Pensieve header: Exp relative to am, bm, cm, dm.

Follows code in Projects/SL2Portfolio/SL2PortfolioProgram.nb.

Startup

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
(*Once[<< KnotTheory`];*)
Once[<< "../Profile/Profile.m";];
<< "Engine-Speedy.m";
<< "Objects.m";
$k = 3;
HL[ℰ_] := Style[ℰ, Background → Green];
```

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: June 2018. Original version: July 1994.
```

Exponentials as needed.

Task. Define $\mathrm{Exp}_{m,i,k}[\lambda, P]$ which computes $e^{\lambda\,\mathbb{O}(P)}$ to ϵ^k in the using the $m_{i,i\to i}$ multiplication, where λ is a scalar and P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.
Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\lambda\,\mathbb{O}(P)} = \mathbb{O}\left(e^{\lambda P_0} F(\lambda)\right)$, then $F(\lambda = 0) = 1$ and we have:
$\mathbb{O}\left(e^{\lambda P_0}(P_0 F(\lambda) + \partial_\lambda F)\right) = \mathbb{O}\left(\partial_\lambda e^{\lambda P_0} F(\lambda)\right) = \partial_\lambda \mathbb{O}\left(e^{\lambda P_0} F(\lambda)\right) = \partial_\lambda e^{\lambda\,\mathbb{O}(P)} = e^{\lambda\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}\left(e^{\lambda P_0} F(\lambda)\right)\mathbb{O}(P)$.
This is an ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ.

```
In[ ]:= (* Bug: The first line is valid only if 𝕆(e^P0) == e^𝕆(P0). *)
(* Bug: λ must be a symbol. *)
Exp_{m_,i_,0}[λ_, P_] := Module[{LQ = Normal@P /. ϵ → 0},
   𝔼[λ LQ /. (x | y)_i → 0, λ LQ /. (b | a | t)_i → 0, 1]];
Exp_{m_,i_,k_}[λ_, P_] := Block[{$k = k},
  Module[{P0, φ, φs, F, j, rhs, at0, atλ},
   P0 = Normal@P /. ϵ → 0;
   φs = Flatten@Table[φ_{j1,j2,j3,j4}[λ], {j2, 0, k},
      {j3, 0, k - j2}, {j1, 0, 2 k + 1 - j2 - j3}, {j4, 0, 2 k + 1 - j3 - j2 - j1}];
   F = Normal@Last@Exp_{m,i,k-1}[λ, P] + ϵ^k φs.(φs /. φ_js__[λ] :→ Times @@ {y_i, b_i, a_i, x_i}^{js});
   rhs = Normal@Last@m_{i,j→i}[𝔼_{{}→{i}}[λ P0 /. (x | y)_i → 0, λ P0 /. (b | a | t)_i → 0, F + O[ϵ]^(k+1)]
        𝔼_{{}→{j}}[0, 0, P + O[ϵ]^(k+1) /. u_i :→ u_j]];
   at0 = (# == 0) & /@ Flatten@CoefficientList[F - 1 /. λ → 0, {y_i, b_i, a_i, x_i}];
   atλ = (# == 0) & /@ Flatten@CoefficientList[(∂_λ F) + P0 F - rhs, {y_i, b_i, a_i, x_i}];
   𝔼_{{}→{i}}[λ P0 /. (x | y)_i → 0, λ P0 /. (b | a | t)_i → 0, F + O[ϵ]^(k+1)] /.
    DSolve[And @@ (at0 ⋃ atλ), φs, λ]⟦1⟧]
 ]
```

In[∘]:= **Exp$_{dm,1,2}$[ξ, x$_1$ + ϵ y$_1$]**

Out[∘]= $\mathbb{E}_{\{\}\to\{1\}}\left[0,\ \xi\,x_1,\ 1+\left(\frac{\xi^2-\xi^2 B_1}{2\hbar}+\xi\,y_1\right)\epsilon+\left(\frac{\xi^4\left(-1+B_1\right)^2}{8\hbar^2}+\frac{1}{2}\xi^2 a_1 B_1-\frac{1}{6}\gamma\,\xi^3\left(-1+3B_1\right)x_1-\frac{\xi^3\left(-1+B_1\right)y_1}{2\hbar}+\frac{1}{2}\gamma\,\xi^2\hbar\,x_1 y_1+\frac{1}{2}\xi^2 y_1^2\right)\epsilon^2+O[\epsilon]^3\right]$

