Pensieve header: Direct formulas for the sl2 logoi.

Prolog

In[∘]:=
```
wdir = SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio"];
<< "SL2PortfolioProgram.m"
```

In[∘]:=
```
$p = 2; $k = 2; $U = QU;
```

In[∘]:=
```
HL[ℰ_] := Style[ℰ, Background → Yellow];
```

$t == \epsilon a - \gamma b$ and $b == -t/\gamma + \epsilon a/\gamma$.

In[∘]:=
```
t2b_{i_→j_} := 𝔼[α_i a_j - τ_i γ b_j, ξ_i x_j + η_i y_j, e^{ϵ τ_i a_j} + O[ϵ]^{$k+1}];
b2t_{i_→j_} := 𝔼[α_i a_j - β_i t_j / γ, ξ_i x_j + η_i y_j, e^{ϵ β_i a_j/γ} + O[ϵ]^{$k+1}];
t2b_{i_} := t2b_{i→i}; b2t_{i_} := b2t_{i→i};
```

In[∘]:= `{t2b_1, b2t_1}`

Out[∘]= $\{\mathbb{E}[a_1 \alpha_1 - \gamma b_1 \tau_1,\ y_1 \eta_1 + x_1 \xi_1,\ 1 + a_1 \tau_1 \epsilon + \frac{1}{2} a_1^2 \tau_1^2 \epsilon^2 + O[\epsilon]^3],$
$\mathbb{E}[a_1 \alpha_1 - \frac{t_1 \beta_1}{\gamma},\ y_1 \eta_1 + x_1 \xi_1,\ 1 + \frac{a_1 \beta_1 \epsilon}{\gamma} + \frac{a_1^2 \beta_1^2 \epsilon^2}{2 \gamma^2} + O[\epsilon]^3]\}$

In[∘]:= `{t2b_{1→2}, b2t_{2→3}}`

Out[∘]= $\{\mathbb{E}[a_2 \alpha_1 - \gamma b_2 \tau_1,\ y_2 \eta_1 + x_2 \xi_1,\ 1 + a_2 \tau_1 \epsilon + \frac{1}{2} a_2^2 \tau_1^2 \epsilon^2 + O[\epsilon]^3],$
$\mathbb{E}[a_3 \alpha_2 - \frac{t_3 \beta_2}{\gamma},\ y_3 \eta_2 + x_3 \xi_2,\ 1 + \frac{a_3 \beta_2 \epsilon}{\gamma} + \frac{a_3^2 \beta_2^2 \epsilon^2}{2 \gamma^2} + O[\epsilon]^3]\}$

In[∘]:= `t2b_{1→2} ~ B_2 ~ b2t_{2→3}`

Out[∘]= $\mathbb{E}[a_3 \alpha_1 + t_3 \tau_1,\ y_3 \eta_1 + x_3 \xi_1,\ 1 + O[\epsilon]^3]$

In[∘]:= `t2b_1 ~ B_1 ~ b2t_1`

Out[∘]= $\mathbb{E}[a_1 \alpha_1 + t_1 \tau_1,\ y_1 \eta_1 + x_1 \xi_1,\ 1 + O[\epsilon]^3]$

In[∘]:= `b2t_1 ~ B_1 ~ t2b_1`

Out[∘]= $\mathbb{E}[a_1 \alpha_1 + b_1 \beta_1,\ y_1 \eta_1 + x_1 \xi_1,\ 1 + O[\epsilon]^3]$

m

In[∘]:=
```
am_{i_,j_->k_} := 𝔼[(α_i + α_j) a_k, (e^{-γ α_j} ξ_i + ξ_j) x_k, 1];
bm_{i_,j_→k_} := 𝔼[(β_i + β_j) b_k, (η_i + η_j) y_k, e^{(e^{-ϵ β_i} - 1) η_j y_k} + O[ϵ]^{$k+1}];
```

In[∘]:= `Timing@Block[{$k = 3}, {am_{i,j→k}, bm_{i,j→k}}]`

Out[∘]= $\left\{0.,\ \left\{\mathbb{E}\left[a_k\ (\alpha_i+\alpha_j),\ x_k\ \left(e^{-\gamma\,\alpha_j}\,\xi_i+\xi_j\right),\ 1\right],\ \mathbb{E}\left[b_k\ (\beta_i+\beta_j),\ y_k\ (\eta_i+\eta_j),\ 1-y_k\,\beta_i\,\eta_j\,\epsilon+\frac{1}{2}\left(y_k\,\beta_i^2\,\eta_j+y_k^2\,\beta_i^2\,\eta_j^2\right)\epsilon^2+\frac{1}{6}\left(-y_k\,\beta_i^3\,\eta_j-3\,y_k^2\,\beta_i^3\,\eta_j^2-y_k^3\,\beta_i^3\,\eta_j^3\right)\epsilon^3+O[\epsilon]^4\right]\right\}\right\}$

