Pensieve header: The CU definitions.

Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
Once[<< KnotTheory`];
<< "Engine-Speedy.m";
<< "Objects.m";
```

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[∘]:= `U2l`

Out[∘]= $\{B^{p\_\cdot}_{i\_} \to e^{-p\gamma\hbar b_i},\ B^{p\_\cdot} \to e^{-bp\gamma\hbar},\ T^{p\_\cdot}_{i\_} \to e^{p\hbar t_i},\ T^{p\_\cdot} \to e^{pt\hbar},\ \mathcal{A}^{p\_\cdot}_{i\_} \to e^{p\gamma\alpha_i},\ \mathcal{A}^{p\_\cdot} \to e^{p\alpha\gamma}\}$

In[∘]:=
ρ@CU[y] = $\begin{pmatrix} 0 & 0 \\ \epsilon & 0 \end{pmatrix}$;
ρ@CU[b] = $\begin{pmatrix} 0 & 0 \\ 0 & -\epsilon \end{pmatrix}$;
ρ@CU[a] = $\begin{pmatrix} \gamma & 0 \\ 0 & 0 \end{pmatrix}$;
ρ@CU[x] = $\begin{pmatrix} 0 & \gamma \\ 0 & 0 \end{pmatrix}$;
$\rho[e^{\mathcal{E}\_}]$:= MatrixExp[ρ[$\mathcal{E}$]];
$\rho[\mathcal{E}\_]$:= ($\mathcal{E}$ /. U2l /. t → $\gamma\epsilon$ /. (U : CU | QU)[u___] :→ Fold[Dot, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, ρ /@ U /@ {u}])

In[∘]:= $\rho[\frac{\epsilon}{\gamma}$ CU[a]$] - \rho[\frac{t}{\gamma}$ CU[]$]$ // Simplify // MatrixForm

Out[∘]//MatrixForm=
$\begin{pmatrix} 0 & 0 \\ 0 & -\epsilon \end{pmatrix}$

In[∘]:= ρ@CU[b, x] − ρ@CU[x, b] == ϵ ρ@CU[x]

Out[∘]= True

In[∘]:= $\lambda_{alt,k\_}$[CU] := If[k == 0, 1, Module[{eq, d, b, c, so},
eq = ρ@$e^{\xi x_{CU}}$.ρ@$e^{\eta y_{CU}}$ == ρ@$e^{d y_{CU}}$.ρ@$e^{c (t 1_{CU} - 2\epsilon a_{CU})}$.$\rho$@$e^{b x_{CU}}$;
{so} = Solve[Thread[Flatten /@ eq], {d, b, c}] /. C@1 → 0;
Series[$e^{-\eta y - \xi x + \eta\xi t + c t + d y - 2\epsilon c a + b x}$ /. so, {ϵ, 0, k}]]];

In[∘]:= $tm_{i\_,j\_\to k\_}$:= Module[{tk},
$\mathbb{E}$[($\tau_i + \tau_j$) t_k + α_i a_k + α_j a_k, η_i y_k + ξ_j x_k, 1]
($tSW_{xy,i,j\to tk}$ /. {$t_{tk} \to t_k$, $T_{tk} \to T_k$, $y_{tk} \to e^{-\gamma\alpha_i} y_k$, $a_{tk} \to a_k$, $x_{tk} \to e^{-\gamma\alpha_j} x_k$})];
$m_{j\_\to k\_}$[$\mathcal{E}$_$\mathbb{E}$] := $\mathcal{E}$ ~ $B_{j,k}$ ~ $tm_{j,k\to k}$;

In[∘]:= eq = MatrixExp[X $\begin{pmatrix} 0 & \gamma \\ 0 & 0 \end{pmatrix}$].MatrixExp[Y $\begin{pmatrix} 0 & 0 \\ \epsilon & 0 \end{pmatrix}$] ==
MatrixExp[η $\begin{pmatrix} 0 & 0 \\ \epsilon & 0 \end{pmatrix}$].MatrixExp[$\beta$ $\begin{pmatrix} 0 & 0 \\ 0 & -\epsilon \end{pmatrix}$].MatrixExp[$\alpha$ $\begin{pmatrix} \gamma & 0 \\ 0 & 0 \end{pmatrix}$].MatrixExp[$\xi$ $\begin{pmatrix} 0 & \gamma \\ 0 & 0 \end{pmatrix}$]

Out[∘]= $\{\{1 + XY\gamma\epsilon, X\gamma\}, \{Y\epsilon, 1\}\} == \{\{e^{\alpha\gamma}, e^{\alpha\gamma}\gamma\xi\}, \{e^{\alpha\gamma}\epsilon\eta, e^{-\beta\epsilon} + e^{\alpha\gamma}\gamma\epsilon\eta\xi\}\}$

In[∘]:=
```
Solve[Thread[Flatten /@ eq], {η, β, α, ξ}]
```

Out[∘]=
$$\left\{\left\{\eta \to \frac{Y}{1 + X Y \gamma \epsilon}, \xi \to \frac{X}{1 + X Y \gamma \epsilon}, \beta \to \text{ConditionalExpression}\left[-\frac{2 i \pi C[2] + \text{Log}\left[\frac{1}{1+X Y \gamma \epsilon}\right]}{\epsilon}, C[2] \in \mathbb{Z}\right], \alpha \to \text{ConditionalExpression}\left[\frac{2 i \pi C[1] + \text{Log}[1 + X Y \gamma \epsilon]}{\gamma}, C[1] \in \mathbb{Z}\right]\right\}\right\}$$

In[∘]:=
```
Det /@ eq
```

Out[∘]=
$$1 == e^{\alpha \gamma - \beta \epsilon}$$

In[∘]:=
```
{so} = Solve[Thread[Flatten /@ eq /. α → β ϵ / γ], {η, β, ξ}] /. C@1 → 0
```

Out[∘]=
$$\left\{\left\{\eta \to \frac{Y}{1 + X Y \gamma \epsilon}, \xi \to \frac{X}{1 + X Y \gamma \epsilon}, \beta \to \frac{\text{Log}[1 + X Y \gamma \epsilon]}{\epsilon}\right\}\right\}$$