Pensieve header: Invariance of k=2 under Qθ.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio"];
        << KnotTheory`
        << "SL2PortfolioProgram.m"
```

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= $p = 5; $k = 2; $U = QU;
```

```
In[•]:= (# → CF@ Cθ@@𝒪CU[{y, a, x}, #] /. {CU → Times, γ | ħ → 1}) & /@ {1, a, ϵ x y, ϵ^2 x^2 y^2}
Out[•]= {1 → 1, a → -a, x y ϵ → -t ϵ + x y ϵ + 2 a ϵ^2, x^2 y^2 ϵ^2 → 2 t^2 ϵ^2 - 4 t x y ϵ^2 + x^2 y^2 ϵ^2}
```

```
In[•]:= (# → Simplify@ Qθ@@𝒪QU[{y, a, x}, #] /. {QU → Times, γ | ħ → 1}) & /@ {1, a, ϵ x y, ϵ^2 x^2 y^2}
Out[•]= {1 → 1, a → -a, x y ϵ → 1/T ϵ (1 - T + x y (1 - 2 ϵ) - ϵ + 2 a ϵ + T ϵ + 2 a x y ϵ),
         x^2 y^2 ϵ^2 → (2 (-1 + T)^2 - 4 (-1 + T) x y + x^2 y^2) ϵ^2 / T^2}
```

```
In[•]:= SCθ[p_] := Collect[Cθ@@𝒪CU[{y, a, x}, p] /. {CU → Times, γ | ħ → 1}, ϵ, Simplify];
        SQθ[p_] := Collect[Qθ@@𝒪QU[{y, a, x}, p] /. {QU → Times, γ | ħ → 1}, ϵ, Simplify];
```

```
In[•]:= (# → SQθ[#]) & /@ {1, a, ϵ x y, ϵ^2 x^2 y^2}
Out[•]= {1 → 1, a → -a, x y ϵ → (1 - T + x y) ϵ / T + (-1 + T - 2 x y + 2 a (1 + x y)) ϵ^2 / T,
         x^2 y^2 ϵ^2 → (2 + 2 T^2 + 4 x y + x^2 y^2 - 4 T (1 + x y)) ϵ^2 / T^2}
```

```
In[•]:= 𝔼[L_, Q_, P_]$k_ := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
        𝔼d→r_[L_, Q_, P_]$k_ := 𝔼d→r@@𝔼[L, Q, P]$k;
        𝔼3@𝔼[ω_, L_, Q_, Ps_] := CF /@ 𝔼[L, ω^-1 Q, ω^-1 (ω^-4 ϵ)^(-1+Range@Length@Ps . Ps)]$k;
        𝔼4@𝔼[L_, Q_, P_] := Module[
            {ω = Normal[P]^-1 /. ϵ → 0, Ps = CoefficientList[P, ϵ]},
            CF /@ 𝔼[ω, L, ω Q, ω^(-3+4 Range@Length@Ps Ps)]];
        𝔼3@𝔼sp___[as___] := 𝔼3@𝔼[as] /. 𝔼 → 𝔼sp;
        𝔼4@𝔼sp___[as___] := 𝔼4@𝔼[as] /. 𝔼 → 𝔼sp;
```

```
In[•]:= P[Knot[n_, k_]] := P[Knot[n, k]] = Module[{fname},
            fname = "../SL2Invariant/k=2/Data/" <> ToString[n] <> "_" <> ToString[k] <> ".m";
            Collect[𝔼3[Get[fname]][[2, 2]][[3]] // Normal, ϵ, Simplify]
          ]
```

In[•]:=
```
P[Knot[3, 1]]
```

Out[•]=
$$\frac{T}{1-T+T^2}+\left(T\left(-2+3T-2T^2+T^3+2a\left(-1+T-T^3+T^4\right)-2xy-2T^3xy\right)\epsilon\right)\Big/\left(1-T+T^2\right)^3+$$
$$\frac{1}{2\left(1-T+T^2\right)^5}T\Big(4+4a^2\left(1-T+T^2\right)^2\left(1+T-6T^2+T^3+T^4\right)+4xy+6x^2y^2+$$
$$T^7\left(1+4xy\right)+2T^3xy\left(-2+15xy\right)+T^2\left(6-12x^2y^2\right)+T^5\left(4+6x^2y^2\right)+$$
$$T\left(-11-8xy+6x^2y^2\right)+T^6\left(-2-8xy+6x^2y^2\right)-2T^4\left(1+2xy+6x^2y^2\right)-4a\left(1-T+T^2\right)$$
$$\left(T\left(2-4xy\right)-2\left(1+xy\right)-8T^3\left(1+xy\right)+T^4\left(5+2xy\right)+T^5\left(-2+4xy\right)+T^2\left(7+10xy\right)\right)\Big)\epsilon^2$$

