Pensieve header: Integration with Γ-calculus.

Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
<< KnotTheory`;
<< "../Profile/Profile.m";
<< "Engine-Speedy.m";
<< "Objects.m";
<< "KT.m";
BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
 Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: June 2018. Original version: July 1994.
```

» Warning: On Sep 4 2019 I swapped the operations ϵ and η. Some incompatibilities may arise in older notebooks.

Out[∘]= Show Profile Monitor

In[∘]:=
```
$k = 0; ℏ = γ = 1;
```

In[∘]:=
```
s; h; t; Γ; dL; V;
Once[Begin["MetaCalculi`"]; << "../MetaCalculi/MetaCalculi.m"; End[];]
ΓSimp = MetaCalculi`ΓSimp;
ΓR[i_, j_] := Γ[Xp[i, j]]; Γ̄R[i_, j_] := Γ[Xm[i, j]];
```

```
MetaCalculi` loading...
```

In[∘]:=
```
?MetaCalculi`*
```

Out[∘]=

MetaCalculi`

A	S1$	β1$	μ
A$	S2	γ1	μ1
dA	S2$	Γ1	μ2
else	simp	Γ1Collect	μ$
expr	SXForm	Γ1Form	ν
FullStitch	S$	γ1p	ν$
heads	tA	γ1p$	Ξ
heads$	Ta	Γ1Simp	Ξ$
hL	tails	γ1$	σ
hm	tails$	γ2	Σ
Hs	Ta$	γ2p	σ1
Hs$	tha	γ2p$	σ2
hΔ	tL	Γb	σa
hη	tm	ΓbCollect	σa$
hσ	tr	ΓbForm	σ$
len	tS	ΓbSimp	ϕ
len$	tΔ	ΓCollect	ϕ$
M	tη	ΓForm	χ
MVA	tσ	ΓSimp	ψ
M$	Vi	δ	ψ$
pl	α1	δ$	ω
qΔ	α2	θ	ω1
rest	αCollect	Θ	ω2
rules	αForm	θ$	ω$
S	αSimplify	λ1	
S1	β1	λ2	

Utilities

In[∘]:=
```
HL[ℰ_] := Style[ℰ, Background → Green];
```

Conversions

```
Γ[𝔼_{}→ss_[LL_, QQ_, PP_]] := Module[{L, Q, P, σ, i, j},
  {L, Q, P} = List @@ (𝔼_{}→ss[LL, QQ, PP] // Composition @@ (dS_# & /@ ss));
  ΓSimp[Γ[
     (Normal[P] /. ϵ → 0)^-1,
     σ = Sum[h_i Product[T_j^Coefficient[L,ħ a_i b_j], {j, ss}], {i, ss}],
     (σ /. h_i_ :→ t_i h_i) + Sum[h_i t_j (1 - T_j) Coefficient[Q, ħ x_i y_j], {i, ss}, {j, ss}]
    ] /. B → T]
 ]
```

```
𝔼[Γ[ω_, σ_, λ_]] := Module[{ss, L, Q, P, i, j},
  ss = dL@Γ[ω, σ, λ];
  P = ω^-1 + O[ϵ];
  L = Sum[ħ a_i b_j Exponent[Coefficient[σ, h_i], T_j], {i, ss}, {j, ss}];
  Q = (λ - (σ /. h_i_ :→ t_i h_i)) /. {h_i_ :→ x_i, t_j_ :→ ħ/(1 - T_j) y_j};
  (𝔼_{}→ss[L, Q, P] /. T → B) // Composition @@ (dS_# & /@ ss)
 ]
```

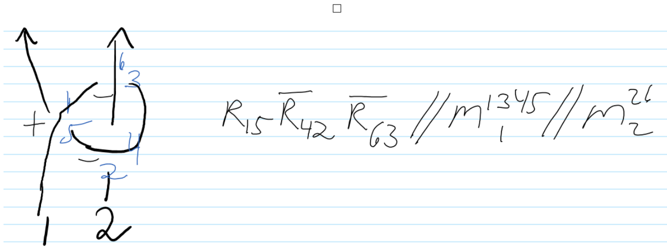
Testing

In[•]:=

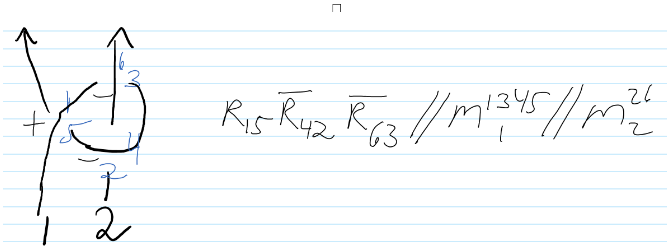
```
{ℰ = R_{1,2}, lhs = ℰ // Γ, rhs = ΓR[1, 2], HL[lhs == rhs]}
```

Out[•]= $\{\mathbb{E}_{\{\}\to\{1,2\}}[\hbar\, a_2 b_1, \hbar\, x_2 y_1, 1], \begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & 1 & 1-T_1 \\ s_2 & 0 & T_1 \\ \Gamma & 1 & T_1 \end{pmatrix}, \begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & 1 & 1-T_1 \\ s_2 & 0 & T_1 \\ \Gamma & 1 & T_1 \end{pmatrix}, \text{True}\}$