In[∘]:= **dS$_1$[𝔼$_{\{\}\to\{1\}}$[0, 0, #]] & /@ {y$_1$, x$_1$}**

Out[∘]= $\left\{\mathbb{E}_{\{\}\to\{1\}}\left[0,\ 0,\ -\frac{y_1}{B_1}+\frac{\gamma\,\hbar\,y_1\,\epsilon}{B_1}-\frac{\left(\gamma^2\hbar^2 y_1\right)\epsilon^2}{2B_1}+\frac{\gamma^3\hbar^3 y_1\,\epsilon^3}{6B_1}+O[\epsilon]^4\right],\ \mathbb{E}_{\{\}\to\{1\}}\left[0,\ 0,\ -x_1-\hbar\,a_1 x_1\,\epsilon-\frac{1}{2}\left(\hbar^2 a_1^2 x_1\right)\epsilon^2-\frac{1}{6}\left(\hbar^3 a_1^3 x_1\right)\epsilon^3+O[\epsilon]^4\right]\right\}$

In[∘]:= **Timing@{lhs = 𝔼$_{\{1\}\to\{1\}}$[0, ξ$_1$ x$_1$, 1] // dS$_1$,**
rhs = Exp$_{dm,1,\$k}$[ξ, Last@dS$_1$[𝔼$_{\{\}\to\{1\}}$[0, 0, x$_1$]]] /. {ξ → ξ$_1$, {} → {1}}; HL[lhs ≡ rhs]}

Out[∘]= $\left\{2.95313,\ \left\{\mathbb{E}_{\{1\}\to\{1\}}\left[0,\ -x_1\xi_1,\ 1+\left(-\hbar\,a_1 x_1\xi_1-\frac{1}{2}\gamma\,\hbar\,x_1^2\xi_1^2\right)\epsilon+\left(-\frac{1}{2}\hbar^2 a_1^2 x_1\xi_1+\frac{1}{4}\gamma^2\hbar^2 x_1^2\xi_1^2-\gamma\,\hbar^2 a_1 x_1^2\xi_1^2+\frac{1}{2}\hbar^2 a_1^2 x_1^2\xi_1^2-\frac{1}{2}\gamma^2\hbar^2 x_1^3\xi_1^3+\frac{1}{2}\gamma\,\hbar^2 a_1 x_1^3\xi_1^3+\frac{1}{8}\gamma^2\hbar^2 x_1^4\xi_1^4\right)\epsilon^2+\left(-\frac{1}{6}\hbar^3 a_1^3 x_1\xi_1-\frac{1}{12}\gamma^3\hbar^3 x_1^2\xi_1^2+\frac{1}{2}\gamma^2\hbar^3 a_1 x_1^2\xi_1^2-\gamma\,\hbar^3 a_1^2 x_1^2\xi_1^2+\frac{1}{2}\hbar^3 a_1^3 x_1^2\xi_1^2+\frac{2}{3}\gamma^3\hbar^3 x_1^3\xi_1^3-\frac{7}{4}\gamma^2\hbar^3 a_1 x_1^3\xi_1^3+\frac{5}{4}\gamma\,\hbar^3 a_1^2 x_1^3\xi_1^3-\frac{1}{6}\hbar^3 a_1^3 x_1^3\xi_1^3-\frac{19}{24}\gamma^3\hbar^3 x_1^4\xi_1^4+\gamma^2\hbar^3 a_1 x_1^4\xi_1^4-\frac{1}{4}\gamma\,\hbar^3 a_1^2 x_1^4\xi_1^4+\frac{1}{4}\gamma^3\hbar^3 x_1^5\xi_1^5-\frac{1}{8}\gamma^2\hbar^3 a_1 x_1^5\xi_1^5-\frac{1}{48}\gamma^3\hbar^3 x_1^6\xi_1^6\right)\epsilon^3+O[\epsilon]^4\right],\ \text{True}\right\}\right\}$