In[•]:=
```
SCΘ@P[Knot[3, 1]]
```

Out[•]=
$$\frac{T}{1-T+T^2}+$$
$$\left(T\left(2a\left(-1+T-T^3+T^4\right)+T\left(1-2T+3T^2+2t\left(1+T^3\right)-2xy-2T^3\left(1+xy\right)\right)\right)\epsilon\right)\Big/\left(1-T+T^2\right)^3+$$
$$\frac{1}{2\left(1-T+T^2\right)^5}T$$
$$\Big(4a^2\left(1-T+T^2\right)^2\left(1+T-6T^2+T^3+T^4\right)+4aT\left(1-T+T^2\right)\left(-4+2t\left(-2-T+4T^2-5T^3+2T^4+T^5\right)+$$
$$4xy+T^4\left(4-4xy\right)-2T^5\left(2+xy\right)+5T^3\left(1+2xy\right)+T\left(7+2xy\right)-2T^2\left(5+4xy\right)\right)+$$
$$T\Big(1+12t^2T\left(1-T+T^2\right)^2\left(1+3T+T^2\right)-28xy+2T^4xy\left(86+15xy\right)-$$
$$6T^5\left(-1+16xy+2x^2y^2\right)+T^6\left(-11+8xy+6x^2y^2\right)+T\left(-2+8xy+6x^2y^2\right)+$$
$$T^7\left(4+20xy+6x^2y^2\right)+T^2\left(4+48xy+6x^2y^2\right)-2T^3\left(1+82xy+6x^2y^2\right)-12t\left(1-T+T^2\right)$$
$$\left(-1-2T^3\left(2+xy\right)+T\left(-1+2xy\right)+T^5\left(1+2xy\right)+T^4\left(1+4xy\right)+T^2\left(2+4xy\right)\right)\Big)\Big)\epsilon^2$$

In[•]:=
```
SQΘ@P[Knot[3, 1]]
```

Out[•]=
$$\frac{T}{1-T+T^2}+\left(T\left(-2+3T-2T^2+T^3+2a\left(-1+T-T^3+T^4\right)-2xy-2T^3xy\right)\epsilon\right)\Big/\left(1-T+T^2\right)^3+$$
$$\frac{1}{2\left(1-T+T^2\right)^5}T\Big(4+4a^2\left(1-T+T^2\right)^2\left(1+T-6T^2+T^3+T^4\right)+4xy+6x^2y^2+$$
$$T^7\left(1+4xy\right)+2T^3xy\left(-2+15xy\right)+T^2\left(6-12x^2y^2\right)+T^5\left(4+6x^2y^2\right)+$$
$$T\left(-11-8xy+6x^2y^2\right)+T^6\left(-2-8xy+6x^2y^2\right)-2T^4\left(1+2xy+6x^2y^2\right)-4a\left(1-T+T^2\right)$$
$$\left(T\left(2-4xy\right)-2\left(1+xy\right)-8T^3\left(1+xy\right)+T^4\left(5+2xy\right)+T^5\left(-2+4xy\right)+T^2\left(7+10xy\right)\right)\Big)\epsilon^2$$

In[•]:=
```
Test[K_Knot] := Simplify[P[K] == SQΘ[P[K]]]
```

In[•]:=
```
Test[Knot[3, 1]]
```

Out[•]=
```
True
```

In[•]:=
```
Test /@ AllKnots[{3, 8}]
```

Out[•]=
```
{True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True}
```

In[•]:=
```
(CF@SST[SD@OQU[{y, a, x}, P@#] - CΘ@SD@OQU[{y, a, x}, P@#]] /. γ | ħ → 1) & /@
 AllKnots[{3, 7}]
```

Out[•]=
$$\left\{-\frac{1777}{12}t^6\epsilon^2\,CU[a],\ -\frac{43\,217}{180}t^6\epsilon^2\,CU[a],\ -\frac{345\,985}{36}t^6\epsilon^2\,CU[a],\right.$$
$$-\frac{244\,111}{90}t^6\epsilon^2\,CU[a],\ -\frac{103\,451}{30}t^6\epsilon^2\,CU[a],\ -\frac{66\,131}{60}t^6\epsilon^2\,CU[a],\ \frac{26\,281}{180}t^6\epsilon^2\,CU[a],$$
$$-\frac{2\,611\,777}{18}t^6\epsilon^2\,CU[a],\ -\frac{859\,727}{60}t^6\epsilon^2\,CU[a],\ -\frac{5\,231\,161}{60}t^6\epsilon^2\,CU[a],$$
$$\left.-\frac{231\,207}{5}t^6\epsilon^2\,CU[a],\ -\frac{263\,783}{9}t^6\epsilon^2\,CU[a],\ -\frac{148\,711}{180}t^6\epsilon^2\,CU[a],\ \frac{42\,839}{180}t^6\epsilon^2\,CU[a]\right\}$$