Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
Once["<< KnotTheory`"];
<< "../Profile/Profile.m";
<< "Engine-Speedy.m";
<< "Objects.m";
<< "KT.m";
$k = 0; γ = 1;
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
 Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]
```

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.

» Warning: On Sep 4 2019 I swapped the operations ∊ and η. Some incompatibilities may arise in older notebooks.

Out[∘]= Show Profile Monitor

Latin Canonical Form:

In[∘]:=
```
LCF[ℰ_List] := LCF /@ ℰ;
LCF[sd_SeriesData] := MapAt[LCF, sd, 3];
LCF[ℰ_] := PP_LCF@Module[
   {vs = Cases[ℰ, (y | b | t | a | x)_, ∞] ⋃ {y, b, t, a, x}},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :→ CF[c] (Times @@ vs^ps)]
  ];
LCF[ℰ_𝔼] := LCF /@ ℰ;
LCF[𝔼_sp___[L_, Q_, P_]] := LCF /@ 𝔼_sp[L, Q, P];
```

In[∘]:= $R_{1,2}$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[\hbar\, a_2 b_1,\ \hbar\, x_2 y_1,\ 1]$

In[∘]:= B_i /. U2l

Out[∘]= $e^{-\hbar\, b_i}$

In[∘]:= $cm_{1,2\to 3}$

Out[∘]= $\mathbb{E}_{\{1,2\}\to\{3\}}\left[a_3\,\alpha_1 + a_3\,\alpha_2 + b_3\,\beta_1 + b_3\,\beta_2,\ y_3\,\eta_1 + \frac{y_3\,\eta_2}{\mathcal{A}_1} + \frac{x_3\,\xi_1}{\mathcal{A}_2} + b_3\,\eta_2\,\xi_1 + x_3\,\xi_2,\ 1 + O[\epsilon]^1\right]$

200906 With $\mathfrak{g} = sl^0_{2+} = \langle y,b,a,x\rangle/([a,x]=x, [a,y]=-y, [x,y]=b, [b,-]=0)$, $\mathcal{U}(\mathfrak{g})_{\mathfrak{g}}$ is freely generated by $\{b^k a^n\}_{k\leqslant n}$, also $\{y^k a^n x^k\}_{k,n\geqslant 0}$, for $[a, y^l a^m x^n] = (n-l)y^l a^m x^n$ forces $l=n$ and $xa=(a-1)x$ implies $xf(a)=f(a-1)x$ so $[x,f(a)]=(f(a-1)-f(a))x$ so with $a^{(k)} := \binom{a+k}{k} = (a+1)(a+2)\cdots(a+k)/k!$ and $a^{(-1)} := 0$, $\left[x,a^{(k)}\right] = -a^{(k-1)}x$ so $\left[x, y^n a^{(k)} x^{n-1}\right] = nby^{n-1}a^{(k)}x^{n-1} - y^n a^{(k-1)}x^n$. Thus in $\mathcal{U}(\mathfrak{g})_{\mathfrak{g}}$, $by^n x^n = 0$ and $y^n a^{(k)} x^n = n! b^n a^{(k+n)}$. So $\{b^k a^n\}$ generates and $b^{n+1}a^n = 0$. The same relations also follow from $[y^{n-1}a^{(k)}x^n, y] = -y^n a^{(k-1)}x^n + nby^{n-1}a^{(k)}x^{n-1}$, and these are all the relations in $\mathcal{U}(\mathfrak{g})_{\mathfrak{g}}$.

Goal: Implement the above going into $\{y^k a^n x^k\}$ and renaming $y\,x \to t$ in the target space.

