Pensieve header: Benchmarking: QU testing at LazyEval, Knot[10,100] at k=1.

Header

Benchmarking in QU, LazyEval

Startup

In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
<< KnotTheory`;
<< "../Profile/Profile.m";
<< "Engine-LazyEval.m";
<< "Objects-LazyEval.m";
<< "KT.m";
BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
 Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]
```

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: June 2018. Original version: July 1994.
```

Out[ ]= Show Profile Monitor

In[ ]:=
```
$k = 2; (*ℏ=γ=1;*)
```

Utilities

In[ ]:=
```
HL[ℰ_] := Style[ℰ, Background → Green];
```

Testing

In[ ]:=
```
$QZipFail = True;
```

In[•]:=
```
Block[{$k = 1}, {
   am → am_{i,j→k}, bm → bm_{i,j→k}, dm → dm_{i,j→k}, R → R_{i,j}, R̄ → R̄_{i,j}, P → P_{i,j},
   aS → aS_i, āS → āS_i, bS → bS_i, b̄S → b̄S_i, dS → dS_i, aΔ → aΔ_{i→j,k}, bΔ → bΔ_{i→j,k},
   dΔ → dΔ_{i→j,k}, C → C_i, C̄ → C̄_i, Kink → Kink_i, K̄ink → K̄ink_i, b2t → b2t_i, t2b → t2b_i
  }] //
 Column
```

Out[•]=

$$\text{am} \to \mathbb{E}_{\{i,j\}\to\{k\}}\left[\{\},\ a_k\ (\alpha_i+\alpha_j),\ x_k\left(\frac{\xi_i}{\mathcal{A}_j}+\xi_j\right),\ 1\right]$$

$$\text{bm} \to \mathbb{E}_{\{i,j\}\to\{k\}}\left[\{\},\ b_k\ (\beta_i+\beta_j),\ y_k\ (\eta_i+\eta_j),\ 1-y_k\,\beta_i\,\eta_j\,\epsilon+O[\epsilon]^2\right]$$

$$\text{dm} \to \mathbb{E}_{\{i,j\}\to\{k\}}\left[\{\},\ a_k\,\alpha_i+a_k\,\alpha_j+b_k\,\beta_i+b_k\,\beta_j,\ y_k\,\eta_i+\frac{y_k\,\eta_j}{\mathcal{A}_i}+\frac{x_k\,\xi_i}{\mathcal{A}_j}+\frac{(1-B_k)\,\eta_j\,\xi_i}{\hbar}+x_k\,\xi_j,\right.$$
$$\left.1+\left(-\frac{y_k\,\beta_i\,\eta_j}{\mathcal{A}_i}-\frac{x_k\,\beta_j\,\xi_i}{\mathcal{A}_j}+a_k\,B_k\,\eta_j\,\xi_i+\frac{\gamma\,\hbar\,x_k\,y_k\,\eta_j\,\xi_i}{\mathcal{A}_i\,\mathcal{A}_j}+\frac{(\gamma-3\,\gamma\,B_k)\,y_k\,\eta_j^2\,\xi_i}{2\,\mathcal{A}_i}+\frac{(\gamma-3\,\gamma\,B_k)\,x_k\,\eta_j\,\xi_i^2}{2\,\mathcal{A}_j}+\frac{(\gamma-4\,\gamma\,B_k+3\,\gamma\,B_k^2)\,\eta_j^2\,\xi_i^2}{4\,\hbar}\right)\epsilon+\right.$$
$$\left.O[\epsilon]^2\right]$$

$$\text{R} \to \mathbb{E}_{\{\}\to\{i,j\}}\left[\{\},\ \hbar\,a_j\,b_i,\ \hbar\,x_j\,y_i,\ 1-\frac{1}{4}\left(\gamma\,\hbar^3\,x_j^2\,y_i^2\right)\epsilon+O[\epsilon]^2\right]$$

$$\bar{\text{R}} \to \mathbb{E}_{\{\}\to\{i,j\}}\left[\{\},\ -\hbar\,a_j\,b_i,\ -\frac{\hbar\,x_j\,y_i}{B_i},\ 1+\left(-\frac{\hbar^2\,a_j\,x_j\,y_i}{B_i}-\frac{3\,\gamma\,\hbar^3\,x_j^2\,y_i^2}{4\,B_i^2}\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{P} \to \mathbb{E}_{\{i,j\}\to\{\}}\left[\{\},\ \frac{\alpha_j\,\beta_i}{\hbar},\ \frac{\eta_i\,\xi_j}{\hbar},\ 1+\frac{\gamma\,\eta_i^2\,\xi_j^2\,\epsilon}{4\,\hbar}+O[\epsilon]^2\right]$$

$$\text{aS} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ -a_i\,\alpha_i,\ -x_i\,\mathcal{A}_i\,\xi_i,\ 1+\left(-\hbar\,a_i\,x_i\,\mathcal{A}_i\,\xi_i-\frac{1}{2}\,\gamma\,\hbar\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2\right)\epsilon+O[\epsilon]^2\right]$$

$$\bar{\text{aS}} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ -a_i\,\alpha_i,\ -x_i\,\mathcal{A}_i\,\xi_i,\ 1+\left(\gamma\,\hbar\,x_i\,\mathcal{A}_i\,\xi_i-\hbar\,a_i\,x_i\,\mathcal{A}_i\,\xi_i-\frac{1}{2}\,\gamma\,\hbar\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{bS} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ -b_i\,\beta_i,\ -\frac{y_i\,\eta_i}{B_i},\ 1+\left(-\frac{y_i\,\beta_i\,\eta_i}{B_i}-\frac{\gamma\,\hbar\,y_i^2\,\eta_i^2}{2\,B_i^2}\right)\epsilon+O[\epsilon]^2\right]$$