In[•]:=



Out[•]=



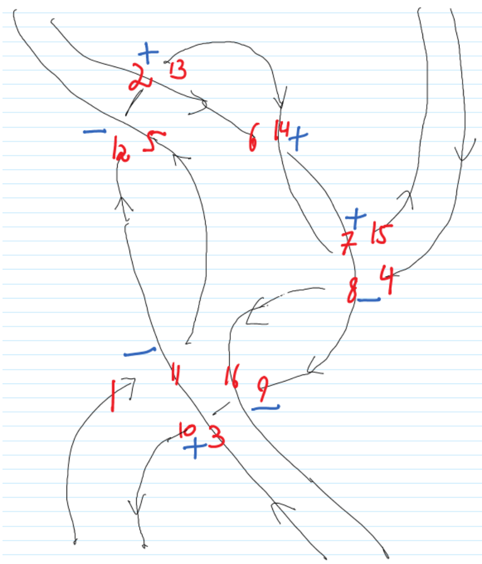
In[∘]:=
```
{ℰ = R_{1,5} R̄_{4,2} R̄_{6,3} // dm_{1,3→1} // dm_{1,4→1} // dm_{1,5→1} // dm_{2,6→2},
 lhs = ℰ // Γ,
 rhs = ΓR[1, 5] ΓR̄[4, 2] ΓR̄[6, 3] // dm[1, 3, 1] // dm[1, 4, 1] // dm[1, 5, 1] // dm[2, 6, 2],
 HL@Simplify[lhs == rhs]}
```

Out[∘]=

$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\hbar a_1 b_1 - \hbar a_2 b_1 - \hbar a_1 b_2, \frac{\left(\hbar B_1 - \hbar B_2 + \hbar B_2^2\right) x_1 y_1}{-B_2 + B_1 B_2 + B_2^2} - \frac{\hbar x_2 y_1}{-1 + B_1 + B_2} - \frac{\hbar B_1 x_1 y_2}{-1 + B_1 + B_2} + \frac{(\hbar - \hbar B_1) x_2 y_2}{-B_1 + B_1^2 + B_1 B_2}, \frac{B_1 B_2}{-1 + B_1 + B_2} + O[\epsilon]^1\right],\right.$$

$$\left(\begin{array}{ccc} \frac{-1+T_1+T_2}{T_2} & s_1 & s_2 \\ s_1 & \frac{T_1}{-1+T_1+T_2} & \frac{(-1+T_1) T_2}{-1+T_1+T_2} \\ s_2 & \frac{-1+T_2}{-1+T_1+T_2} & -\frac{1-T_1-2 T_2+T_1 T_2}{-1+T_1+T_2} \\ \Gamma & \frac{T_1}{T_2} & \frac{1}{T_1} \end{array}\right), \left(\begin{array}{ccc} \frac{-1+T_1+T_2}{T_2} & s_1 & s_2 \\ s_1 & \frac{T_1}{-1+T_1+T_2} & \frac{(-1+T_1) T_2}{-1+T_1+T_2} \\ s_2 & \frac{-1+T_2}{-1+T_1+T_2} & -\frac{1-T_1-2 T_2+T_1 T_2}{-1+T_1+T_2} \\ \Gamma & \frac{T_1}{T_2} & \frac{1}{T_1} \end{array}\right), \text{True}\Big\}$$

From 2014-05/RibbonPropertyExample.nb:



In[∘]:=
```
{ℰ = R̄11,1 R̄5,12 R2,13 R14,6 R7,15 R̄8,4 R̄16,9 R3,10 // dm1,5→1 // dm2,6→2 // dm2,7→2 // dm2,8→2 // dm2,9→2 //
      dm2,10→2 // dm3,11→3 // dm3,12→3 // dm3,13→3 // dm3,14→3 // dm3,15→3 // dm4,16→4,
  lhs = ℰ // Γ,
  rhs =
   Xm[11, 1] Xm[5, 12] Xp[2, 13] Xp[14, 6] Xp[7, 15] Xm[8, 4] Xm[16, 9] Xp[3, 10] // Γ // dm[
          1, 5, 1] // dm[2, 6, 2] // dm[2, 7, 2] // dm[2, 8, 2] //
       dm[2, 9, 2] // dm[2, 10, 2] // dm[3, 11, 3] // dm[3, 12, 3] //
     dm[3, 13, 3] // dm[3, 14, 3] // dm[3, 15, 3] // dm[4, 16, 4],
  HL@Simplify[lhs == rhs]} // Column
```

$\mathbb{E}_{\{\}\to\{1,2,3,4\}}\Big[-\hbar a_3 b_1 + 2\hbar a_3 b_2 - \hbar a_4 b_2 - \hbar a_1 b_3 + 2\hbar a_2 b_3 - \hbar a_2 b_4,$
$\frac{(-\hbar+\hbar B_3+\hbar B_4-\hbar B_2 B_4-\hbar B_3 B_4+\hbar B_2 B_3 B_4)\, x_2 y_1}{B_1 B_4} + \frac{(-\hbar B_2+\hbar B_2 B_3-\hbar B_2^2 B_3)\, x_3 y_1}{B_1 B_3} + \frac{(\hbar-\hbar B_2-\hbar B_3+\hbar B_2 B_3)\, x_4 y_1}{B_1 B_3} + \frac{(\hbar B_3-\hbar B_3 B_4+\hbar B_2 B_3 B_4-\hbar B_2 B_3^2 \ldots}{B_2 B_4}$