In[∘]:= **Timing@{lhs = 𝔼$_{\{1\}\to\{1\}}$[0, η$_1$ y$_1$, 1] // dS$_1$,**
rhs = Exp$_{dm,1,\$k}$[η, Last@dS$_1$[𝔼$_{\{\}\to\{1\}}$[0, 0, y$_1$]]] /. {η → η$_1$, {} → {1}}; HL[lhs ≡ rhs]}

Out[∘]= $\left\{14.5938,\ \left\{\mathbb{E}_{\{1\}\to\{1\}}\left[0,\ -\frac{y_1\eta_1}{B_1},\ 1+\left(\frac{\gamma\,\hbar\,y_1\eta_1}{B_1}-\frac{\gamma\,\hbar\,y_1^2\eta_1^2}{2B_1^2}\right)\epsilon+\left(-\frac{\gamma^2\hbar^2 y_1\eta_1}{2B_1}+\frac{7\gamma^2\hbar^2 y_1^2\eta_1^2}{4B_1^2}-\frac{\gamma^2\hbar^2 y_1^3\eta_1^3}{B_1^3}+\frac{\gamma^2\hbar^2 y_1^4\eta_1^4}{8B_1^4}\right)\epsilon^2+\left(\frac{\gamma^3\hbar^3 y_1\eta_1}{6B_1}-\frac{25\gamma^3\hbar^3 y_1^2\eta_1^2}{12B_1^2}+\frac{23\gamma^3\hbar^3 y_1^3\eta_1^3}{6B_1^3}-\frac{49\gamma^3\hbar^3 y_1^4\eta_1^4}{24B_1^4}+\frac{3\gamma^3\hbar^3 y_1^5\eta_1^5}{8B_1^5}-\frac{\gamma^3\hbar^3 y_1^6\eta_1^6}{48B_1^6}\right)\epsilon^3+O[\epsilon]^4\right],\ \text{True}\right\}\right\}$

In[∘]:= **Timing@{lhs = 𝔼$_{\{1\}\to\{1\}}$[0, ξ$_1$ x$_1$, 1] // cS$_1$,**
rhs = Exp$_{cm,1,\$k}$[ξ, Last@cS$_1$[𝔼$_{\{\}\to\{1\}}$[0, 0, x$_1$]]] /. {ξ → ξ$_1$, {} → {1}}; HL[lhs ≡ rhs]}

Out[∘]= $\left\{4.85938,\ \left\{\mathbb{E}_{\{1\}\to\{1\}}\left[0,\ -x_1\xi_1,\ 1+O[\epsilon]^4\right],\ \text{True}\right\}\right\}$

In[∘]:= **Timing@{lhs = 𝔼$_{\{1\}\to\{1\}}$[0, η$_1$ y$_1$, 1] // cS$_1$,**
rhs = Exp$_{cm,1,\$k}$[η, Last@cS$_1$[𝔼$_{\{\}\to\{1\}}$[0, 0, y$_1$]]] /. {η → η$_1$, {} → {1}}; HL[lhs ≡ rhs]}

··· DSolve: There are fewer dependent variables than equations, so the system is overdetermined.

ReplaceAll: {≪1≫} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

Out[∘]= $\{5.,\ \{\mathbb{E}_{\{1\}\to\{1\}}[0,\ -y_1\,\eta_1,\ 1+O[\epsilon]^4],$