$$\bar{\text{bS}} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ -b_i\,\beta_i,\ -\frac{y_i\,\eta_i}{B_i},\ 1+\left(\frac{\gamma\,\hbar\,y_i\,\eta_i}{B_i}-\frac{y_i\,\beta_i\,\eta_i}{B_i}-\frac{\gamma\,\hbar\,y_i^2\,\eta_i^2}{2\,B_i^2}\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{dS} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ -a_i\,\alpha_i-b_i\,\beta_i,\ -\frac{y_i\,\mathcal{A}_i\,\eta_i}{B_i}-x_i\,\mathcal{A}_i\,\xi_i+\frac{(\mathcal{A}_i-B_i\,\mathcal{A}_i)\,\eta_i\,\xi_i}{\hbar\,B_i},\right.$$
$$\left.1+\left(\frac{\gamma\,\hbar\,y_i\,\mathcal{A}_i\,\eta_i}{B_i}-\frac{y_i\,\mathcal{A}_i\,\beta_i\,\eta_i}{B_i}-\frac{\gamma\,\hbar\,y_i^2\,\mathcal{A}_i^2\,\eta_i^2}{2\,B_i^2}-\hbar\,a_i\,x_i\,\mathcal{A}_i\,\xi_i-x_i\,\mathcal{A}_i\,\beta_i\,\xi_i+\frac{a_i\,\mathcal{A}_i\,\eta_i\,\xi_i}{B_i}-\right.\right.$$
$$\frac{\gamma\,\hbar\,x_i\,y_i\,\mathcal{A}_i^2\,\eta_i\,\xi_i}{B_i}+\frac{(-\gamma\,\mathcal{A}_i+\gamma\,B_i\,\mathcal{A}_i)\,\eta_i\,\xi_i}{B_i}+\frac{(\mathcal{A}_i-B_i\,\mathcal{A}_i)\,\beta_i\,\eta_i\,\xi_i}{\hbar\,B_i}+\frac{y_i\,(3\,\gamma\,\mathcal{A}_i^2-\gamma\,B_i\,\mathcal{A}_i^2)\,\eta_i^2\,\xi_i}{2\,B_i^2}-$$
$$\left.\left.\frac{1}{2}\,\gamma\,\hbar\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2+\frac{x_i\,(3\,\gamma\,\mathcal{A}_i^2-\gamma\,B_i\,\mathcal{A}_i^2)\,\eta_i\,\xi_i^2}{2\,B_i}+\frac{(-3\,\gamma\,\mathcal{A}_i^2+4\,\gamma\,B_i\,\mathcal{A}_i^2-\gamma\,B_i^2\,\mathcal{A}_i^2)\,\eta_i^2\,\xi_i^2}{4\,\hbar\,B_i^2}\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{a}\Delta \to \mathbb{E}_{\{i\}\to\{j,k\}}\left[\{\},\ a_j\,\alpha_i+a_k\,\alpha_i,\ x_j\,\xi_i+x_k\,\xi_i,\ 1+\left(-\hbar\,a_j\,x_k\,\xi_i+\frac{1}{2}\,\gamma\,\hbar\,x_j\,x_k\,\xi_i^2\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{b}\Delta \to \mathbb{E}_{\{i\}\to\{j,k\}}\left[\{\},\ b_j\,\beta_i+b_k\,\beta_i,\ B_k\,y_j\,\eta_i+y_k\,\eta_i,\ 1+\frac{1}{2}\,\gamma\,\hbar\,B_k\,y_j\,y_k\,\eta_i^2\,\epsilon+O[\epsilon]^2\right]$$

$$\text{d}\Delta \to \mathbb{E}_{\{i\}\to\{j,k\}}\left[\{\},\ a_j\,\alpha_i+a_k\,\alpha_i+b_j\,\beta_i+b_k\,\beta_i,\right.$$
$$\left.y_j\,\eta_i+B_j\,y_k\,\eta_i+x_j\,\xi_i+x_k\,\xi_i,\ 1+\left(\frac{1}{2}\,\gamma\,\hbar\,B_j\,y_j\,y_k\,\eta_i^2-\hbar\,a_j\,x_k\,\xi_i+\frac{1}{2}\,\gamma\,\hbar\,x_j\,x_k\,\xi_i^2\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{C} \to \mathbb{E}_{\{\}\to\{i\}}\left[\{\},\ 0,\ 0,\ \sqrt{B_i}-\frac{1}{2}\left(\hbar\,a_i\,\sqrt{B_i}\right)\epsilon+O[\epsilon]^2\right]$$

$$\bar{\text{C}} \to \mathbb{E}_{\{\}\to\{i\}}\left[\{\},\ 0,\ 0,\ \frac{1}{\sqrt{B_i}}+\frac{\hbar\,a_i\,\epsilon}{2\,\sqrt{B_i}}+O[\epsilon]^2\right]$$

$$\text{Kink} \to \mathbb{E}_{\{\}\to\{i\}}\left[\{\},\ \hbar\,a_i\,b_i,\ \hbar\,x_i\,y_i,\ \frac{1}{\sqrt{B_i}}+\left(\frac{\hbar\,a_i}{2\,\sqrt{B_i}}-\frac{\gamma\,\hbar^3\,x_i^2\,y_i^2}{4\,\sqrt{B_i}}\right)\epsilon+O[\epsilon]^2\right]$$

$$\bar{\text{Kink}} \to \mathbb{E}_{\{\}\to\{i\}}\left[\{\},\ -\hbar\,a_i\,b_i,\ -\frac{\hbar\,x_i\,y_i}{B_i},\ \sqrt{B_i}+\left(-\frac{1}{2}\,\hbar\,a_i\,\sqrt{B_i}-\frac{\hbar^2\,a_i\,x_i\,y_i}{\sqrt{B_i}}-\frac{3\,\gamma\,\hbar^3\,x_i^2\,y_i^2}{4\,B_i^{3/2}}\right)\epsilon+O[\epsilon]^2\right]$$

$$\text{b2t} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ a_i\,\alpha_i-\frac{t_i\,\beta_i}{\gamma},\ y_i\,\eta_i+x_i\,\xi_i,\ 1+\frac{a_i\,\beta_i\,\epsilon}{\gamma}+O[\epsilon]^2\right]$$

$$\text{t2b} \to \mathbb{E}_{\{i\}\to\{i\}}\left[\{\},\ a_i\,\alpha_i-\gamma\,b_i\,\tau_i,\ y_i\,\eta_i+x_i\,\xi_i,\ 1+a_i\,\tau_i\,\epsilon+O[\epsilon]^2\right]$$

Check that on the generators this agrees with our conventions in the handout:

```
In[•]:= Timing@{{
          "[a,x]" →
           Zip[𝔼{}→{1,2}[0, 0, a2 x1] // am1,2→1][-1] - Zip[𝔼{}→{1,2}[0, 0, a1 x2] // am1,2→1][-1],
          "[b,y]" → Zip[𝔼{}→{1,2}[0, 0, y2 b1] // bm1,2→1][-1] -
            Zip[𝔼{}→{1,2}[0, 0, y1 b2] // bm1,2→1][-1]} /. z_1 → z,
        {"Δ[y]" → Last@Zip[𝔼{}→{1}[0, 0, y1] // bΔ1→1,2],
         "Δ[b]" → Last@Zip[𝔼{}→{1}[0, 0, b1] // bΔ1→1,2],
         "Δ[a]" → Last@Zip[𝔼{}→{1}[0, 0, a1] // aΔ1→1,2],
         "Δ[x]" → Last@Zip[𝔼{}→{1}[0, 0, x1] // aΔ1→1,2]},
        {
          "S(a)" → Last@Zip[𝔼{}→{1}[0, 0, a1] // aS1],
          "S(x)" → Last@Zip[𝔼{}→{1}[0, 0, x1] // aS1],
          "S(b)" → Last@Zip[𝔼{}→{1}[0, 0, b1] // bS1],
          "S(y)" → Last@Zip[𝔼{}→{1}[0, 0, y1] // bS1]
         } /. z_1 → z}
```

Out[•]=

$$\left\{1.45313,\ \left\{\{\text{[a,x]} \to -x\,\gamma,\ \text{[b,y]} \to -y\,\epsilon + 0[\epsilon]^3\right\},\ \left\{\Delta[y] \to (B_2\,y_1 + y_2) + 0[\epsilon]^3,\ \Delta[b] \to \left(b_1 + b_2\right) + 0[\epsilon]^3,\right.\right.$$
$$\left.\Delta[a] \to (a_1 + a_2) + 0[\epsilon]^3,\ \Delta[x] \to (x_1 + x_2) - \hbar\,a_1\,x_2\,\epsilon + \frac{1}{2}\,\hbar^2\,a_1^2\,x_2\,\epsilon^2 + 0[\epsilon]^3\right\},\ \left\{S(a) \to -a + 0[\epsilon]^3,$$
$$\left.\left.S(x) \to -x - a\,x\,\hbar\,\epsilon - \frac{1}{2}\left(a^2\,x\,\hbar^2\right)\epsilon^2 + 0[\epsilon]^3,\ S(b) \to -b + 0[\epsilon]^3,\ S(y) \to -\frac{y}{B} + 0[\epsilon]^3\right\}\right\}\right\}$$

Hopf algebra axioms on both sides separately.

