

In[*]:= **CS₁**[**E**{ \rightarrow {1}}][**0**, **0**, **y₁**]

Out[*]:= **E**{ \rightarrow {1}}[**0**, **0**, **-y₁** + **O**[ϵ]⁴]

In[*]:= **{m**, **i**, **k}**

Out[*]:= **{m**, **i**, **k}**

In[*]:= **CS₁**[**E**{ \rightarrow {1}}][**0**, **0**, **y₁**]

Out[*]:= **E**{ \rightarrow {1}}[**0**, **0**, **-y₁** + **O**[ϵ]⁴]

In[*]:= **m = cm**

Out[*]:= **cm**

In[*]:= **CS₁**[**E**{ \rightarrow {1}}][**0**, **0**, **y₁**]

Out[*]:= **E**{ \rightarrow {1}}[**0**, **0**, **-y₁** + **O**[ϵ]⁴]

In[*]:= **i = 1**

Out[*]:= **1**

In[*]:= **CS₁**[**E**{ \rightarrow {1}}][**0**, **0**, **y₁**]

Out[*]:= **E**{ \rightarrow {1}}[**0**, **0**, **-y₁** + **O**[ϵ]⁴]

In[*]:= **{m**, **i**, **k} = {cm**, **1**, **1}**

Out[*]:= **{cm**, **1**, **1}**

In[*]:= **{ λ** , **P}** = **{ η** , **Last@CS₁**[**E**{ \rightarrow {1}}][**0**, **0**, **y₁**]]

Out[*]:= **{ η** , **-y₁** + **O**[ϵ]⁴}

In[*]:= **P0 = Normal@P** /. **$\epsilon \rightarrow 0$**

Out[*]:= **-y₁**

In[*]:= **φ s = Flatten@Table**[**φ _{j₁,j₂,j₃,j₄}[λ], **{j₂**, **0**, **k}**,
{j₃, **0**, **k - j₂**}, **{j₁**, **0**, **2 k + 1 - j₂ - j₃**}, **{j₄**, **0**, **2 k + 1 - j₃ - j₂ - j₁**]]**

Out[*]:= **{ $\varphi_{0,0,0,0}[\eta]$** , **$\varphi_{0,0,0,1}[\eta]$** , **$\varphi_{0,0,0,2}[\eta]$** , **$\varphi_{0,0,0,3}[\eta]$** , **$\varphi_{1,0,0,0}[\eta]$** , **$\varphi_{1,0,0,1}[\eta]$** , **$\varphi_{1,0,0,2}[\eta]$** , **$\varphi_{2,0,0,0}[\eta]$** ,
 $\varphi_{2,0,0,1}[\eta]$, **$\varphi_{3,0,0,0}[\eta]$** , **$\varphi_{0,0,1,0}[\eta]$** , **$\varphi_{0,0,1,1}[\eta]$** , **$\varphi_{0,0,1,2}[\eta]$** , **$\varphi_{1,0,1,0}[\eta]$** , **$\varphi_{1,0,1,1}[\eta]$** ,
 $\varphi_{2,0,1,0}[\eta]$, **$\varphi_{0,1,0,0}[\eta]$** , **$\varphi_{0,1,0,1}[\eta]$** , **$\varphi_{0,1,0,2}[\eta]$** , **$\varphi_{1,1,0,0}[\eta]$** , **$\varphi_{1,1,0,1}[\eta]$** , **$\varphi_{2,1,0,0}[\eta]$** }

In[*]:= **Length**[**φ s**]

Out[*]:= **22**

In[*]:= **F = Normal@Last@Exp_{m,i,k-1}**[**λ** , **P**] + **ϵ^k φ s**. (**φ s** /. **$\varphi_{js_}$** [λ] **\Rightarrow Times@@{**y_i**, **b_i**, **a_i**, **x_i**}^{js})**

