}{720}\left(-39\,a\,t^3-39\,t^3\,\gamma+9\,t^2\,\omega\right)\hbar^4+\frac{\left(-16\,a\,t^4-16\,t^4\,\gamma+4\,t^3\,\omega\right)\hbar^5}{1440}\right)\epsilon+$$
$$\left(\frac{1}{12}\left(7\,a^2+13\,a\,\gamma+6\,\gamma^2\right)\hbar^2+\frac{1}{24}\left(9\,a^2\,t+17\,a\,t\,\gamma+8\,t\,\gamma^2-2\,a\,\omega-2\,\gamma\,\omega\right)\hbar^3+\right.$$
$$\frac{1}{720}\left(101\,a^2\,t^2+192\,a\,t^2\,\gamma+91\,t^2\,\gamma^2-32\,a\,t\,\omega-33\,t\,\gamma\,\omega+2\,\omega^2\right)\hbar^4+$$
$$\left.\frac{\left(54\,a^2\,t^3+103\,a\,t^3\,\gamma+49\,t^3\,\gamma^2-20\,a\,t^2\,\omega-21\,t^2\,\gamma\,\omega+2\,t\,\omega^2\right)\hbar^5}{1440}\right)\epsilon^2+O[\epsilon]^3$$

In[•]:=
```
Series[AD$f, {a, 0, 1}] // Simplify
```

Out[•]=
$$-\frac{1}{\omega\,\hbar}e^{\frac{1}{2}(t-2\gamma\epsilon)\hbar}\gamma\left(\text{Cosh}\left[\frac{1}{2}\left(t-\gamma\epsilon\right)\hbar\right]-\text{Cosh}\left[\sqrt{\frac{1}{4}\left(t-\gamma\epsilon\right)^2+\epsilon\,\omega}\;\hbar\right]\right)\text{Csch}\left[\frac{\gamma\epsilon\hbar}{2}\right]-$$
$$\frac{1}{\omega^2\,\hbar}e^{\frac{1}{2}(t-2\gamma\epsilon)\hbar}\gamma\,\text{Csch}\left[\frac{\gamma\epsilon\hbar}{2}\right]\left(-\left(t-\gamma\epsilon+\epsilon\,\omega\,\hbar\right)\text{Cosh}\left[\frac{1}{2}\left(t-\gamma\epsilon\right)\hbar\right]+\right.$$
$$\left.\left(t-\gamma\epsilon+\epsilon\,\omega\,\hbar\right)\text{Cosh}\left[\sqrt{\frac{1}{4}\left(t-\gamma\epsilon\right)^2+\epsilon\,\omega}\;\hbar\right]-\epsilon\,\omega\,\hbar\,\text{Sinh}\left[\frac{1}{2}\left(t-\gamma\epsilon\right)\hbar\right]\right)a+O[a]^2$$

In[•]:=
```
Series[AD$f, {ω, 0, 1}] // Simplify
```

Out[•]=
$$\frac{1}{a\left(t-(a+\gamma)\,\epsilon\right)\hbar}$$
$$e^{\frac{1}{2}(t-2(a+\gamma)\epsilon)\hbar}\gamma\left(\text{Cosh}\left[\frac{1}{2}\sqrt{\left(t-\gamma\epsilon\right)^2}\;\hbar\right]-\text{Cosh}\left[\frac{1}{2}\left(t-\left(2\,a+\gamma\right)\epsilon\right)\hbar\right]\right)\text{Csch}\left[\frac{\gamma\epsilon\hbar}{2}\right]+$$
$$\left(e^{\frac{1}{2}(t-2(a+\gamma)\epsilon)\hbar}\gamma\,\text{Csch}\left[\frac{\gamma\epsilon\hbar}{2}\right]\left(-\text{Cosh}\left[\frac{1}{2}\sqrt{\left(t-\gamma\epsilon\right)^2}\;\hbar\right]+\text{Cosh}\left[\frac{1}{2}\left(t-\left(2\,a+\gamma\right)\epsilon\right)\hbar\right]+\right.\right.$$
$$\left.\left.\frac{a\epsilon\left(t-(a+\gamma)\,\epsilon\right)\hbar\,\text{Sinh}\left[\frac{1}{2}\sqrt{\left(t-\gamma\epsilon\right)^2}\;\hbar\right]}{\sqrt{\left(t-\gamma\epsilon\right)^2}}\right)\omega\right)\Big/\left(a^2\left(t-(a+\gamma)\,\epsilon\right)^2\hbar\right)+O[\omega]^2$$