$\frac{\hbar x_1 y_3}{B_3} + \frac{(\hbar-\hbar B_1+\hbar B_1 B_3-\hbar B_4+2\hbar B_1 B_4+\hbar B_2 B_4-\hbar B_1 B_2 B_4-\hbar B_1 B_3 B_4+\hbar B_1 B_2 B_3 B_4)\, x_2 y_3}{B_1 B_4} + \frac{(\hbar B_2-\hbar B_1 B_2+\hbar B_1 B_2 B_3-\hbar B_1 B_2^2 B_3)\, x_3 y_3}{B_1 B_3} + \frac{(-\hbar+\hbar \ldots}{\ldots}$

Out[∘]=

$\frac{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}{T_1T_4}$	S_1	S_2
S_1	$\frac{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_4+T_3T_4-T_1T_3T_4}{T_3(1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4)}$	$-\frac{(-1+T_1)(-1+T_3)(1-T_2+T_2T_3)}{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}$ …
S_2	$\frac{(-1+T_2)(-1+T_3)(1+T_2T_3)}{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4}$	$-\frac{T_1T_3(1-T_2+T_2T_3)}{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4}$ …
S_3	$-\frac{T_2(-1+T_3)(1-T_3+T_2T_3)T_4}{T_3(1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4)}$	$\frac{T_2(-1+T_3)(T_2T_3+T_1T_4)}{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4}$ …
S_4	$\frac{(-1+T_2)(-1+T_3)(1+T_2T_3)(-1+T_4)}{T_3(1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4)}$	$\frac{T_1(1-T_2+T_2T_3)(-1+T_4)}{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}$ …
Γ	$\frac{1}{T_3}$	$\frac{T_3^2}{T_4}$

$\frac{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}{T_1T_4}$	S_1	S_2
S_1	$\frac{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_4+T_3T_4-T_1T_3T_4}{T_3(1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4)}$	$-\frac{(-1+T_1)(-1+T_3)(1-T_2+T_2T_3)}{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}$ …
S_2	$\frac{(-1+T_2)(-1+T_3)(1+T_2T_3)}{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4}$	$\frac{T_1T_3(1-T_2+T_2T_3)}{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}$
S_3	$-\frac{T_2(-1+T_3)(1-T_3+T_2T_3)T_4}{T_3(1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4)}$	$\frac{T_2(-1+T_3)(T_2T_3+T_1T_4)}{1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4}$ …
S_4	$\frac{(-1+T_2)(-1+T_3)(1+T_2T_3)(-1+T_4)}{T_3(1-T_2-T_3+2T_2T_3-T_2^2T_3-T_2T_3^2+T_2^2T_3^2-T_1T_4)}$	$\frac{T_1(1-T_2+T_2T_3)(-1+T_4)}{-1+T_2+T_3-2T_2T_3+T_2^2T_3+T_2T_3^2-T_2^2T_3^2+T_1T_4}$
Γ	$\frac{1}{T_3}$	$\frac{T_3^2}{T_4}$