$(\epsilon\,\varphi\$47215_{0,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,2}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,0,3}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,1}[0] == 0\ \&\&$
$\gamma\,\epsilon\,\varphi\$47215_{0,0,1,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,2}[0] == 0\ \&\&$
$2\,\gamma\,\epsilon\,\varphi\$47215_{0,0,1,2}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,1,0,0}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,1,0,1}[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{0,1,0,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,1,0,2}[0] == 0\ \&\&$
$2\,\gamma\,\epsilon\,\varphi\$47215_{0,1,0,2}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,0,1}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{1,0,0,2}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,1,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,1,1}[0] == 0\ \&\&$
$\gamma\,\epsilon\,\varphi\$47215_{1,0,1,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,1,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,1,0,1}[0] == 0\ \&\&$
$\gamma\,\epsilon\,\varphi\$47215_{1,1,0,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{2,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{2,0,0,1}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{2,0,1,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{2,1,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{3,0,0,0}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,0,0}'[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,1}'[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,2}'[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,0,3}'[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,0}'[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,1}'[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,1,2}'[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{0,0,0,1}[0] + \epsilon\,\varphi\$47215_{0,1,0,0}'[0] == 0\ \&\&$
$2\,\gamma\,\epsilon\,\varphi\$47215_{0,0,0,2}[0] + \epsilon\,\varphi\$47215_{0,1,0,1}'[0] == 0\ \&\&$
$3\,\gamma\,\epsilon\,\varphi\$47215_{0,0,0,3}[0] + \epsilon\,\varphi\$47215_{0,1,0,2}'[0] == 0\ \&\&$
$-\gamma\,\epsilon\,\varphi\$47215_{0,0,1,0}[0] + \epsilon\,\varphi\$47215_{1,0,0,0}'[0] == 0\ \&\&$
$-\gamma\,\epsilon\,\varphi\$47215_{0,0,1,1}[0] + \epsilon\,\varphi\$47215_{1,0,0,1}'[0] == 0\ \&\&$
$-\gamma\,\epsilon\,\varphi\$47215_{0,0,1,2}[0] + \epsilon\,\varphi\$47215_{1,0,0,2}'[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,1,0}'[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{1,0,1,1}'[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{1,0,0,1}[0] + \epsilon\,\varphi\$47215_{1,1,0,0}'[0] == 0\ \&\&$
$2\,\gamma\,\epsilon\,\varphi\$47215_{1,0,0,2}[0] + \epsilon\,\varphi\$47215_{1,1,0,1}'[0] == 0\ \&\&$
$-\gamma\,\epsilon\,\varphi\$47215_{1,0,1,0}[0] + \epsilon\,\varphi\$47215_{2,0,0,0}'[0] == 0\ \&\&$
$-\gamma\,\epsilon\,\varphi\$47215_{1,0,1,1}[0] + \epsilon\,\varphi\$47215_{2,0,0,1}'[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{2,0,1,0}'[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{2,0,0,1}[0] + \epsilon\,\varphi\$47215_{2,1,0,0}'[0] == 0\ \&\&$
$-\gamma\,\epsilon\,\varphi\$47215_{2,0,1,0}[0] + \epsilon\,\varphi\$47215_{3,0,0,0}'[0] == 0) -$
$(\epsilon\,\varphi\$47215_{0,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,2}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,0,3}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,1}[0] == 0\ \&\&$
$\gamma\,\epsilon\,\varphi\$47215_{0,0,1,1}[\eta_1] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,2}[0] == 0\ \&\&\ 2\,\gamma\,\epsilon\,\varphi\$47215_{0,0,1,2}[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,1,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,1,0,1}[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{0,1,0,1}[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,1,0,2}[0] == 0\ \&\&\ 2\,\gamma\,\epsilon\,\varphi\$47215_{0,1,0,2}[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{1,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,0,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,0,2}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{1,0,1,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,1,1}[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{1,0,1,1}[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{1,1,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,1,0,1}[0] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{1,1,0,1}[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{2,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{2,0,0,1}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{2,0,1,0}[0] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{2,1,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{3,0,0,0}[0] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,0}'[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,0,1}'[\eta_1] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,0,2}'[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,0,3}'[\eta_1] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,0}'[\eta_1] == 0\ \&\&\ \epsilon\,\varphi\$47215_{0,0,1,1}'[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{0,0,1,2}'[\eta_1] == 0\ \&\&\ \gamma\,\epsilon\,\varphi\$47215_{0,0,0,1}[\eta_1] + \epsilon\,\varphi\$47215_{0,1,0,0}'[\eta_1] == 0\ \&\&$
$2\,\gamma\,\epsilon\,\varphi\$47215_{0,0,0,2}[\eta_1] + \epsilon\,\varphi\$47215_{0,1,0,1}'[\eta_1] == 0\ \&\&$
$3\,\gamma\,\epsilon\,\varphi\$47215_{0,0,0,3}[\eta_1] + \epsilon\,\varphi\$47215_{0,1,0,2}'[\eta_1] == 0\ \&\&\ -\epsilon\,\varphi\$47215_{0,0,0,0}[\eta_1] -$
$\quad \epsilon\,(-\varphi\$47215_{0,0,0,0}[\eta_1] + \gamma\,\varphi\$47215_{0,0,1,0}[\eta_1]) + \epsilon\,\varphi\$47215_{1,0,0,0}'[\eta_1] == 0\ \&\&$
$-\epsilon\,\varphi\$47215_{0,0,0,1}[\eta_1] - \epsilon\,(-\varphi\$47215_{0,0,0,1}[\eta_1] + \gamma\,\varphi\$47215_{0,0,1,1}[\eta_1]) +$
$\quad \epsilon\,\varphi\$47215_{1,0,0,1}'[\eta_1] == 0\ \&\&\ -\epsilon\,\varphi\$47215_{0,0,0,2}[\eta_1] -$
$\quad \epsilon\,(-\varphi\$47215_{0,0,0,2}[\eta_1] + \gamma\,\varphi\$47215_{0,0,1,2}[\eta_1]) + \epsilon\,\varphi\$47215_{1,0,0,2}'[\eta_1] == 0\ \&\&$
$\epsilon\,\varphi\$47215_{1,0,1,0}'[\eta_1] == 0\ \&\&\ \epsilon\,\varphi\$47215_{1,0,1,1}'[\eta_1] == 0\ \&\&\ -\epsilon\,\varphi\$47215_{0,1,0,0}[\eta_1] -$
$\quad \epsilon\,(-\varphi\$47215_{0,1,0,0}[\eta_1] - \gamma\,\varphi\$47215_{1,0,0,1}[\eta_1]) + \epsilon\,\varphi\$47215_{1,1,0,0}'[\eta_1] == 0\ \&\&$
$-\epsilon\,\varphi\$47215_{0,1,0,1}[\eta_1] - \epsilon\,(-\varphi\$47215_{0,1,0,1}[\eta_1] - 2\,\gamma\,\varphi\$47215_{1,0,0,2}[\eta_1]) +$