Out[*]:= **1 + ϵ** (**$\varphi_{0,0,0,0}[\eta]$** + **x₁ $\varphi_{0,0,0,1}[\eta]$** + **x₁² $\varphi_{0,0,0,2}[\eta]$** + **x₁³ $\varphi_{0,0,0,3}[\eta]$** +
a₁ $\varphi_{0,0,1,0}[\eta]$ + **a₁ x₁ $\varphi_{0,0,1,1}[\eta]$** + **a₁ x₁² $\varphi_{0,0,1,2}[\eta]$** + **b₁ $\varphi_{0,1,0,0}[\eta]$** + **b₁ x₁ $\varphi_{0,1,0,1}[\eta]$** +
b₁ x₁² $\varphi_{0,1,0,2}[\eta]$ + **y₁ $\varphi_{1,0,0,0}[\eta]$** + **x₁ y₁ $\varphi_{1,0,0,1}[\eta]$** + **x₁² y₁ $\varphi_{1,0,0,2}[\eta]$** +
a₁ y₁ $\varphi_{1,0,1,0}[\eta]$ + **a₁ x₁ y₁ $\varphi_{1,0,1,1}[\eta]$** + **b₁ y₁ $\varphi_{1,1,0,0}[\eta]$** + **b₁ x₁ y₁ $\varphi_{1,1,0,1}[\eta]$** +
y₁² $\varphi_{2,0,0,0}[\eta]$ + **x₁ y₁² $\varphi_{2,0,0,1}[\eta]$** + **a₁ y₁² $\varphi_{2,0,1,0}[\eta]$** + **b₁ y₁² $\varphi_{2,1,0,0}[\eta]$** + **y₁³ $\varphi_{3,0,0,0}[\eta]$**)