True

In[∘]:=
```
HL[ℰ ≡ (ℰ // Γ // 𝔼)]
```

Out[∘]= True

In[∘]:= **Γ[V]**

Out[∘]=
$$\left(\begin{array}{ccc} \dfrac{\left(\frac{-1+T_1}{\mathrm{Log}[T_1]}\right)^{1/4}\left(\frac{-1+T_2}{\mathrm{Log}[T_2]}\right)^{1/4}}{\left(\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}\right)^{1/4}} & s_1 & s_2 \\ s_1 & \dfrac{\mathrm{Log}[T_1]\left(\mathrm{Log}[T_2]\sqrt{\frac{(-1+T_1)\,T_1}{\mathrm{Log}[T_1]}}\sqrt{\frac{-1+T_2}{\mathrm{Log}[T_2]}}-\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}+T_1\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}\right)}{\mathrm{Log}[T_1 T_2]\,(-1+T_1)\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}} & -\dfrac{\mathrm{Log}[T_1]\left(\sqrt{\frac{(-1+T_1)\,T_1}{\mathrm{Log}[T_1]}}\,T_2-\sqrt{\frac{-1+T_2}{\mathrm{Log}[T_2]}}\sqrt{\ldots}\right.}{\mathrm{Log}[T_1 T_2]\sqrt{\frac{-1+T_2}{\mathrm{Log}[T_2]}}\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}} \\ s_2 & \dfrac{\mathrm{Log}[T_2]\left(-T_1\sqrt{\frac{-1+T_2}{\mathrm{Log}[T_2]}}+\sqrt{\frac{(-1+T_1)\,T_1}{\mathrm{Log}[T_1]}}\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}\right)}{\mathrm{Log}[T_1 T_2]\sqrt{\frac{(-1+T_1)\,T_1}{\mathrm{Log}[T_1]}}\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}} & \dfrac{\mathrm{Log}[T_1]\sqrt{\frac{(-1+T_1)\,T_1}{\mathrm{Log}[T_1]}}\,T_2+\mathrm{Log}[T_2]\sqrt{\frac{-1+T_2}{\mathrm{Log}[T_2]}}\,\ldots}{\mathrm{Log}[T_1 T_2]\sqrt{\frac{-1+T_2}{\mathrm{Log}[T_2]}}\sqrt{\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}}} \\ \Gamma & 1 & \sqrt{T_1} \end{array}\right.$$

In[∘]:= **γ2 = Γ[ω, σ₁ h₁ + σ₂ h₂, {t₁, t₂}.(α θ; ψ Ξ).{h₁, h₂}]**

$$\gamma 2=\Gamma\left[\omega,\ \sigma_1 h_1+\sigma_2 h_2,\ \{t_1,t_2\}.\begin{pmatrix}\alpha & \theta\\ \psi & \Xi\end{pmatrix}.\{h_1,h_2\}\right]$$

Out[∘]=
$$\begin{pmatrix} \omega & s_1 & s_2 \\ s_1 & \alpha & \theta \\ s_2 & \psi & \Xi \\ \Gamma & \sigma_1 & \sigma_2 \end{pmatrix}$$

In[∘]:= **γ2 // MetaCalculi`tr[1] // 𝔼**

Out[∘]=
$$\mathbb{E}_{\{\}\to\{2\}}\Big[0,\ \frac{-\Xi\hbar x_2 y_2+\alpha\Xi\hbar x_2 y_2-\theta\psi\hbar x_2 y_2+\hbar x_2 y_2\sigma_2-\alpha\hbar x_2 y_2\sigma_2}{1-\alpha+\Xi-\alpha\Xi+\theta\psi-B_2+\alpha B_2-\Xi B_2+\alpha\Xi B_2-\theta\psi B_2-\sigma_2+\alpha\sigma_2+B_2\sigma_2-\alpha B_2\sigma_2},$$