$\in\varphi\$47215_{1,1,0,1}'[\eta_1]==0\;\&\&\;-\in\varphi\$47215_{1,0,0,0}[\eta_1]-$
$\in\left(-\varphi\$47215_{1,0,0,0}[\eta_1]+\gamma\,\varphi\$47215_{1,0,1,0}[\eta_1]\right)+\in\varphi\$47215_{2,0,0,0}'[\eta_1]==0\;\&\&$
$-\in\varphi\$47215_{1,0,0,1}[\eta_1]-\in\left(-\varphi\$47215_{1,0,0,1}[\eta_1]+\gamma\,\varphi\$47215_{1,0,1,1}[\eta_1]\right)+$
$\in\varphi\$47215_{2,0,0,1}'[\eta_1]==0\;\&\&\in\varphi\$47215_{2,0,1,0}'[\eta_1]==0\;\&\&$
$-\in\varphi\$47215_{1,1,0,0}[\eta_1]-\in\left(-\varphi\$47215_{1,1,0,0}[\eta_1]-\gamma\,\varphi\$47215_{2,0,0,1}[\eta_1]\right)+$
$\in\varphi\$47215_{2,1,0,0}'[\eta_1]==0\;\&\&\;-\in\varphi\$47215_{2,0,0,0}[\eta_1]-$
$\in\left(-\varphi\$47215_{2,0,0,0}[\eta_1]+\gamma\,\varphi\$47215_{2,0,1,0}[\eta_1]\right)+\in\varphi\$47215_{3,0,0,0}'[\eta_1]==0\Big)+$