```
In[◦]:= tr_i_ := 𝔼_{i}→{}[0, 0, 1 + α_i a_i c_1 + β_i α_i t_i c_2 + α_i^2 a_i^2 c_3 + ξ_i η_i t_i c_4 + β_i α_i^2 (a_i t_i c_5)]
lhs = cm_{1,2→0} // tr_0 // LCF // Last // Normal
rhs = cm_{2,1→0} // tr_0 // LCF // Last // Normal
lhs - rhs // LCF
```

Out[◦]=
$$1 + a_0 (c_1\alpha_1 + c_1\alpha_2) + a_0^2 \left(c_3\alpha_1^2 + 2c_3\alpha_1\alpha_2 + c_3\alpha_2^2\right) + a_0 t_0 \left(c_5\alpha_1^2\beta_1 + 2c_5\alpha_1\alpha_2\beta_1 + c_5\alpha_2^2\beta_1 + c_5\alpha_1^2\beta_2 + 2c_5\alpha_1\alpha_2\beta_2 + c_5\alpha_2^2\beta_2 + c_5\alpha_1^2\eta_2\xi_1 + 2c_5\alpha_1\alpha_2\eta_2\xi_1 + c_5\alpha_2^2\eta_2\xi_1\right) + t_0 \left(c_2\alpha_1\beta_1 + c_2\alpha_2\beta_1 + c_2\alpha_1\beta_2 + c_2\alpha_2\beta_2 + \frac{c_4\eta_1\xi_1}{\mathcal{A}_2} + \frac{c_4\eta_2\xi_1}{\mathcal{A}_1\mathcal{A}_2} + c_2\alpha_1\eta_2\xi_1 + c_2\alpha_2\eta_2\xi_1 + c_4\eta_1\xi_2 + \frac{c_4\eta_2\xi_2}{\mathcal{A}_1}\right)$$

Out[◦]=
$$1 + a_0 (c_1\alpha_1 + c_1\alpha_2) + a_0^2 \left(c_3\alpha_1^2 + 2c_3\alpha_1\alpha_2 + c_3\alpha_2^2\right) + a_0 t_0 \left(c_5\alpha_1^2\beta_1 + 2c_5\alpha_1\alpha_2\beta_1 + c_5\alpha_2^2\beta_1 + c_5\alpha_1^2\beta_2 + 2c_5\alpha_1\alpha_2\beta_2 + c_5\alpha_2^2\beta_2 + c_5\alpha_1^2\eta_1\xi_2 + 2c_5\alpha_1\alpha_2\eta_1\xi_2 + c_5\alpha_2^2\eta_1\xi_2\right) + t_0 \left(c_2\alpha_1\beta_1 + c_2\alpha_2\beta_1 + c_2\alpha_1\beta_2 + c_2\alpha_2\beta_2 + \frac{c_4\eta_1\xi_1}{\mathcal{A}_2} + c_4\eta_2\xi_1 + \frac{c_4\eta_1\xi_2}{\mathcal{A}_1\mathcal{A}_2} + c_2\alpha_1\eta_1\xi_2 + c_2\alpha_2\eta_1\xi_2 + \frac{c_4\eta_2\xi_2}{\mathcal{A}_1}\right)$$

Out[◦]=
$$t_0 \left(\frac{(c_4 - c_4\mathcal{A}_1\mathcal{A}_2)\,\eta_2\xi_1}{\mathcal{A}_1\mathcal{A}_2} + c_2\alpha_1\eta_2\xi_1 + c_2\alpha_2\eta_2\xi_1 + \frac{(-c_4 + c_4\mathcal{A}_1\mathcal{A}_2)\,\eta_1\xi_2}{\mathcal{A}_1\mathcal{A}_2} - c_2\alpha_1\eta_1\xi_2 - c_2\alpha_2\eta_1\xi_2\right) + a_0 t_0 \left(c_5\alpha_1^2\eta_2\xi_1 + 2c_5\alpha_1\alpha_2\eta_2\xi_1 + c_5\alpha_2^2\eta_2\xi_1 - c_5\alpha_1^2\eta_1\xi_2 - 2c_5\alpha_1\alpha_2\eta_1\xi_2 - c_5\alpha_2^2\eta_1\xi_2\right)$$