Associativity of am and bm:

```
In[•]:= Timing@Block[{$k = 3},
          HL /@ {(am1,2→1 // am1,3→1) ≡ (am2,3→2 // am1,2→1), (bm1,2→1 // bm1,3→1) ≡ (bm2,3→2 // bm1,2→1)}
         ]
```

Out[•]= {0.359375, {True, True}}

R and *P* are inverses:

```
In[•]:= Timing@Block[{$k = 3}, {Ri,j, Pi,k, HL[(Ri,j // Pi,k) ≡ 𝔼{k}→{j}[aj αk, xj ξk, 1]]}]
```

Out[•]=

$$\left\{0.203125,\ \left\{\mathbb{E}_{\{\}\to\{i,j\}}\left[\{\},\ \hbar\,a_j\,b_i,\ \hbar\,x_j\,y_i,\ 1 - \frac{1}{4}\left(\gamma\,\hbar^3\,x_j^2\,y_i^2\right)\epsilon + \left(\frac{1}{9}\,\gamma^2\,\hbar^5\,x_j^3\,y_i^3 + \frac{1}{32}\,\gamma^2\,\hbar^6\,x_j^4\,y_i^4\right)\epsilon^2 + \frac{1}{1152}\right.\right.\right.$$
$$\left.\left.\left.\left(24\,\gamma^3\,\hbar^5\,x_j^2\,y_i^2 - 72\,\gamma^3\,\hbar^7\,x_j^4\,y_i^4 - 32\,\gamma^3\,\hbar^8\,x_j^5\,y_i^5 - 3\,\gamma^3\,\hbar^9\,x_j^6\,y_i^6\right)\epsilon^3 + 0[\epsilon]^4\right]\right.,$$
$$\mathbb{E}_{\{i,k\}\to\{\}}\left[\{\},\ \frac{\alpha_k\,\beta_i}{\hbar},\ \frac{\eta_i\,\xi_k}{\hbar},\ 1 + \frac{\gamma\,\eta_i^2\,\xi_k^2\,\epsilon}{4\,\hbar} + \frac{\left(36\,\gamma^2\,\hbar^2\,\eta_i^2\,\xi_k^2 + 40\,\gamma^2\,\hbar\,\eta_i^3\,\xi_k^3 + 9\,\gamma^2\,\eta_i^4\,\xi_k^4\right)\epsilon^2}{288\,\hbar^2} + \right.$$
$$\left.\left.\left.\left(\frac{1}{24}\,\gamma^3\,\hbar\,\eta_i^2\,\xi_k^2 + \frac{1}{6}\,\gamma^3\,\eta_i^3\,\xi_k^3 + \frac{13\,\gamma^3\,\eta_i^4\,\xi_k^4}{96\,\hbar} + \frac{5\,\gamma^3\,\eta_i^5\,\xi_k^5}{144\,\hbar^2} + \frac{\gamma^3\,\eta_i^6\,\xi_k^6}{384\,\hbar^3}\right)\epsilon^3 + 0[\epsilon]^4\right]\right.,\ \text{True}\right\}\right\}$$

as and $\overline{\text{aS}}$ are inverses, bs and $\overline{\text{bS}}$ are inverses:

```
In[•]:= Timing[HL /@ {(aS1 // aS1) ≡ 𝔼{1}→{1}[a1 α1, x1 ξ1, 1], (bS1 // bS1) ≡ 𝔼{1}→{1}[b1 β1, y1 η1, 1]}]
```

Out[•]= {0.46875, {True, True}}

(co)-associativity on both sides

```
In[•]:= Timing[
  HL /@ {(aΔ1→1,2 // aΔ2→2,3) ≡ (aΔ1→1,3 // aΔ1→1,2), (bΔ1→1,2 // bΔ2→2,3) ≡ (bΔ1→1,3 // bΔ1→1,2),
    (am1,2→1 // am1,3→1) ≡ (am2,3→2 // am1,2→1), (bm1,2→1 // bm1,3→1) ≡ (bm2,3→2 // bm1,2→1)}]
Out[•]= {0.484375, {True, True, True, True}}
```

Δ is an algebra morphism

```
In[•]:= Timing[HL /@ {(am1,2→1 // aΔ1→1,2) ≡ ((aΔ1→1,3 aΔ2→2,4) // (am3,4→2 am1,2→1)),
    (bm1,2→1 // bΔ1→1,2) ≡ ((bΔ1→1,3 bΔ2→2,4) // (bm3,4→2 bm1,2→1))}]
Out[•]= {0.453125, {True, True}}
```

An explicit formula for aS_i

```
In[•]:= Timing@Block[{$k = 4}, HL[aSi ≡ (𝔼{i}→{i,j}[-αi aj, -ξi xi,
      Sum[Expand[e^(ξi xi) (-ħ γ ϵ)^k / (2^k k!) Nest[Expand[xi^2 ∂{xi,2} #] &, e^(-ξi e^(ħ ϵ ai) xi), k]], {k, 0, $k}]]$k //
    ami,j→i)]]
Out[•]= {115.281, True}
```

S is convolution inverse of id

```
In[•]:= Timing[HL[# ≡ 𝔼{1}→{1}[0, 0, 1]] & /@ {
    (aΔ1→1,2 ~ B1 ~ aS1) ~ B1,2 ~ am1,2→1, (aΔ1→1,2 ~ B2 ~ aS2) ~ B1,2 ~ am1,2→1,
    (bΔ1→1,2 ~ B1 ~ bS1) ~ B1,2 ~ bm1,2→1, (bΔ1→1,2 ~ B2 ~ bS2) ~ B1,2 ~ bm1,2→1}]
Out[•]= {0.515625, {True, True, True, True}}
```

But not with the opposite product:

```
In[•]:= Timing[Short[# ≡ 𝔼{1}→{1}[0, 0, 1]] & /@ {
    (aΔ1→1,2 ~ B1 ~ aS1) ~ B1,2 ~ am2,1→1, (aΔ1→1,2 ~ B2 ~ aS2) ~ B1,2 ~ am2,1→1,
    (bΔ1→1,2 ~ B1 ~ bS1) ~ B1,2 ~ bm2,1→1, (bΔ1→1,2 ~ B2 ~ bS2) ~ B1,2 ~ bm2,1→1}]
Out[•]= {0.5625, {1/2 (-2 γ ϵ ħ x1 A1 ξ1 + γ^2 ϵ^2 <<3>> ξ1 - <<1>> + 2 γ^2 ϵ^2 ħ^2 x1^2 A1^2 ξ1^2) == 0,
  1/2 (-2 γ ϵ ħ x1 ξ1 - γ^2 ϵ^2 ħ^2 x1 ξ1 + 2 γ^2 ϵ^2 ħ^2 x1^2 ξ1^2) == 0,
  1/2 (-2 γ ϵ ħ y1 η1 - γ^2 ϵ^2 ħ^2 y1 η1 + 2 γ^2 ϵ^2 ħ^2 y1^2 η1^2) == 0,
  (-2 γ ϵ ħ B1 y1 η1 + <<3>> + 2 <<4>> <<1>>) / (2 B1^2) == 0}}
```