$$-\frac{1}{-\omega+\alpha\omega-\Xi\omega+\alpha\Xi\omega-\theta\psi\omega+\omega\sigma_2-\alpha\omega\sigma_2}+O[\epsilon]^1\Big]$$

In[∘]:= **𝔼{}→{1,2}[l₁₁ b₁ a₁ + l₁₂ b₁ a₂ + l₂₁ b₂ a₁ + l₂₂ b₂ a₂, q₁₁ y₁ x₁ + q₁₂ y₁ x₂ + q₂₁ y₂ x₁ + q₂₂ y₂ x₂, ω⁻¹] // Γ // Simplify**

$$\mathbb{E}_{\{\}\to\{1,2\}}\left[l_{11} b_1 a_1+l_{12} b_1 a_2+l_{21} b_2 a_1+l_{22} b_2 a_2,\ q_{11} y_1 x_1+q_{12} y_1 x_2+q_{21} y_2 x_1+q_{22} y_2 x_2,\ \omega^{-1}\right]\ //\ \Gamma\ //\ \mathrm{Simplify}$$

Out[∘]=
$$\left(\begin{array}{c} \dfrac{\omega\left(\frac{1}{T_1}\right)^{1-l_{12}}\left(\left(\frac{1}{T_1}\right)^{-1+l_{12}}T_2+q_{11}(-1+T_1)\left(\frac{1}{T_1}\right)^{l_{12}}T_1^{l_{11}}T_2^{1+l_{21}}-q_{22}T_1T_2^{l_{22}}+q_{22}T_1T_2^{1+l_{22}}+(q_{12}q_{21}-q_{11}q_{22})(-1+T_1)T_1^{l_{11}}T_2^{l_{21}+l_{22}}-(q_{12}q_{21}-q_{11}q_{22})(-1+T_1)T_1^{\ldots}\right.}{T_2} \\ s_1 \\ s_2 \\ \Gamma \end{array}\right.$$

In[•]:= $\mathbb{E}_{\{\}\to\{1,2\}}[l_{11}\,b_1\,a_1 + l_{12}\,b_1\,a_2 + l_{21}\,b_2\,a_1 + l_{22}\,b_2\,a_2,\ q_{11}\,y_1\,x_1 + q_{12}\,y_1\,x_2 + q_{21}\,y_2\,x_1 + q_{22}\,y_2\,x_2,\ \omega^{-1}]$ // Γ // MetaCalculi`tr[1] // $\mathbb{E}$ // Simplify

Out[•]=

$$\mathbb{E}_{\{\}\to\{2\}}\Big[a_2\,b_2\,l_{22},$$

$$\left(\left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{2\,l_{22}} \left(-B_1\,q_{22} + B_1^{1+l_{11}} \left(\frac{1}{B_2}\right)^{l_{21}} \left(q_{12}\,q_{21} - (-1+q_{11})\,q_{22}\right) + B_1^{l_{11}} \left(\frac{1}{B_2}\right)^{l_{21}} \left(-q_{12}\,q_{21} + q_{11}\,q_{22}\right)\right)\right.$$

$$\left.x_2\,y_2\right) \Big/ \left(-B_1\,(-1+B_2)\left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{l_{22}}\,q_{22} + \left(\frac{1}{B_1}\right)^{-1+l_{12}} \left(-\left(\frac{1}{B_1}\right)^{l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} q_{22}\right) + \right.$$

$$B_1^{1+l_{11}} \left(\frac{1}{B_2}\right)^{l_{21}} \left(\left(\left(\frac{1}{B_1}\right)^{2\,l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(-\left(\frac{1}{B_1}\right)^{l_{12}} + \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{12}\,q_{21} - \right.\right.$$

$$\left(\frac{1}{B_1}\right)^{l_{12}} \left(\frac{1}{B_2}\right)^{-1+l_{22}} q_{22} + \left(\frac{1}{B_1}\right)^{l_{12}} \left(\frac{1}{B_2}\right)^{l_{22}} q_{22} - \left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{l_{22}}\,q_{22} + \left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{1+l_{22}}\,q_{22} -$$