In[•]:=
```
Simplify[Series[AD$f, {ϵ, 0, 2}], Assumptions → {t > 0}]
```

Out[•]=
$$\frac{-1+e^{t\hbar}}{t\hbar}-\frac{e^{\frac{t\hbar}{2}}\left(t\left(a\,t+t\,\gamma-\omega\right)\hbar\,\text{Cosh}\left[\frac{t\hbar}{2}\right]+2\left(-t\,\gamma+\omega+t^2\,\gamma\,\hbar+a\,t\left(-1+t\,\hbar\right)\right)\text{Sinh}\left[\frac{t\hbar}{2}\right]\right)\epsilon}{t^3\hbar}+$$
$$\frac{1}{24\,t^5\left(a\,t+\omega\right)^2\hbar}e^{\frac{t\hbar}{2}}\left(-2\,t^4\,\gamma^2\left(a\,t+\omega\right)^2\hbar^2\,\text{Sinh}\left[\frac{t\hbar}{2}\right]+\right.$$
$$24\,t^4\,(a+\gamma)^2\left(\omega^2\hbar^2+2\,a\,\omega\,\hbar\left(-1+t\,\hbar\right)+a^2\left(2-2\,t\,\hbar+t^2\hbar^2\right)\right)\text{Sinh}\left[\frac{t\hbar}{2}\right]+6\,t^2\,(a+\gamma)$$
$$\left(\omega\hbar+a\left(-1+t\,\hbar\right)\right)\left(4\,t\left(a\,t+t\,\gamma-\omega\right)\left(a\,t+\omega\right)\hbar\,\text{Cosh}\left[\frac{t\hbar}{2}\right]+8\,\omega\left(-t\,\gamma+\omega\right)\text{Sinh}\left[\frac{t\hbar}{2}\right]\right)-$$
$$\left(a\,t+\omega\right)\left(24\,t\left(t\,\gamma-2\,\omega\right)\left(t\,\gamma-\omega\right)\omega\,\hbar\,\text{Cosh}\left[\frac{t\hbar}{2}\right]-48\left(t\,\gamma-2\,\omega\right)\left(t\,\gamma-\omega\right)\omega\,\text{Sinh}\left[\frac{t\hbar}{2}\right]-\right.$$
$$\left.\left.t^5\left(2\,a+\gamma\right)^3\hbar^2\,\text{Sinh}\left[\frac{t\hbar}{2}\right]+t^2\left(t\,\gamma-2\,\omega\right)^3\hbar^2\,\text{Sinh}\left[\frac{t\hbar}{2}\right]\right)\right)\epsilon^2+O[\epsilon]^3$$

In[•]:=
```
Limit[AD$f, ϵ → 0]
```

Out[•]=
```
$Aborted
```

In[•]:=
```
1 + e^(t ħ) - 2 e^(t ħ/2) Cosh[t ħ/2] // TrigExpand
```

Out[•]=
```
0
```

In[•]:=
```
1/(24 t^5 (a t + ω)^2 ħ) e^(t ħ/2) (-2 t^4 γ^2 (a t + ω)^2 ħ^2 Sinh[t ħ/2] +
    24 t^4 (a + γ)^2 (ω^2 ħ^2 + 2 a ω ħ (-1 + t ħ) + a^2 (2 - 2 t ħ + t^2 ħ^2)) Sinh[t ħ/2] + 6 t^2 (a + γ)
     (ω ħ + a (-1 + t ħ)) (4 t (a t + t γ - ω) (a t + ω) ħ Cosh[t ħ/2] + 8 ω (-t γ + ω) Sinh[t ħ/2]) -
    (a t + ω) (24 t (t γ - 2 ω) (t γ - ω) ω ħ Cosh[t ħ/2] - 48 (t γ - 2 ω) (t γ - ω) ω Sinh[t ħ/2] -
      t^5 (2 a + γ)^3 ħ^2 Sinh[t ħ/2] + t^2 (t γ - 2 ω)^3 ħ^2 Sinh[t ħ/2])) ϵ^2 // TrigExpand // Together
```