In[]:= $(\partial_\lambda F) + P0 F - rhs$

Out[]:= $y_1 -$

$$\begin{aligned} & \in \left(-\gamma b_1 \varphi_{0,0,0,1}[\eta] - 2\gamma b_1 x_1 \varphi_{0,0,0,2}[\eta] - 3\gamma b_1 x_1^2 \varphi_{0,0,0,3}[\eta] - x_1^3 y_1 \varphi_{0,0,0,3}[\eta] - a_1 y_1 \varphi_{0,0,1,0}[\eta] + \right. \\ & y_1 (-\varphi_{0,0,0,0}[\eta] + \gamma \varphi_{0,0,1,0}[\eta]) - \gamma a_1 b_1 \varphi_{0,0,1,1}[\eta] - a_1 x_1 y_1 \varphi_{0,0,1,1}[\eta] + \\ & x_1 y_1 (-\varphi_{0,0,0,1}[\eta] + \gamma \varphi_{0,0,1,1}[\eta]) - 2\gamma a_1 b_1 x_1 \varphi_{0,0,1,2}[\eta] - a_1 x_1^2 y_1 \varphi_{0,0,1,2}[\eta] + \\ & x_1^2 y_1 (-\varphi_{0,0,0,2}[\eta] + \gamma \varphi_{0,0,1,2}[\eta]) - \gamma b_1^2 \varphi_{0,1,0,1}[\eta] - 2\gamma b_1^2 x_1 \varphi_{0,1,0,2}[\eta] - \\ & b_1 x_1^2 y_1 \varphi_{0,1,0,2}[\eta] + b_1 y_1 (-\varphi_{0,1,0,0}[\eta] - \gamma \varphi_{1,0,0,1}[\eta]) - x_1^2 y_1^2 \varphi_{1,0,0,2}[\eta] + \\ & b_1 x_1 y_1 (-\varphi_{0,1,0,1}[\eta] - 2\gamma \varphi_{1,0,0,2}[\eta]) - a_1 y_1^2 \varphi_{1,0,1,0}[\eta] + y_1^2 (-\varphi_{1,0,0,0}[\eta] + \gamma \varphi_{1,0,1,0}[\eta]) - \\ & \gamma a_1 b_1 y_1 \varphi_{1,0,1,1}[\eta] - a_1 x_1 y_1^2 \varphi_{1,0,1,1}[\eta] + x_1 y_1^2 (-\varphi_{1,0,0,1}[\eta] + \gamma \varphi_{1,0,1,1}[\eta]) - \\ & \gamma b_1^2 y_1 \varphi_{1,1,0,1}[\eta] - b_1 x_1 y_1^2 \varphi_{1,1,0,1}[\eta] - x_1 y_1^3 \varphi_{2,0,0,1}[\eta] + b_1 y_1^2 (-\varphi_{1,1,0,0}[\eta] - \gamma \varphi_{2,0,0,1}[\eta]) - \\ & a_1 y_1^3 \varphi_{2,0,1,0}[\eta] + y_1^3 (-\varphi_{2,0,0,0}[\eta] + \gamma \varphi_{2,0,1,0}[\eta]) - b_1 y_1^3 \varphi_{2,1,0,0}[\eta] - y_1^4 \varphi_{3,0,0,0}[\eta] \left. \right) - \\ & y_1 \left(1 + \in \left(\varphi_{0,0,0,0}[\eta] + x_1 \varphi_{0,0,0,1}[\eta] + x_1^2 \varphi_{0,0,0,2}[\eta] + x_1^3 \varphi_{0,0,0,3}[\eta] + a_1 \varphi_{0,0,1,0}[\eta] + \right. \right. \\ & a_1 x_1 \varphi_{0,0,1,1}[\eta] + a_1 x_1^2 \varphi_{0,0,1,2}[\eta] + b_1 \varphi_{0,1,0,0}[\eta] + b_1 x_1 \varphi_{0,1,0,1}[\eta] + \\ & b_1 x_1^2 \varphi_{0,1,0,2}[\eta] + y_1 \varphi_{1,0,0,0}[\eta] + x_1 y_1 \varphi_{1,0,0,1}[\eta] + x_1^2 y_1 \varphi_{1,0,0,2}[\eta] + a_1 y_1 \varphi_{1,0,1,0}[\eta] + \\ & a_1 x_1 y_1 \varphi_{1,0,1,1}[\eta] + b_1 y_1 \varphi_{1,1,0,0}[\eta] + b_1 x_1 y_1 \varphi_{1,1,0,1}[\eta] + y_1^2 \varphi_{2,0,0,0}[\eta] + \\ & x_1 y_1^2 \varphi_{2,0,0,1}[\eta] + a_1 y_1^2 \varphi_{2,0,1,0}[\eta] + b_1 y_1^2 \varphi_{2,1,0,0}[\eta] + y_1^3 \varphi_{3,0,0,0}[\eta] \left. \right) \left. \right) + \\ & \in \left(\varphi_{0,0,0,0'}[\eta] + x_1 \varphi_{0,0,0,1'}[\eta] + x_1^2 \varphi_{0,0,0,2'}[\eta] + x_1^3 \varphi_{0,0,0,3'}[\eta] + a_1 \varphi_{0,0,1,0'}[\eta] + \right. \\ & a_1 x_1 \varphi_{0,0,1,1'}[\eta] + a_1 x_1^2 \varphi_{0,0,1,2'}[\eta] + b_1 \varphi_{0,1,0,0'}[\eta] + b_1 x_1 \varphi_{0,1,0,1'}[\eta] + \\ & b_1 x_1^2 \varphi_{0,1,0,2'}[\eta] + y_1 \varphi_{1,0,0,0'}[\eta] + x_1 y_1 \varphi_{1,0,0,1'}[\eta] + x_1^2 y_1 \varphi_{1,0,0,2'}[\eta] + \\ & a_1 y_1 \varphi_{1,0,1,0'}[\eta] + a_1 x_1 y_1 \varphi_{1,0,1,1'}[\eta] + b_1 y_1 \varphi_{1,1,0,0'}[\eta] + b_1 x_1 y_1 \varphi_{1,1,0,1'}[\eta] + \\ & y_1^2 \varphi_{2,0,0,0'}[\eta] + x_1 y_1^2 \varphi_{2,0,0,1'}[\eta] + a_1 y_1^2 \varphi_{2,0,1,0'}[\eta] + b_1 y_1^2 \varphi_{2,1,0,0'}[\eta] + y_1^3 \varphi_{3,0,0,0'}[\eta] \left. \right) \end{aligned}$$

