Pensieve header: The Objects.

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Echo@"Warning: On Sep 4 2019 I swapped the operations
   ϵ and η. Some incompatibilities may arise in older notebooks."
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Program

The Objects

Program

Symmetric Algebra Objects

Program

$$sm_{i\_,j\_\to k\_} := \mathbb{E}_{\{i,j\}\to\{k\}}[b_k(\beta_i+\beta_j)+t_k(\tau_i+\tau_j)+a_k(\alpha_i+\alpha_j)+y_k(\eta_i+\eta_j)+x_k(\xi_i+\xi_j)];$$
$$s\Delta_{i\_\to j\_,k\_} := \mathbb{E}_{\{i\}\to\{j,k\}}\left[\beta_i\left(b_j+b_k\right)+\tau_i\left(t_j+t_k\right)+\alpha_i(a_j+a_k)+\eta_i(y_j+y_k)+\xi_i(x_j+x_k)\right];$$
$$sS_{i\_} := \mathbb{E}_{\{i\}\to\{i\}}[-\beta_i b_i-\tau_i t_i-\alpha_i a_i-\eta_i y_i-\xi_i x_i];$$
$$s\eta_{i\_} := \mathbb{E}_{\{\}\to\{i\}}[0];$$
$$s\epsilon_{i\_} := \mathbb{E}_{\{i\}\to\{\}}[0];$$

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$$s\sigma_{i\_\to j\_} := \mathbb{E}_{\{i\}\to\{j\}}[\beta_i b_j+\tau_i t_j+\alpha_i a_j+\eta_i y_j+\xi_i x_j];$$
$$s\Upsilon_{i\_\to j\_,k\_,l\_,m\_} := \mathbb{E}_{\{i\}\to\{j,k,l,m\}}[\beta_i b_k+\tau_i t_k+\alpha_i a_l+\eta_i y_j+\xi_i x_m];$$

Program

The CU Definitions

Program

$$c\Lambda = \left(\eta_i+\frac{e^{-\gamma\alpha_i-\epsilon\beta_i}\eta_j}{1+\gamma\epsilon\eta_j\xi_i}\right)y_k+\left(\beta_i+\beta_j+\frac{\text{Log}[1+\gamma\epsilon\eta_j\xi_i]}{\epsilon}\right)b_k+$$
$$\left(\alpha_i+\alpha_j+\frac{\text{Log}[1+\gamma\epsilon\eta_j\xi_i]}{\gamma}\right)a_k+\left(\frac{e^{-\gamma\alpha_j-\epsilon\beta_j}\xi_i}{1+\gamma\epsilon\eta_j\xi_i}+\xi_j\right)x_k;$$
$$\text{Define}[cm_{i,j\to k}=\mathbb{E}_{\{i,j\}\to\{k\}}[c\Lambda]]$$

Program

$$\text{Define}[c\sigma_{i\to j}=s\sigma_{i,j}\;/.\;\tau_i\to 0,\ c\epsilon_i=s\epsilon_i,\ c\eta_i=s\eta_i,\ c\Delta_{i\to j,k}=s\Delta_{i\to j,k},$$
$$cS_i=sS_i\;//\;s\Upsilon_{i\to 1,2,3,4}\;//\;cm_{4,3\to i}\;//\;cm_{i,2\to i}\;//\;cm_{i,1\to i}];$$

Program

Booting Up QU

Program

$$\text{Define}[a\sigma_{i\to j}=\mathbb{E}_{\{i\}\to\{j\}}[a_j\alpha_i+x_j\xi_i],\ b\sigma_{i\to j}=\mathbb{E}_{\{i\}\to\{j\}}[b_j\beta_i+y_j\eta_i]]$$

Program

$$\text{Define}\left[am_{i,j\to k}=\mathbb{E}_{\{i,j\}\to\{k\}}\left[(\alpha_i+\alpha_j)a_k+\left(\mathcal{A}_j^{-1}\xi_i+\xi_j\right)x_k\right],\right.$$
$$\left.bm_{i,j\to k}=\mathbb{E}_{\{i,j\}\to\{k\}}\left[(\beta_i+\beta_j)b_k+\left(\eta_i+e^{-\epsilon\beta_i}\eta_j\right)y_k\right]\right]$$

Three types of inverses appear below!
$\overline{R}$ is the inverse of R in the algebra $\mathbb{B}\otimes\mathbb{A}$.
P is the inverse of R as a quadratic form, like how an element of $V^*\otimes V^*$ can be the inverse of an element of $V\otimes V$.
$\overline{\text{aS}}$ is the inverse of aS as an operator form, like how an element of $V^*\otimes V$ can be the inverse of another element of $V^*\otimes V$.