S is an algebra anti-(co)morphism

```
In[•]:= Timing[HL /@ {am1,2→1 ~ B1 ~ aS1 ≡ (aS1 aS2) ~ B1,2 ~ am2,1→1, bm1,2→1 ~ B1 ~ bS1 ≡ (bS1 bS2) ~ B1,2 ~ bm2,1→1,
    aS1 ~ B1 ~ aΔ1→1,2 ≡ aΔ1→2,1 ~ B1,2 ~ (aS1 aS2), bS1 ~ B1 ~ bΔ1→1,2 ≡ bΔ1→2,1 ~ B1,2 ~ (bS1 bS2)}]
Out[•]= {0.921875, {True, True, True, True}}
```

Pairing axioms

```
In[•]:= Timing[HL /@ {(bm_{1,2→1} 𝔼_{3}→{3}[α_3 a_3, ξ_3 x_3, 1]) ~B_{1,3}~P_{1,3} ≡
          (𝔼_{1}→{1}[β_1 b_1, η_1 y_1, 1] 𝔼_{2}→{2}[β_2 b_2, η_2 y_2, 1] aΔ_{3→4,5}) ~B_{1,4}~P_{1,4}~B_{2,5}~P_{2,5},
        (bΔ_{1→1,2} 𝔼_{3}→{3}[α_3 a_3, ξ_3 x_3, 1] 𝔼_{4}→{4}[α_4 a_4, ξ_4 x_4, 1]) ~B_{1,3}~P_{1,3}~B_{2,4}~P_{2,4} ≡
          (𝔼_{1}→{1}[β_1 b_1, η_1 y_1, 1] am_{3,4→3}) ~B_{1,3}~P_{1,3} }]
Out[•]= {0.390625, {True, True}}

In[•]:= Timing[HL /@ {((bS_1 𝔼_{2}→{2}[α_2 a_2, ξ_2 x_2, 1]) // P_{1,2}) ≡ ((𝔼_{1}→{1}[β_1 b_1, η_1 y_1, 1] aS_2) // P_{1,2}),
        (bS̄_1 𝔼_{2}→{2}[α_2 a_2, ξ_2 x_2, 1]) ~B_{1,2}~P_{1,2} ≡ (𝔼_{1}→{1}[β_1 b_1, η_1 y_1, 1] aS̄_2) ~B_{1,2}~P_{1,2}}]
Out[•]= {0.34375, {True, True}}
```

Tests for the double.

Check the double formulas on the generators agree with SL2Portfolio.pdf:

```
In[•]:= Timing@{{
          "[a,y]" → Last@Zip[𝔼_{}→{1,2}[0, 0, y_2 a_1] ~B_{1,2}~dm_{1,2→1}] -
            Last@Zip[𝔼_{}→{1,2}[0, 0, y_1 a_2] ~B_{1,2}~dm_{1,2→1}],
          "[b,x]"  → Last@Zip[𝔼_{}→{1,2}[0, 0, x_2 b_1] ~B_{1,2}~dm_{1,2→1}] -
            Last@Zip[𝔼_{}→{1,2}[0, 0, x_1 b_2] ~B_{1,2}~dm_{1,2→1}],
          "xy-qyx" → Last@Zip[𝔼_{}→{1,2}[0, 0, x_1 y_2] ~B_{1,2}~dm_{1,2→1}] -
            (1 + ϵ) Last@Zip[𝔼_{}→{1,2}[0, 0, y_1 x_2] ~B_{1,2}~dm_{1,2→1}]
        } /. {z__1 → z} // Expand // Factor,
       {
         "Δ(a)" → Last@Zip[𝔼_{}→{1}[0, 0, a_1] ~B_1~dΔ_{1→1,2}],
         "Δ(x)" → Last@Zip[𝔼_{}→{1}[0, 0, x_1] ~B_1~dΔ_{1→1,2}],
         "Δ(b)" → Last@Zip[𝔼_{}→{1}[0, 0, b_1] ~B_1~dΔ_{1→1,2}],
         "Δ(y)" → Last@Zip[𝔼_{}→{1}[0, 0, y_1] ~B_1~dΔ_{1→1,2}]
        } // Simplify,
       {
          "S(a)" → Last@Zip[𝔼_{}→{1}[0, 0, a_1] ~B_1~dS_1],
          "S(x)" → Last@Zip[𝔼_{}→{1}[0, 0, x_1] ~B_1~dS_1],
          "S(b)" → Last@Zip[𝔼_{}→{1}[0, 0, b_1] ~B_1~dS_1],
          "S(y)" → Last@Zip[𝔼_{}→{1}[0, 0, y_1] ~B_1~dS_1]
        } /. {z__1 → z} // Simplify
       }
Out[•]= {6.5, {{[a,y] → -y γ + O[ϵ]^3, [b,x] → x ϵ + O[ϵ]^3,
     xy-qyx → (1 - B)/ħ + (a B - x y + x y γ ħ) ϵ + (-1/2 a^2 B ħ + 1/2 x y γ^2 ħ^2) ϵ^2 + O[ϵ]^3},
    {Δ(a) → (a_1 + a_2) + O[ϵ]^3, Δ(x) → (x_1 + x_2) - ħ a_1 x_2 ϵ + 1/2 ħ^2 a_1^2 x_2 ϵ^2 + O[ϵ]^3,
     Δ(b) → (b_1 + b_2) + O[ϵ]^3, Δ(y) → (y_1 + B_1 y_2) + O[ϵ]^3},
    {S(a) → -a + O[ϵ]^3, S(x) → -x - a x ħ ϵ - 1/2 (a^2 x ħ^2) ϵ^2 + O[ϵ]^3,
     S(b) → -b + O[ϵ]^3, S(y) → -y/B + (y γ ħ ϵ)/B - ((y γ^2 ħ^2) ϵ^2)/(2 B) + O[ϵ]^3}}}
```

(co)-associativity

```
In[•]:= Timing[
          HL /@ {(dΔ1→1,2 // dΔ2→2,3) ≡ (dΔ1→1,3 // dΔ1→1,2), (dm1,2→1 // dm1,3→1) ≡ (dm2,3→2 // dm1,2->1)}]
Out[•]= {3.40625, {True, True}}
```

Δ is an algebra morphism

```
In[•]:= Timing@HL[dm1,2→1 ~ B1 ~ dΔ1→1,2 ≡ (dΔ1→1,3 dΔ2→2,4) ~ B1,2,3,4 ~ (dm3,4→2 dm1,2→1)]
Out[•]= {3.90625, True}
```