Out[•]=
$$\frac{1}{12\,t^5\,\hbar}$$
$$\left(-12\,a^2\,t^2\,\epsilon^2+12\,a^2\,e^{t\hbar}\,t^2\,\epsilon^2-24\,a\,t^2\,\gamma\,\epsilon^2+24\,a\,e^{t\hbar}\,t^2\,\gamma\,\epsilon^2-12\,t^2\,\gamma^2\,\epsilon^2+12\,e^{t\hbar}\,t^2\,\gamma^2\,\epsilon^2+24\,a\,t\,\epsilon^2\,\omega-\right.$$
$$24\,a\,e^{t\hbar}\,t\,\epsilon^2\,\omega+36\,t\,\gamma\,\epsilon^2\,\omega-36\,e^{t\hbar}\,t\,\gamma\,\epsilon^2\,\omega-24\,\epsilon^2\,\omega^2+24\,e^{t\hbar}\,\epsilon^2\,\omega^2+6\,a^2\,t^3\,\epsilon^2\,\hbar-18\,a^2\,e^{t\hbar}\,t^3\,\epsilon^2\,\hbar+$$
$$12\,a\,t^3\,\gamma\,\epsilon^2\,\hbar-36\,a\,e^{t\hbar}\,t^3\,\gamma\,\epsilon^2\,\hbar+6\,t^3\,\gamma^2\,\epsilon^2\,\hbar-18\,e^{t\hbar}\,t^3\,\gamma^2\,\epsilon^2\,\hbar+24\,a\,e^{t\hbar}\,t^2\,\epsilon^2\,\omega\,\hbar+$$
$$6\,t^2\,\gamma\,\epsilon^2\,\omega\,\hbar+30\,e^{t\hbar}\,t^2\,\gamma\,\epsilon^2\,\omega\,\hbar-12\,t\,\epsilon^2\,\omega^2\,\hbar-12\,e^{t\hbar}\,t\,\epsilon^2\,\omega^2\,\hbar-2\,a^2\,t^4\,\epsilon^2\,\hbar^2+14\,a^2\,e^{t\hbar}\,t^4\,\epsilon^2\,\hbar^2-$$
$$3\,a\,t^4\,\gamma\,\epsilon^2\,\hbar^2+27\,a\,e^{t\hbar}\,t^4\,\gamma\,\epsilon^2\,\hbar^2-t^4\,\gamma^2\,\epsilon^2\,\hbar^2+13\,e^{t\hbar}\,t^4\,\gamma^2\,\epsilon^2\,\hbar^2-4\,a\,t^3\,\epsilon^2\,\omega\,\hbar^2-$$
$$\left.8\,a\,e^{t\hbar}\,t^3\,\epsilon^2\,\omega\,\hbar^2-3\,t^3\,\gamma\,\epsilon^2\,\omega\,\hbar^2-9\,e^{t\hbar}\,t^3\,\gamma\,\epsilon^2\,\omega\,\hbar^2-2\,t^2\,\epsilon^2\,\omega^2\,\hbar^2+2\,e^{t\hbar}\,t^2\,\epsilon^2\,\omega^2\,\hbar^2\right)$$

In[•]:=
```
AD$ω = γ CU[y, x] + ϵ CU[a, a] - (t - γ ϵ) CU[a];
```

In[•]:=
```
Exp_{cm,1,0}[γ y1 x1 + ϵ a1^2 - (t1 - γ ϵ) a1]
```

Out[•]=
$$\mathbb{E}\left[-a_1\,t_1,\ \gamma\,x_1\,y_1,\ 1\right]$$

In[•]:= $\mathbf{Exp_{cm,1,1}}\left[\gamma\, y_1\, x_1 + \epsilon\, a_1^2 - \left(t_1 - \gamma\, \epsilon\right) a_1\right]$

Out[•]= $Aborted