$$\left.q_{11}\left(\left(\frac{1}{B_1}\right)^{2\,l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(-\left(\frac{1}{B_1}\right)^{l_{12}} + \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{22}\right)\right) +$$

$$B_1^{l_{11}} \left(\frac{1}{B_2}\right)^{l_{21}} \left((-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(\left(\frac{1}{B_1}\right)^{l_{12}} - \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{12}\,q_{21} + \right.$$

$$\left.\left.q_{11}\left(\left(\frac{1}{B_1}\right)^{2\,l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(-\left(\frac{1}{B_1}\right)^{l_{12}} + \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{22}\right)\right)\right),$$

$$\left(\frac{1}{B_1}\right)^{-1+2\,l_{12}} \Big/ \left(\omega\left(B_1\,(-1+B_2)\left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{l_{22}}\,q_{22} + \left(\frac{1}{B_1}\right)^{-1+l_{12}} \left(\left(\frac{1}{B_1}\right)^{l_{12}} - (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} q_{22}\right) - \right.\right.$$

$$B_1^{l_{11}} \left(\frac{1}{B_2}\right)^{l_{21}} \left((-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(\left(\frac{1}{B_1}\right)^{l_{12}} - \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{12}\,q_{21} + \right.$$

$$\left.q_{11}\left(\left(\frac{1}{B_1}\right)^{2\,l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(-\left(\frac{1}{B_1}\right)^{l_{12}} + \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{22}\right)\right) +$$

$$B_1^{1+l_{11}} \left(\frac{1}{B_2}\right)^{l_{21}} \left(-\left(\frac{1}{B_1}\right)^{2\,l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(\left(\frac{1}{B_1}\right)^{l_{12}} - \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{12}\,q_{21} + \right.$$

$$\left(\frac{1}{B_1}\right)^{l_{12}} \left(\frac{1}{B_2}\right)^{-1+l_{22}} q_{22} - \left(\frac{1}{B_1}\right)^{l_{12}} \left(\frac{1}{B_2}\right)^{l_{22}} q_{22} + \left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{l_{22}}\,q_{22} - \left(\frac{1}{B_2}\right)^{2\,l_{22}} B_2^{1+l_{22}}\,q_{22} +$$

$$\left.\left.\left.q_{11}\left(\left(\frac{1}{B_1}\right)^{2\,l_{12}} + (-1+B_2)\left(\frac{1}{B_2}\right)^{l_{22}} \left(-\left(\frac{1}{B_1}\right)^{l_{12}} + \left(\frac{1}{B_2}\right)^{l_{22}} B_2^{l_{22}}\right) q_{22}\right)\right)\right)\right) + O[\epsilon]^1\Big]$$