S_2 inverts R, but not S_1:

```
In[•]:= Timing@{R1,2 ~ B1 ~ dS1 ≡ R̄1,2, HL[R1,2 ~ B2 ~ dS2 ≡ R̄1,2]}
Out[•]= {0.671875, {1/(4 B1^3) (4 γ ∈ ħ^2 B1^2 x2 y1 - 2 γ^2 ∈^2 ħ^3 B1^2 x2 y1 + 4 γ ∈^2 ħ^3 a2 B1^2 x2 y1 +
              8 γ^2 ∈^2 ħ^4 B1 x2^2 y1^2 - 4 γ ∈^2 ħ^4 a2 B1 x2^2 y1^2 - 3 γ^2 ∈^2 ħ^5 x2^3 y1^3) == 0, True}}
```

S is convolution inverse of id

```
In[•]:= Timing[HL[# ≡ 𝔼{1}→{1}[0, 0, 1]] & /@
            {(dΔ1→1,2 ~ B1 ~ dS1) ~ B1,2 ~ dm1,2→1, (dΔ1→1,2 ~ B2 ~ dS2) // dm1,2→1}]
Out[•]= {5.8125, {True, True}}
```

S is a (co)-algebra anti-morphism

```
In[•]:= Timing[HL /@
            Expand /@ {dm1,2→1 ~ B1 ~ dS1 ≡ (dS1 dS2) ~ B1,2 ~ dm2,1→1, dS1 ~ B1 ~ dΔ1→1,2 ≡ dΔ1→2,1 ~ B1,2 ~ (dS1 dS2)}]
Out[•]= {14.4375, {True, True}}
```

Quasi-triangular axiom 1:

```
In[•]:= Timing@HL[R1,2 ~ B1 ~ dΔ1->1,3 ≡ (R1,4 R3,2) ~ B2,4 ~ dm2,4→2]
Out[•]= {0.28125, True}
```

Quasi-triangular axiom 2:

```
In[•]:= Timing@HL[((dΔ1->1,2 R3,4) ~ B1,2,3,4 ~ (dm1,3→1 dm2,4→2)) ≡ ((dΔ1→2,1 R3,4) ~ B1,2,3,4 ~ (dm3,1→1 dm4,2→2))]
Out[•]= {3.14063, True}
```

The Drinfel'd element inverse property, $(u_1 \bar{u}_2) \sim B_{1,2} \sim \mathrm{dm}_{1,2\to1} \equiv \mathbb{E}[0, 0, 1]$:

```
In[•]:= Timing@HL[((R1,2 ~ B1 ~ dS1 ~ B1,2 ~ dm2,1→i) (R1,2 ~ B2 ~ dS2 ~ B2 ~ dS2 ~ B1,2 ~ dm2,1→j)) ~ Bi,j ~ dmi,j→i ≡
              𝔼{}→{i}[0, 0, 1]]
Out[•]= {992.703, True}
```

The ribbon element v satisfies $v^2 = S(u)\,u$. The spinner $C = uv^{-1}$. It is convenient to compute $z = S(u)\,u^{-1}$ which is something easy.

```
In[•]:= Timing@Block[{$k = 2}, Zip[
          (((R_{1,2} ~ B_1 ~ dS_1 ~ B_{1,2} ~ dm_{2,1→i}) ~ B_i ~ dS_i) (R_{1,2} ~ B_2 ~ dS_2 ~ B_2 ~ dS_2 ~ B_{1,2} ~ dm_{2,1→j})) ~ B_{i,j} ~ dm_{i,j→i}]]
Out[•]= {265.922, 𝔼_{}→{i}[{}, 0, 0, 1/B_i + ħ a_i ∈/B_i + ħ^2 a_i^2 ∈^2/(2 B_i) + O[∈]^3]}

In[•]:= Timing@Block[{$k = 2}, HL /@ {(C_i C̄_j) ~ B_{i,j} ~ dm_{i,j→i} ≡ 𝔼_{}→{i}[0, 0, 1], (C̄_i C̄_j) ~ B_{i,j} ~ dm_{i,j→i} ≡
          (((R_{1,2} ~ B_1 ~ dS_1 ~ B_{1,2} ~ dm_{2,1→i}) ~ B_i ~ dS_i) (R_{1,2} ~ B_2 ~ dS_2 ~ B_2 ~ dS_2 ~ B_{1,2} ~ dm_{2,1→j})) ~ B_{i,j} ~ dm_{i,j→i}}]
Out[•]= {280.297, {True, True}}
```

Reidemeister 2:

```
In[•]:= Timing[HL[# ≡ 𝔼_{}→{1,2}[0, 0, 1]] & /@
          {(R̄_{1,2} R_{3,4}) ~ B_{1,2,3,4} ~ (dm_{1,3→1} dm_{2,4→2}), (R_{1,2} R̄_{3,4}) ~ B_{1,2,3,4} ~ (dm_{1,3→1} dm_{2,4→2})}]
Out[•]= {2.57813, {True, True}}
```

Cyclic Reidemeister 2:

```
In[•]:= Timing@HL[(R_{1,4} R̄_{5,2} C̄_3) ~ B_{2,4} ~ dm_{2,4→2} ~ B_{1,3} ~ dm_{1,3→1} ~ B_{1,5} ~ dm_{1,5→1} ≡ C̄_1 𝔼_{}→{2}[0, 0, 1]]
Out[•]= {4.07813, True}
```

Reidemeister 3:

```
In[•]:= Timing@HL[((R_{1,2} R_{4,3} R_{5,6}) ~ B_{1,4} ~ dm_{1,4→1} ~ B_{2,5} ~ dm_{2,5→2} ~ B_{3,6} ~ dm_{3,6→3}) ≡
          ((R_{1,6} R_{2,3} R_{4,5}) ~ B_{1,4} ~ dm_{1,4→1} ~ B_{2,5} ~ dm_{2,5→2} ~ B_{3,6} ~ dm_{3,6→3})]
Out[•]= {8.75, True}
```

Relations between the four kinks:

```
In[•]:= Timing[HL /@ {Kink_i ≡ (R_{3,1} C_2) ~ B_{1,2} ~ dm_{1,2→1} ~ B_{1,3} ~ dm_{1,3→i},
          Kink̄_j ≡ (R̄_{3,1} C̄_2) ~ B_{1,2} ~ dm_{1,2→1} ~ B_{1,3} ~ dm_{1,3→j}, (Kink_i Kink̄_j) ~ B_{i,j} ~ dm_{i,j→1} ≡ 𝔼_{}→{1}[0, 0, 1]}]
Out[•]= {11.0313, {True, True, True}}
```