$\in^2\int_1^0-\frac{1}{\in^2}\Big(\gamma\in\varphi\$47215_{0,0,1,1}'[K[1]]==0\;\&\&\;2\gamma\in\varphi\$47215_{0,0,1,2}'[K[1]]==0\;\&\&$
$\gamma\in\varphi\$47215_{0,1,0,1}'[K[1]]==0\;\&\&\;2\gamma\in\varphi\$47215_{0,1,0,2}'[K[1]]==0\;\&\&\;\gamma\in\varphi\$47215_{1,0,1,1}'[$
$K[1]]==0\;\&\&\;\gamma\in\varphi\$47215_{1,1,0,1}'[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,0,0}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{0,0,0,1}''[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,0,2}''[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,0,3}''[K[1]]==$
$0\;\&\&\in\varphi\$47215_{0,0,1,0}''[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,1,1}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{0,0,1,2}''[K[1]]==0\;\&\&\;\gamma\in\varphi\$47215_{0,0,0,1}'[K[1]]+\in\varphi\$47215_{0,1,0,0}''[K[1]]==0\;\&\&$
$2\gamma\in\varphi\$47215_{0,0,0,2}'[K[1]]+\in\varphi\$47215_{0,1,0,1}''[K[1]]==0\;\&\&$
$3\gamma\in\varphi\$47215_{0,0,0,3}'[K[1]]+\in\varphi\$47215_{0,1,0,2}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{0,0,0,0}'[K[1]]-$
$\in\left(-\varphi\$47215_{0,0,0,0}'[K[1]]+\gamma\,\varphi\$47215_{0,0,1,0}'[K[1]]\right)+\in\varphi\$47215_{1,0,0,0}''[K[1]]==0\;\&\&$
$-\in\varphi\$47215_{0,0,0,1}'[K[1]]-\in\left(-\varphi\$47215_{0,0,0,1}'[K[1]]+\gamma\,\varphi\$47215_{0,0,1,1}'[K[1]]\right)+$
$\in\varphi\$47215_{1,0,0,1}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{0,0,0,2}'[K[1]]-$
$\in\left(-\varphi\$47215_{0,0,0,2}'[K[1]]+\gamma\,\varphi\$47215_{0,0,1,2}'[K[1]]\right)+\in\varphi\$47215_{1,0,0,2}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{1,0,1,0}''[K[1]]==0\;\&\&\in\varphi\$47215_{1,0,1,1}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{0,1,0,0}'[K[1]]-$
$\in\left(-\varphi\$47215_{0,1,0,0}'[K[1]]-\gamma\,\varphi\$47215_{1,0,0,1}'[K[1]]\right)+\in\varphi\$47215_{1,1,0,0}''[K[1]]==0\;\&\&$
$-\in\varphi\$47215_{0,1,0,1}'[K[1]]-\in\left(-\varphi\$47215_{0,1,0,1}'[K[1]]-2\gamma\,\varphi\$47215_{1,0,0,2}'[K[1]]\right)+$
$\in\varphi\$47215_{1,1,0,1}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{1,0,0,0}'[K[1]]-$
$\in\left(-\varphi\$47215_{1,0,0,0}'[K[1]]+\gamma\,\varphi\$47215_{1,0,1,0}'[K[1]]\right)+\in\varphi\$47215_{2,0,0,0}''[K[1]]==0\;\&\&$
$-\in\varphi\$47215_{1,0,0,1}'[K[1]]-\in\left(-\varphi\$47215_{1,0,0,1}'[K[1]]+\gamma\,\varphi\$47215_{1,0,1,1}'[K[1]]\right)+$
$\in\varphi\$47215_{2,0,0,1}''[K[1]]==0\;\&\&\in\varphi\$47215_{2,0,1,0}''[K[1]]==0\;\&\&$
$-\in\varphi\$47215_{1,1,0,0}'[K[1]]-\in\left(-\varphi\$47215_{1,1,0,0}'[K[1]]-\gamma\,\varphi\$47215_{2,0,0,1}'[K[1]]\right)+$
$\in\varphi\$47215_{2,1,0,0}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{2,0,0,0}'[K[1]]-\in\Big(-\varphi\$47215_{2,0,0,0}'[K[1]]+$
$\gamma\,\varphi\$47215_{2,0,1,0}'[K[1]]\Big)+\in\varphi\$47215_{3,0,0,0}''[K[1]]==0\Big)\,dK[1]-$