In[]:= **CoefficientRules**[($\partial_\lambda F$) + P0 F - rhs, { y_i , b_i , a_i , x_i }]

Out[]:= { {3, 0, 0, 0} → $-\gamma \in \varphi_{2,0,1,0}[\eta] + \in \varphi_{3,0,0,0'}[\eta]$, {2, 1, 0, 0} → $\gamma \in \varphi_{2,0,0,1}[\eta] + \in \varphi_{2,1,0,0'}[\eta]$,
 {2, 0, 1, 0} → $\in \varphi_{2,0,1,0'}[\eta]$, {2, 0, 0, 1} → $-\gamma \in \varphi_{1,0,1,1}[\eta] + \in \varphi_{2,0,0,1'}[\eta]$,
 {2, 0, 0, 0} → $-\gamma \in \varphi_{1,0,1,0}[\eta] + \in \varphi_{2,0,0,0'}[\eta]$, {1, 2, 0, 0} → $\gamma \in \varphi_{1,1,0,1}[\eta]$,
 {1, 1, 1, 0} → $\gamma \in \varphi_{1,0,1,1}[\eta]$, {1, 1, 0, 1} → $2\gamma \in \varphi_{1,0,0,2}[\eta] + \in \varphi_{1,1,0,1'}[\eta]$,
 {1, 1, 0, 0} → $\gamma \in \varphi_{1,0,0,1}[\eta] + \in \varphi_{1,1,0,0'}[\eta]$, {1, 0, 1, 1} → $\in \varphi_{1,0,1,1'}[\eta]$,
 {1, 0, 1, 0} → $\in \varphi_{1,0,1,0'}[\eta]$, {1, 0, 0, 2} → $-\gamma \in \varphi_{0,0,1,2}[\eta] + \in \varphi_{1,0,0,2'}[\eta]$,
 {1, 0, 0, 1} → $-\gamma \in \varphi_{0,0,1,1}[\eta] + \in \varphi_{1,0,0,1'}[\eta]$, {1, 0, 0, 0} → $-\gamma \in \varphi_{0,0,1,0}[\eta] + \in \varphi_{1,0,0,0'}[\eta]$,
 {0, 2, 0, 1} → $2\gamma \in \varphi_{0,1,0,2}[\eta]$, {0, 2, 0, 0} → $\gamma \in \varphi_{0,1,0,1}[\eta]$,
 {0, 1, 1, 1} → $2\gamma \in \varphi_{0,0,1,2}[\eta]$, {0, 1, 1, 0} → $\gamma \in \varphi_{0,0,1,1}[\eta]$,
 {0, 1, 0, 2} → $3\gamma \in \varphi_{0,0,0,3}[\eta] + \in \varphi_{0,1,0,2'}[\eta]$, {0, 1, 0, 1} → $2\gamma \in \varphi_{0,0,0,2}[\eta] + \in \varphi_{0,1,0,1'}[\eta]$,
 {0, 1, 0, 0} → $\gamma \in \varphi_{0,0,0,1}[\eta] + \in \varphi_{0,1,0,0'}[\eta]$, {0, 0, 1, 2} → $\in \varphi_{0,0,1,2'}[\eta]$,
 {0, 0, 1, 1} → $\in \varphi_{0,0,1,1'}[\eta]$, {0, 0, 1, 0} → $\in \varphi_{0,0,1,0'}[\eta]$, {0, 0, 0, 3} → $\in \varphi_{0,0,0,3'}[\eta]$,
 {0, 0, 0, 2} → $\in \varphi_{0,0,0,2'}[\eta]$, {0, 0, 0, 1} → $\in \varphi_{0,0,0,1'}[\eta]$, {0, 0, 0, 0} → $\in \varphi_{0,0,0,0'}[\eta]$ }

In[*]:= DSolve[And@@(at0 ∪ atλ) /. ε → 1, φs, λ]

DSolve: There are fewer dependent variables than equations, so the system is overdetermined.