The Trefoil

```
In[•]:= Timing@Block[{$k = 1},
          Z31 = R_{1,5} R_{6,2} R_{3,7} C̄_4 Kink̄_8 Kink̄_9 Kink̄_{10};
          Do[Z31 = Z31 ~ B_{1,r} ~ dm_{1,r→1}, {r, 2, 10}];
          {CF /@ Z31, CF /@ (Z31 ~ B_1 ~ b2t_1 /. T_1 → T)}] // Short
Out[•]//Short= {2.03125, {𝔼_{}→{1}[ <<1>> ], 𝔼_{}→{1}[ <<1>> ]}}

In[•]:= Timing[Zip@Z31]
Out[•]= {11.0313, 𝔼_{}→{1}[{}, 0, 0, B_1/(1 - B_1 + B_1^2) +
          (a_1 (-ħ B_1 + ħ B_1^3)/(1 - 2 B_1 + 3 B_1^2 - 2 B_1^3 + B_1^4) + (-γ ħ B_1^2 + 2 γ ħ B_1^3 - 3 γ ħ B_1^4 + 2 γ ħ B_1^5)/(1 - 3 B_1 + 6 B_1^2 - 7 B_1^3 + 6 B_1^4 - 3 B_1^5 + B_1^6) + (-2 γ ħ^2 B_1 - 2 γ ħ^2 B_1^2) x_1 y_1/(1 - 2 B_1 + 3 B_1^2 - 2 B_1^3 + B_1^4)) ∈ +
          O[∈]^2]}
```

```
In[•]:= Timing@Block[{$k = 1},
          Z31 = kR1,5 kR6,2 kR3,7 kC4 kKink8 kKink9 kKink10;
          Do[Z31 = Z31 ~B1,r~ km1,r→1, {r, 2, 10}];
          CF /@ Z31] // Short
```

Out[•]//Short= {1.4375, 𝔼{}→{1}[<<1>>]}

```
In[•]:= Timing[Zip@Z31]
```

Out[•]= $\left\{11.3438, \mathbb{E}_{\{\}\to\{1\}}\left[\{\}, 0, 0, \frac{T}{1-T+T^2} + \left(\frac{-T^2\gamma\hbar + 2T^3\gamma\hbar - 3T^4\gamma\hbar + 2T^5\gamma\hbar}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} + \frac{\left(-2T\hbar + 2T^3\hbar\right)a_1}{1-2T+3T^2-2T^3+T^4} + \frac{\left(-2T\gamma\hbar^2 - 2T^2\gamma\hbar^2\right)x_1 y_1}{1-2T+3T^2-2T^3+T^4}\right)\in + O[\in]^2\right]\right\}$

Knot

```
In[•]:= $k = 1; Timing@Zip@Z@Knot[10, 100]
```

Knot

KnotTheory: Loading precomputed data in PD4Knots`.

```
In[•]:= EndProfile[];
```