Comparisons with 2018-05/ybax.nb:

In[∘]:= $\{\mathrm{HL}[am_{i,j\to k}\equiv\mathbb{E}[(\alpha_i+\alpha_j)\,a_k,\ (e^{-\alpha_j}\,\xi_i+\xi_j)\,x_k,\ 1+O[\epsilon]^2]\ /.\ \mathrm{l2U}\ /.\ \hbar\mid\gamma\to 1],$
$\mathrm{HL}[bm_{i,j\to k}\equiv\mathbb{E}[(\beta_i+\beta_j)\,b_k,\ (\eta_i+\eta_j)\,y_k,\ 1-\epsilon\,\eta_j\,y_k\,\beta_i+O[\epsilon]^2]]\}$

Out[∘]= {True, True}

Comparisons with tm:

In[∘]:= `Timing@Block[{$k = 3},`
$\quad \mathrm{HL}\ /@\ \{\mathbb{E}[\alpha_i\,a_i+\alpha_j\,a_j,\ \xi_i\,x_i+\xi_j\,x_j,\ 1]\sim B_{i,j}\sim tm_{i,j\to k}\equiv(am_{i,j\to k}\ /.\ \mathrm{l2U}),$
$\quad\quad bm_{i,j\to k}\equiv\mathbb{E}[\beta_i\,b_i+\beta_j\,b_j,\ \eta_i\,y_i+\eta_j\,y_j,\ 1]\sim B_{i,j}\sim(b2t_i\ b2t_j)\sim B_{i,j}\sim tm_{i,j\to k}\sim B_k\sim t2b_k\}$
`]`

Out[∘]= {34.7188, {True, True}}

Associativity on both sides

In[∘]:= `Timing@Block[{$k = 3},`
$\quad \mathrm{HL}\ /@\ \{(am_{1,2\to1}\sim B_1\sim am_{1,3\to1})\equiv(am_{2,3\to2}\sim B_2\sim am_{1,2\to1}),\ (bm_{1,2\to1}\sim B_1\sim bm_{1,3\to1})\equiv(bm_{2,3\to2}\sim B_2\sim bm_{1,2\to1})\}$
`]`

Out[∘]= {0.125, {True, True}}

R

In[∘]:= $e_{q\_,k\_}[x\_] := \mathrm{Module}\left[\{j\},\ e\,\hat{}\left(\sum_{j=1}^{k+1}\frac{(1-q)^j\,x^j}{j\,(1-q^j)}\right)\right];\ e_{q\_}[x\_] := e_{q,\$k}[x]$

In[∘]:= `Series[e_{q,5}[x], {x, 0, 5}]`

Out[∘]= $1+x+\frac{x^2}{1+q}+\frac{x^3}{(1+q)\,(1+q+q^2)}+\frac{x^4}{(1+q)^2\,(1+q^2)\,(1+q+q^2)}+x^5\Big/\left((1+q)^2\,(1+q^2)\,(1+q+q^2)\,(1+q+q^2+q^3+q^4)\right)+O[x]^6$

In[∘]:= e^{-x} `Series[e_{e^ε,5}[x], {ε, 0, 5}]`

Out[∘]= $1-\frac{x^2\,\epsilon}{4}+\frac{1}{288}\,x^3\,(32+9\,x)\,\epsilon^2-\frac{\left(x^2\,(-24+72\,x^2+32\,x^3+3\,x^4)\right)\epsilon^3}{1152}+\frac{1}{4\,147\,200}\,x^3\left(-115\,200-21\,600\,x+165\,888\,x^2+90\,400\,x^3+14\,400\,x^4+675\,x^5\right)\epsilon^4-\frac{1}{16\,588\,800}\left(x^2\left(34\,560-518\,400\,x^2-153\,600\,x^3+450\,000\,x^4+281\,088\,x^5+58\,000\,x^6+4800\,x^7+135\,x^8\right)\right)\epsilon^5+O[\epsilon]^6$