$\in^2\int_1^{\eta_1}-\frac{1}{\in^2}\Big(\gamma\in\varphi\$47215_{0,0,1,1}'[K[1]]==0\;\&\&\;2\gamma\in\varphi\$47215_{0,0,1,2}'[K[1]]==0\;\&\&$
$\gamma\in\varphi\$47215_{0,1,0,1}'[K[1]]==0\;\&\&\;2\gamma\in\varphi\$47215_{0,1,0,2}'[K[1]]==0\;\&\&$
$\gamma\in\varphi\$47215_{1,0,1,1}'[K[1]]==0\;\&\&\;\gamma\in\varphi\$47215_{1,1,0,1}'[K[1]]==0\;\&\&$
$\in\varphi\$47215_{0,0,0,0}''[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,0,1}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{0,0,0,2}''[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,0,3}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{0,0,1,0}''[K[1]]==0\;\&\&\in\varphi\$47215_{0,0,1,1}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{0,0,1,2}''[K[1]]==0\;\&\&\;\gamma\in\varphi\$47215_{0,0,0,1}'[K[1]]+\in\varphi\$47215_{0,1,0,0}''[K[1]]==0\;\&\&$
$2\gamma\in\varphi\$47215_{0,0,0,2}'[K[1]]+\in\varphi\$47215_{0,1,0,1}''[K[1]]==0\;\&\&$
$3\gamma\in\varphi\$47215_{0,0,0,3}'[K[1]]+\in\varphi\$47215_{0,1,0,2}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{0,0,0,0}'[K[1]]-$
$\in\left(-\varphi\$47215_{0,0,0,0}'[K[1]]+\gamma\,\varphi\$47215_{0,0,1,0}'[K[1]]\right)+\in\varphi\$47215_{1,0,0,0}''[K[1]]==0\;\&\&$
$-\in\varphi\$47215_{0,0,0,1}'[K[1]]-\in\left(-\varphi\$47215_{0,0,0,1}'[K[1]]+\gamma\,\varphi\$47215_{0,0,1,1}'[K[1]]\right)+$
$\in\varphi\$47215_{1,0,0,1}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{0,0,0,2}'[K[1]]-$
$\in\left(-\varphi\$47215_{0,0,0,2}'[K[1]]+\gamma\,\varphi\$47215_{0,0,1,2}'[K[1]]\right)+\in\varphi\$47215_{1,0,0,2}''[K[1]]==0\;\&\&$
$\in\varphi\$47215_{1,0,1,0}''[K[1]]==0\;\&\&\in\varphi\$47215_{1,0,1,1}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{0,1,0,0}'[K[1]]-$
$\in\left(-\varphi\$47215_{0,1,0,0}'[K[1]]-\gamma\,\varphi\$47215_{1,0,0,1}'[K[1]]\right)+\in\varphi\$47215_{1,1,0,0}''[K[1]]==0\;\&\&$
$-\in\varphi\$47215_{0,1,0,1}'[K[1]]-\in\left(-\varphi\$47215_{0,1,0,1}'[K[1]]-2\gamma\,\varphi\$47215_{1,0,0,2}'[K[1]]\right)+$
$\in\varphi\$47215_{1,1,0,1}''[K[1]]==0\;\&\&\;-\in\varphi\$47215_{1,0,0,0}'[K[1]]-$

$\in\left(-\varphi\$47215_{1,0,0,0}'[K[1]]+\gamma\,\varphi\$47215_{1,0,1,0}'[K[1]]\right)+\in\varphi\$47215_{2,0,0,0}''[K[1]]==0\,\&\&$
$-\in\varphi\$47215_{1,0,0,1}'[K[1]]-\in\left(-\varphi\$47215_{1,0,0,1}'[K[1]]+\gamma\,\varphi\$47215_{1,0,1,1}'[K[1]]\right)+$
$\in\varphi\$47215_{2,0,0,1}''[K[1]]==0\,\&\&\in\varphi\$47215_{2,0,1,0}''[K[1]]==0\,\&\&$
$-\in\varphi\$47215_{1,1,0,0}'[K[1]]-\in\left(-\varphi\$47215_{1,1,0,0}'[K[1]]-\gamma\,\varphi\$47215_{2,0,0,1}'[K[1]]\right)+$
$\in\varphi\$47215_{2,1,0,0}''[K[1]]==0\,\&\&-\in\varphi\$47215_{2,0,0,0}'[K[1]]-\in\left(-\varphi\$47215_{2,0,0,0}'[K[1]]+\right.$
$\left.\gamma\,\varphi\$47215_{2,0,1,0}'[K[1]]\right)+\in\varphi\$47215_{3,0,0,0}''[K[1]]==0)\;\mathrm{d}K[1]==0\}\}$