Out[*]:= DSolve[φ_{0,0,0,0}[0] == 0 && φ_{0,0,0,1}[0] == 0 && φ_{0,0,0,2}[0] == 0 && φ_{0,0,0,3}[0] == 0 && φ_{0,0,1,0}[0] == 0 &&
 φ_{0,0,1,1}[0] == 0 && γ φ_{0,0,1,1}[η] == 0 && φ_{0,0,1,2}[0] == 0 && 2 γ φ_{0,0,1,2}[η] == 0 && φ_{0,1,0,0}[0] == 0 &&
 φ_{0,1,0,1}[0] == 0 && γ φ_{0,1,0,1}[η] == 0 && φ_{0,1,0,2}[0] == 0 && 2 γ φ_{0,1,0,2}[η] == 0 &&
 φ_{1,0,0,0}[0] == 0 && φ_{1,0,0,1}[0] == 0 && φ_{1,0,0,2}[0] == 0 && φ_{1,0,1,0}[0] == 0 && φ_{1,0,1,1}[0] == 0 &&
 γ φ_{1,0,1,1}[η] == 0 && φ_{1,1,0,0}[0] == 0 && φ_{1,1,0,1}[0] == 0 && γ φ_{1,1,0,1}[η] == 0 && φ_{2,0,0,0}[0] == 0 &&
 φ_{2,0,0,1}[0] == 0 && φ_{2,0,1,0}[0] == 0 && φ_{2,1,0,0}[0] == 0 && φ_{3,0,0,0}[0] == 0 && φ_{0,0,0,0}'[η] == 0 &&
 φ_{0,0,0,1}'[η] == 0 && φ_{0,0,0,2}'[η] == 0 && φ_{0,0,0,3}'[η] == 0 && φ_{0,0,1,0}'[η] == 0 && φ_{0,0,1,1}'[η] == 0 &&
 φ_{0,0,1,2}'[η] == 0 && γ φ_{0,0,0,1}[η] + φ_{0,1,0,0}'[η] == 0 && 2 γ φ_{0,0,0,2}[η] + φ_{0,1,0,1}'[η] == 0 &&
 3 γ φ_{0,0,0,3}[η] + φ_{0,1,0,2}'[η] == 0 && -γ φ_{0,0,1,0}[η] + φ_{1,0,0,0}'[η] == 0 &&
 -γ φ_{0,0,1,1}[η] + φ_{1,0,0,1}'[η] == 0 && -γ φ_{0,0,1,2}[η] + φ_{1,0,0,2}'[η] == 0 && φ_{1,0,1,0}'[η] == 0 &&
 φ_{1,0,1,1}'[η] == 0 && γ φ_{1,0,0,1}[η] + φ_{1,1,0,0}'[η] == 0 && 2 γ φ_{1,0,0,2}[η] + φ_{1,1,0,1}'[η] == 0 &&
 -γ φ_{1,0,1,0}[η] + φ_{2,0,0,0}'[η] == 0 && -γ φ_{1,0,1,1}[η] + φ_{2,0,0,1}'[η] == 0 &&
 φ_{2,0,1,0}'[η] == 0 && γ φ_{2,0,0,1}[η] + φ_{2,1,0,0}'[η] == 0 && -γ φ_{2,0,1,0}[η] + φ_{3,0,0,0}'[η] == 0,
 {φ_{0,0,0,0}[η], φ_{0,0,0,1}[η], φ_{0,0,0,2}[η], φ_{0,0,0,3}[η], φ_{1,0,0,0}[η], φ_{1,0,0,1}[η], φ_{1,0,0,2}[η],
 φ_{2,0,0,0}[η], φ_{2,0,0,1}[η], φ_{3,0,0,0}[η], φ_{0,0,1,0}[η], φ_{0,0,1,1}[η], φ_{0,0,1,2}[η], φ_{1,0,1,0}[η], φ_{1,0,1,1}[η],
 φ_{2,0,1,0}[η], φ_{0,1,0,0}[η], φ_{0,1,0,1}[η], φ_{0,1,0,2}[η], φ_{1,1,0,0}[η], φ_{1,1,0,1}[η], φ_{2,1,0,0}[η]}, η]