Program

$$\texttt{Define}\Big[R_{i,j} = \mathbb{E}_{\{\}\to\{i,j\}}\Big[\hbar\, a_j\, b_i + \sum_{k=1}^{\$k+1} \frac{\left(1-e^{\gamma\,\epsilon\,\hbar}\right)^k (\hbar\, y_i\, x_j)^k}{k\left(1-e^{k\,\gamma\,\epsilon\,\hbar}\right)}\Big],$$
$$\overline{R}_{i,j} = \text{CF}@\mathbb{E}_{\{\}\to\{i,j\}}\Big[-\hbar\, a_j\, b_i,\ -\hbar\, x_j\, y_i / B_i,\ 1 + \text{If}\Big[\$k == 0,\ 0,\ (\overline{R}_{\{i,j\},\$k-1})_{\$k}[3] -$$
$$\Big(\Big(((\overline{R}_{\{i,j\},0})_{\$k}\, R_{1,2}\, (\overline{R}_{\{3,4\},\$k-1})_{\$k}\Big)\ //\ \Big(bm_{i,1\to i}\, am_{j,2\to j}\Big)\ //\ \Big(bm_{i,3\to i}\, am_{j,4\to j}\Big)\Big)[3]\Big]\Big],$$
$$P_{i,j} = \mathbb{E}_{\{i,j\}\to\{\}}[\beta_i\, \alpha_j / \hbar,\ \eta_i\, \xi_j / \hbar,\ 1 + \text{If}[\$k == 0,\ 0,\ (P_{\{i,j\},\$k-1})_{\$k}[3] -$$
$$(R_{1,2}\ //\ ((P_{\{1,j\},0})_{\$k}\ (P_{\{i,2\},\$k-1})_{\$k}))[3]]]\Big]$$

Program

$$\texttt{Define}\Big[aS_i = \left(a\sigma_{i\to 2}\, \overline{R}_{1,i}\right)\ //\ P_{1,2},$$
$$\overline{aS}_i = \mathbb{E}_{\{i\}\to\{i\}}\Big[-a_i\, \alpha_i,\ -x_i\, \mathcal{A}_i\, \xi_i,\ 1 + \text{If}\Big[\$k == 0,\ 0,\ (\overline{aS}_{\{i\},\$k-1})_{\$k}[3] -$$
$$\Big((\overline{aS}_{\{i\},0})_{\$k}\ //\ aS_i\ //\ (\overline{aS}_{\{i\},\$k-1})_{\$k}\Big)[3]\Big]\Big]\Big]$$

(was $\text{aS}_j = \overline{R}_{i,j} \sim B_i \sim P_{i,j}$).

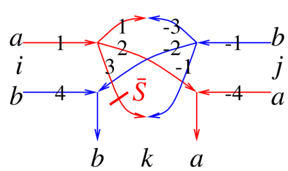
Program

$$\texttt{Define}\Big[bS_i = b\sigma_{i\to 1}\, R_{i,2}\ //\ aS_2\ //\ P_{1,2},$$
$$\overline{bS}_i = b\sigma_{i\to 1}\, R_{i,2}\ //\ \overline{aS}_2\ //\ P_{1,2},$$
$$a\Delta_{i\to j,k} = (R_{1,j}\, R_{2,k})\ //\ bm_{1,2\to 3}\ //\ P_{3,i},$$
$$b\Delta_{i\to j,k} = (R_{j,1}\, R_{k,2})\ //\ am_{1,2\to 3}\ //\ P_{i,3}\Big]$$

(was $\text{bS}_i = R_{i,1} \sim B_1 \sim \text{aS}_1 \sim B_1 \sim P_{i,1}$, $\overline{\text{bS}}_i = R_{i,1} \sim B_1 \sim \overline{\text{aS}}_1 \sim B_1 \sim P_{i,1}$).