Profile

```
In[•]:= PrintProfile[]
```

Profile

```
Out[•]= ProfileRoot is root. Profiled time: 744.33
          (   157)    1.546/   1.546 above B
          (    37)    0.266/  28.985 above Boot
          (   434)    1.110/   3.047 above CF
          (   103)    6.533/ 192.817 above LZip
          (   103)    9.933/ 517.935 above QZip
        Collect: called 1930 times, time in 72.013/72.013
          (   276)    0.750/   0.750 under Zip[1]
          (   276)    3.267/   3.267 under Zip[2]
          (   224)    2.512/   2.512 under Zip[3]
          (   172)    1.749/   1.749 under Zip[4]
          (   134)    5.376/   5.376 under Zip[5]
          (   134)   10.547/  10.547 under Zip[6]
          (    81)    6.777/   6.777 under Zip[7]
          (    81)    5.969/   5.969 under Zip[8]
          (    62)    1.639/   1.639 under Zip[9]
          (    47)    4.610/   4.610 under Zip[10]
          (    47)    5.549/   5.549 under Zip[11]
          (    47)    4.389/   4.389 under Zip[12]
          (    25)    3.202/   3.202 under Zip[13]
          (    25)    3.299/   3.299 under Zip[14]
          (    25)    3.673/   3.673 under Zip[15]
          (    25)    1.108/   1.108 under Zip[16]
          (    21)    1.172/   1.172 under Zip[17]
          (    21)    0.579/   0.579 under Zip[18]
          (    17)    1.735/   1.735 under Zip[19]
          (    17)    0.985/   0.985 under Zip[20]
          (    15)    0.703/   0.703 under Zip[21]
```

```
  (    15)    0.563/   0.563 under Zip[22]
  (    13)    0.422/   0.422 under Zip[23]
  (    13)    0.109/   0.109 under Zip[24]
  (     9)    0.329/   0.329 under Zip[25]
  (     9)    0.141/   0.141 under Zip[26]
  (     9)    0.281/   0.281 under Zip[27]
  (     8)    0.062/   0.062 under Zip[28]
  (     8)    0.344/   0.344 under Zip[29]
  (     8)    0.094/   0.094 under Zip[30]
  (     6)       0/      0 under Zip[31]
  (     6)    0.016/   0.016 under Zip[32]
  (     6)       0/      0 under Zip[33]
  (     4)       0/      0 under Zip[34]
  (     4)    0.015/   0.015 under Zip[35]
  (     4)       0/      0 under Zip[36]
  (     2)       0/      0 under Zip[37]
  (     2)       0/      0 under Zip[38]
  (     2)    0.016/   0.016 under Zip[39]
  (     2)       0/      0 under Zip[40]
  (     2)       0/      0 under Zip[41]
  (     2)       0/      0 under Zip[42]
  (     2)    0.015/   0.015 under Zip[43]
  (     2)       0/      0 under Zip[44]
  (     2)       0/      0 under Zip[45]
  (     2)       0/      0 under Zip[46]
  (     2)       0/      0 under Zip[47]
  (     2)       0/      0 under Zip[48]
  (     2)       0/      0 under Zip[49]
  (     2)       0/      0 under Zip[50]
  (     2)       0/      0 under Zip[51]
  (     2)    0.016/   0.016 under Zip[52]
  (     2)       0/      0 under Zip[53]
  (     2)       0/      0 under Zip[54]
CCF: called 52618 times, time in 58.043/113.504
  ( 52618)   58.043/ 113.504 under CF
  ( 52618)   45.621/  55.461 above Together
CF: called 14923 times, time in 52.72/166.224
  (  1265)    8.873/  43.205 under EEQ
  (  1441)   23.868/  80.692 under LZip
  (   434)    1.110/   3.047 under ProfileRoot
  ( 11783)   18.869/  39.280 under QZip
  ( 52618)   58.043/ 113.504 above CCF
Together: called 52618 times, time in 45.621/55.461
  ( 52618)   45.621/  55.461 under CCF
  ( 52618)    9.840/   9.840 above Exp
Zip[5]: called 134 times, time in 38.031/123.705
  (   134)   38.031/ 123.705 under Zip[6]
  (   134)    5.376/   5.376 above Collect
  (   134)   21.825/  80.298 above Zip[4]
Zip[8]: called 81 times, time in 36.852/251.191
  (    19)    0.175/   3.236 under QZip
```

```
  (    62)   36.677/ 247.955 under Zip[9]
  (    81)    5.969/   5.969 above Collect
  (    81)   35.216/ 208.370 above Zip[7]
Zip[6]: called 134 times, time in 35.628/169.88
  (    38)    0.345/   1.861 under LZip
  (    15)    0.250/   1.642 under QZip
  (    81)   35.033/ 166.377 under Zip[7]
  (   134)   10.547/  10.547 above Collect
  (   134)   38.031/ 123.705 above Zip[5]
Zip[7]: called 81 times, time in 35.216/208.37
  (    81)   35.216/ 208.370 under Zip[8]
  (    81)    6.777/   6.777 above Collect
  (    81)   35.033/ 166.377 above Zip[6]
Zip[11]: called 47 times, time in 32.515/347.872
  (    47)   32.515/ 347.872 under Zip[12]
  (    47)    5.549/   5.549 above Collect
  (    47)   24.870/ 309.808 above Zip[10]
Zip[9]: called 62 times, time in 31.891/281.485
  (    15)    0.141/   1.157 under LZip
  (    47)   31.750/ 280.328 under Zip[10]
  (    62)    1.639/   1.639 above Collect
  (    62)   36.677/ 247.955 above Zip[8]
Zip[14]: called 25 times, time in 26.888/429.608
  (    25)   26.888/ 429.608 under Zip[15]
  (    25)    3.299/   3.299 above Collect
  (    25)   26.264/ 399.421 above Zip[13]
Zip[13]: called 25 times, time in 26.264/399.421
  (    25)   26.264/ 399.421 under Zip[14]
  (    25)    3.202/   3.202 above Collect
  (    25)   23.019/ 369.955 above Zip[12]
Zip[10]: called 47 times, time in 24.87/309.808
  (    47)   24.870/ 309.808 under Zip[11]
  (    47)    4.610/   4.610 above Collect
  (    47)   31.750/ 280.328 above Zip[9]
Zip[12]: called 47 times, time in 23.426/375.687
  (    19)    0.407/   4.748 under LZip
  (     3)       0/   0.984 under QZip
  (    25)   23.019/ 369.955 under Zip[13]
  (    47)    4.389/   4.389 above Collect
  (    47)   32.515/ 347.872 above Zip[11]
Zip[15]: called 25 times, time in 22.922/456.203
  (    25)   22.922/ 456.203 under Zip[16]
  (    25)    3.673/   3.673 above Collect
  (    25)   26.888/ 429.608 above Zip[14]
Zip[4]: called 172 times, time in 22.153/81.607
  (    38)    0.328/   1.309 under QZip
  (   134)   21.825/  80.298 under Zip[5]
  (   172)    1.749/   1.749 above Collect
  (   172)   21.754/  57.705 above Zip[3]
Zip[3]: called 224 times, time in 22.066/58.513
  (    52)    0.312/   0.808 under LZip
```

```
  (   172)   21.754/  57.705 under Zip[4]
  (   224)    2.512/   2.512 above Collect
  (   224)   16.191/  33.935 above Zip[2]
Zip[2]: called 276 times, time in 16.536/34.828
  (    52)    0.345/   0.893 under QZip
  (   224)   16.191/  33.935 under Zip[3]
  (   276)    3.267/   3.267 above Collect
  (   276)   14.275/  15.025 above Zip[1]
Zip[1]: called 276 times, time in 14.275/15.025
  (   276)   14.275/  15.025 under Zip[2]
  (   276)    0.750/   0.750 above Collect
Zip[16]: called 25 times, time in 12.471/469.782
  (     4)    0.032/   3.266 under QZip
  (    21)   12.439/ 466.516 under Zip[17]
  (    25)    1.108/   1.108 above Collect
  (    25)   22.922/ 456.203 above Zip[15]
QZip: called 138 times, time in 11.462/529.634
  (    35)    1.529/  11.699 under Boot
  (   103)    9.933/ 517.935 under ProfileRoot
  ( 11783)   18.869/  39.280 above CF
  (    52)    0.345/   0.893 above Zip[2]
  (    38)    0.328/   1.309 above Zip[4]
  (    15)    0.250/   1.642 above Zip[6]
  (    19)    0.175/   3.236 above Zip[8]
  (     3)       0/   0.984 above Zip[12]
  (     4)    0.032/   3.266 above Zip[16]
  (     1)    0.312/  53.531 above Zip[18]
  (     2)    2.751/ 411.407 above Zip[20]
  (     2)       0/   0.547 above Zip[22]
  (     2)       0/   2.077 above Zip[36]
Exp: called 52618 times, time in 9.84/9.84
  ( 52618)    9.840/   9.840 under Together
Zip[19]: called 17 times, time in 8.563/434.812
  (    17)    8.563/ 434.812 under Zip[20]
  (    17)    1.735/   1.735 above Collect
  (    17)    5.889/ 424.514 above Zip[18]
LZip: called 138 times, time in 7.968/209.097
  (    35)    1.435/  16.280 under Boot
  (   103)    6.533/ 192.817 under ProfileRoot
  (  1265)    4.081/  47.286 above EEQ
  (  1441)   23.868/  80.692 above CF
  (    52)    0.312/   0.808 above Zip[3]
  (    38)    0.345/   1.861 above Zip[6]
  (    15)    0.141/   1.157 above Zip[9]