Program

The Drinfel'd double:



Program

$$\texttt{Define}\Big[$$
$$dm_{i,j\to k} = \Big(\Big(s\Upsilon_{i\to 4,4,1,1}\ //\ a\Delta_{1\to 1,2}\ //\ a\Delta_{2\to 2,3}\ //\ \overline{aS}_3\Big)\ \Big(s\Upsilon_{j\to -1,-1,-4,-4}\ //\ b\Delta_{-1\to -1,-2}\ //\ b\Delta_{-2\to -2,-3}\Big)\Big)\ //$$
$$\Big(P_{-1,3}\, P_{-3,1}\, am_{2,-4\to k}\, bm_{4,-2\to k}\Big)\Big]$$

Program

$$\text{Define}\Big[d\sigma_{i\to j} = a\sigma_{i\to j}\, b\sigma_{i\to j},\; d\epsilon_i = s\epsilon_i,\; d\eta_i = s\eta_i,\; dS_i = s\Upsilon_{i\to 1,1,2,2} \,//\, \left(\overline{bS}_1\, aS_2\right) \,//\, dm_{2,1\to i},\; \overline{dS}_i = s\Upsilon_{i\to 1,1,2,2} \,//\, \left(bS_1\, \overline{aS}_2\right) \,//\, dm_{2,1\to i},\; d\Delta_{i\to j,k} = \left(b\Delta_{i\to 3,1}\, a\Delta_{i\to 2,4}\right) \,//\, \left(dm_{3,4\to k}\, dm_{1,2\to j}\right)\Big]$$

Program

In[∘]:=

$$\text{Define}\Big[C_i = \mathbb{E}_{\{\}\to\{i\}}\left[0,\, 0,\, B_i^{1/2}\, e^{-\hbar\,\epsilon\,a_i/2}\right]_{\$k},\; \overline{C}_i = \mathbb{E}_{\{\}\to\{i\}}\left[0,\, 0,\, B_i^{-1/2}\, e^{\hbar\,\epsilon\,a_i/2}\right]_{\$k},\; \text{Kink}_i = \left(R_{1,3}\, \overline{C}_2\right) \,//\, dm_{1,2\to 1} \,//\, dm_{1,3\to i},\; \overline{\text{Kink}}_i = \left(\overline{R}_{1,3}\, C_2\right) \,//\, dm_{1,2\to 1} \,//\, dm_{1,3\to i}\Big]$$

Program

Note. $t == \epsilon \text{a} - \gamma \text{b}$ and $b == -t/\gamma + \epsilon \text{a}/\gamma$.

Program

$$\text{Define}\Big[b2t_i = \mathbb{E}_{\{i\}\to\{i\}}\left[\alpha_i\, a_i + \beta_i\, \left(\epsilon\, a_i - t_i\right) / \gamma + \xi_i\, x_i + \eta_i\, y_i\right],\; t2b_i = \mathbb{E}_{\{i\}\to\{i\}}\left[\alpha_i\, a_i + \tau_i\, \left(\epsilon\, a_i - \gamma\, b_i\right) + \xi_i\, x_i + \eta_i\, y_i\right]\Big]$$

Program

The Knot Tensors

Program

In[∘]:=

$$\text{Define}\Big[kR_{i,j} = R_{i,j} \,//\, \left(b2t_i\, b2t_j\right) \;/.\; t_{i|j} \to t,\; \overline{kR}_{i,j} = \overline{R}_{i,j} \,//\, \left(b2t_i\, b2t_j\right) \;/.\; \{t_{i|j} \to t,\, T_{i|j} \to T\},\; km_{i,j\to k} = \left(t2b_i\, t2b_j\right) \,//\, dm_{i,j\to k} \,//\, b2t_k \;/.\; \{t_k \to t,\, T_k \to T,\, \tau_{i|j} \to 0\},\; kC_i = C_i \,//\, b2t_i \;/.\; T_i \to T,\; \overline{kC}_i = \overline{C}_i \,//\, b2t_i \;/.\; T_i \to T,\; k\text{Kink}_i = \text{Kink}_i \,//\, b2t_i \;/.\; \{t_i \to t,\, T_i \to T\},\; \overline{k\text{Kink}}_i = \overline{\text{Kink}}_i \,//\, b2t_i \;/.\; \{t_i \to t,\, T_i \to T\}\Big]$$