  (    19)    0.407/   4.748 above Zip[12]
  (     3)    0.250/   3.984 above Zip[18]
  (     4)    0.109/   1.390 above Zip[24]
  (     1)    0.375/   8.266 above Zip[27]
  (     2)    2.406/  48.687 above Zip[30]
  (     2)    0.016/   0.469 above Zip[33]
  (     2)    0.031/   1.781 above Zip[54]
```

```
Zip[17]: called 21 times, time in 7.311/474.999
  (    21)    7.311/ 474.999 under Zip[18]
  (    21)    1.172/   1.172 above Collect
  (    21)   12.439/ 466.516 above Zip[16]
Zip[18]: called 21 times, time in 6.451/482.029
  (     3)    0.250/   3.984 under LZip
  (     1)    0.312/  53.531 under QZip
  (    17)    5.889/ 424.514 under Zip[19]
  (    21)    0.579/   0.579 above Collect
  (    21)    7.311/ 474.999 above Zip[17]
Zip[20]: called 17 times, time in 5.753/441.55
  (     2)    2.751/ 411.407 under QZip
  (    15)    3.002/  30.143 under Zip[21]
  (    17)    0.985/   0.985 above Collect
  (    17)    8.563/ 434.812 above Zip[19]
EEQ: called 1265 times, time in 4.081/47.286
  (  1265)    4.081/  47.286 under LZip
  (  1265)    8.873/  43.205 above CF
Zip[23]: called 13 times, time in 3.375/40.689
  (    13)    3.375/  40.689 under Zip[24]
  (    13)    0.422/   0.422 above Collect
  (    13)    3.251/  36.892 above Zip[22]
Zip[29]: called 8 times, time in 3.358/49.547
  (     8)    3.358/  49.547 under Zip[30]
  (     8)    0.344/   0.344 above Collect
  (     8)    2.298/  45.845 above Zip[28]
Zip[22]: called 15 times, time in 3.251/37.439
  (     2)       0/   0.547 under QZip
  (    13)    3.251/  36.892 under Zip[23]
  (    15)    0.563/   0.563 above Collect
  (    15)    2.779/  33.625 above Zip[21]
Zip[25]: called 9 times, time in 3.093/45.751
  (     9)    3.093/  45.751 under Zip[26]
  (     9)    0.329/   0.329 above Collect
  (     9)    2.812/  42.329 above Zip[24]
Zip[26]: called 9 times, time in 3.031/48.923
  (     9)    3.031/  48.923 under Zip[27]
  (     9)    0.141/   0.141 above Collect
  (     9)    3.093/  45.751 above Zip[25]
Zip[24]: called 13 times, time in 2.921/43.719
  (     4)    0.109/   1.390 under LZip
  (     9)    2.812/  42.329 under Zip[25]
  (    13)    0.109/   0.109 above Collect
  (    13)    3.375/  40.689 above Zip[23]
Zip[21]: called 15 times, time in 2.779/33.625
  (    15)    2.779/  33.625 under Zip[22]
  (    15)    0.703/   0.703 above Collect
  (    15)    3.002/  30.143 above Zip[20]
Zip[27]: called 9 times, time in 2.547/51.751
  (     1)    0.375/   8.266 under LZip
  (     8)    2.172/  43.485 under Zip[28]
```

```
  (     9)    0.281/   0.281 above Collect
  (     9)    3.031/  48.923 above Zip[26]
Zip[30]: called 8 times, time in 2.485/52.126
  (     2)    2.406/  48.687 under LZip
  (     6)    0.079/   3.439 under Zip[31]
  (     8)    0.094/   0.094 above Collect
  (     8)    3.358/  49.547 above Zip[29]
Zip[28]: called 8 times, time in 2.298/45.845
  (     8)    2.298/  45.845 under Zip[29]
  (     8)    0.062/   0.062 above Collect
  (     8)    2.172/  43.485 above Zip[27]
B: called 219 times, time in 1.831/1.831
  (    62)    0.285/   0.285 under Boot
  (   157)    1.546/   1.546 under ProfileRoot
Boot: called 56 times, time in 0.721/47.251
  (    19)    0.455/  18.266 under Boot
  (    37)    0.266/  28.985 under ProfileRoot
  (    62)    0.285/   0.285 above B
  (    19)    0.455/  18.266 above Boot
  (    35)    1.435/  16.280 above LZip
  (    35)    1.529/  11.699 above QZip
Zip[32]: called 6 times, time in 0.062/3.578
  (     6)    0.062/   3.578 under Zip[33]
  (     6)    0.016/   0.016 above Collect
  (     6)    0.061/   3.500 above Zip[31]
Zip[31]: called 6 times, time in 0.061/3.5
  (     6)    0.061/   3.500 under Zip[32]
  (     6)       0/      0 above Collect
  (     6)    0.079/   3.439 above Zip[30]
Zip[40]: called 2 times, time in 0.047/1.329
  (     2)    0.047/   1.329 under Zip[41]
  (     2)       0/      0 above Collect
  (     2)    0.031/   1.282 above Zip[39]
Zip[33]: called 6 times, time in 0.047/3.625
  (     2)    0.016/   0.469 under LZip
  (     4)    0.031/   3.156 under Zip[34]
  (     6)       0/      0 above Collect
  (     6)    0.062/   3.578 above Zip[32]
Zip[46]: called 2 times, time in 0.047/1.547
  (     2)    0.047/   1.547 under Zip[47]
  (     2)       0/      0 above Collect
  (     2)    0.046/   1.500 above Zip[45]
Zip[41]: called 2 times, time in 0.047/1.376
  (     2)    0.047/   1.376 under Zip[42]
  (     2)       0/      0 above Collect
  (     2)    0.047/   1.329 above Zip[40]
Zip[34]: called 4 times, time in 0.047/3.203
  (     4)    0.047/   3.203 under Zip[35]
  (     4)       0/      0 above Collect
  (     4)    0.031/   3.156 above Zip[33]
Zip[48]: called 2 times, time in 0.046/1.625
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  (     2)    0.046/   1.625 under Zip[49]
  (     2)       0/      0 above Collect
  (     2)    0.032/   1.579 above Zip[47]
Zip[45]: called 2 times, time in 0.046/1.5
  (     2)    0.046/   1.500 under Zip[46]
  (     2)       0/      0 above Collect
  (     2)    0.032/   1.454 above Zip[44]
Zip[37]: called 2 times, time in 0.032/1.22
  (     2)    0.032/   1.220 under Zip[38]
  (     2)       0/      0 above Collect
  (     2)    0.031/   1.188 above Zip[36]
Zip[47]: called 2 times, time in 0.032/1.579
  (     2)    0.032/   1.579 under Zip[48]
  (     2)       0/      0 above Collect
  (     2)    0.047/   1.547 above Zip[46]
Zip[50]: called 2 times, time in 0.032/1.673
  (     2)    0.032/   1.673 under Zip[51]
  (     2)       0/      0 above Collect
  (     2)    0.016/   1.641 above Zip[49]
Zip[44]: called 2 times, time in 0.032/1.454
  (     2)    0.032/   1.454 under Zip[45]
  (     2)       0/      0 above Collect
  (     2)    0.031/   1.422 above Zip[43]
Zip[51]: called 2 times, time in 0.031/1.704
  (     2)    0.031/   1.704 under Zip[52]
  (     2)       0/      0 above Collect
  (     2)    0.032/   1.673 above Zip[50]
Zip[43]: called 2 times, time in 0.031/1.422
  (     2)    0.031/   1.422 under Zip[44]
  (     2)    0.015/   0.015 above Collect
  (     2)       0/   1.376 above Zip[42]
Zip[54]: called 2 times, time in 0.031/1.781
  (     2)    0.031/   1.781 under LZip
  (     2)       0/      0 above Collect
  (     2)    0.015/   1.750 above Zip[53]
Zip[39]: called 2 times, time in 0.031/1.282
  (     2)    0.031/   1.282 under Zip[40]
  (     2)    0.016/   0.016 above Collect
  (     2)    0.015/   1.235 above Zip[38]
Zip[36]: called 4 times, time in 0.031/3.265
  (     2)       0/   2.077 under QZip
  (     2)    0.031/   1.188 under Zip[37]
  (     4)       0/      0 above Collect
  (     4)    0.016/   3.234 above Zip[35]
Zip[49]: called 2 times, time in 0.016/1.641
  (     2)    0.016/   1.641 under Zip[50]
  (     2)       0/      0 above Collect
  (     2)    0.046/   1.625 above Zip[48]
Zip[35]: called 4 times, time in 0.016/3.234
  (     4)    0.016/   3.234 under Zip[36]
  (     4)    0.015/   0.015 above Collect
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  (     4)    0.047/   3.203 above Zip[34]
Zip[53]: called 2 times, time in 0.015/1.75
  (     2)    0.015/   1.750 under Zip[54]
  (     2)       0/      0 above Collect
  (     2)    0.015/   1.735 above Zip[52]
Zip[52]: called 2 times, time in 0.015/1.735
  (     2)    0.015/   1.735 under Zip[53]
  (     2)    0.016/   0.016 above Collect
  (     2)    0.031/   1.704 above Zip[51]
Zip[38]: called 2 times, time in 0.015/1.235
  (     2)    0.015/   1.235 under Zip[39]
  (     2)       0/      0 above Collect
  (     2)    0.032/   1.220 above Zip[37]
Zip[42]: called 2 times, time in 0./1.376
  (     2)       0/   1.376 under Zip[43]
  (     2)       0/      0 above Collect
  (     2)    0.047/   1.376 above Zip[41]
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