In[∘]:= $tR_{1,2}$

Out[∘]= $$\mathbb{E}\Big[-\frac{\hbar\, a_2\, t_1}{\gamma},\ \hbar\, x_2\, y_1,\ 1+\left(\frac{\hbar\, a_1\, a_2}{\gamma}-\frac{1}{4}\gamma\,\hbar^3\, x_2^2\, y_1^2\right)\epsilon+\left(\frac{\hbar^2\, a_1^2\, a_2^2}{2\gamma^2}-\frac{1}{4}\hbar^4\, a_1\, a_2\, x_2^2\, y_1^2+\frac{1}{288}\gamma^2\,\hbar^5\, x_2^3\, y_1^3\,(32+9\,\hbar\, x_2\, y_1)\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:=
```
nR_{i_,j_} := 𝔼[ℏ a_j b_i, ℏ x_j y_i, Series[ⅇ^(-ℏ y_i x_j) ⅇ_{q_ℏ}[ℏ y_i x_j], {ϵ, 0, $k}]];
nR_{1,2}
```

Out[∘]= $$\mathbb{E}\Big[\hbar\, a_2\, b_1,\ \hbar\, x_2\, y_1,\ 1-\frac{1}{4}\left(\gamma\,\hbar^3\, x_2^2\, y_1^2\right)\epsilon+\left(\frac{1}{9}\gamma^2\,\hbar^5\, x_2^3\, y_1^3+\frac{1}{32}\gamma^2\,\hbar^6\, x_2^4\, y_1^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:= $nR_{1,2}\sim B_1\sim b2t_1$

Out[∘]= $$\mathbb{E}\Big[-\frac{\hbar\, a_2\, t_1}{\gamma},\ \hbar\, x_2\, y_1,\ 1+\frac{\left(4\,\hbar\, a_1\, a_2-\gamma^2\,\hbar^3\, x_2^2\, y_1^2\right)\epsilon}{4\gamma}+\frac{1}{288\gamma^2}\left(144\,\hbar^2\, a_1^2\, a_2^2-72\,\gamma^2\,\hbar^4\, a_1\, a_2\, x_2^2\, y_1^2+32\,\gamma^4\,\hbar^5\, x_2^3\, y_1^3+9\,\gamma^4\,\hbar^6\, x_2^4\, y_1^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:= $HL[(nR_{1,2}\sim B_1\sim b2t_1)\equiv tR_{1,2}]$

Out[∘]= True

P

In[∘]:=
```
nP_{i_,j_,0} := 𝔼[β_i α_j / ℏ, η_i ξ_j / ℏ, 1];
nP_{i_,j_,k_} := Module[{m, n},
   MapAt[
     (# - ϵ^k Coefficient[(nR_{n,m} ~ B_{n,m} ~ (nP_{n,j,0} nP_{i,m,k-1}))⟦3⟧, ϵ, k] + O[ϵ]^($k+1)) &, nP_{i,j,k-1}, 3]
  ];
nP_{i_,j_} := nP_{i,j,$k};
```

In[∘]:= $nR_{i,j}$

Out[∘]= $$\mathbb{E}\Big[\hbar\, a_j\, b_i,\ \hbar\, x_j\, y_i,\ 1-\frac{1}{4}\left(\gamma\,\hbar^3\, x_j^2\, y_i^2\right)\epsilon+\left(\frac{1}{9}\gamma^2\,\hbar^5\, x_j^3\, y_i^3+\frac{1}{32}\gamma^2\,\hbar^6\, x_j^4\, y_i^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:=
```
Block[{$k = 3}, {nR_{i,j}, nP_{i,k}, HL[nR_{i,j} ~ B_i ~ nP_{i,k} ≡ 𝔼[a_j α_k, x_j ξ_k, 1]]}]
```

Out[∘]= $$\Big\{\mathbb{E}\Big[\hbar\, a_j\, b_i,\ \hbar\, x_j\, y_i,\ 1-\frac{1}{4}\left(\gamma\,\hbar^3\, x_j^2\, y_i^2\right)\epsilon+\left(\frac{1}{9}\gamma^2\,\hbar^5\, x_j^3\, y_i^3+\frac{1}{32}\gamma^2\,\hbar^6\, x_j^4\, y_i^4\right)\epsilon^2+\frac{1}{1152}\left(24\,\gamma^3\,\hbar^5\, x_j^2\, y_i^2-72\,\gamma^3\,\hbar^7\, x_j^4\, y_i^4-32\,\gamma^3\,\hbar^8\, x_j^5\, y_i^5-3\,\gamma^3\,\hbar^9\, x_j^6\, y_i^6\right)\epsilon^3+O[\epsilon]^4\Big],$$
$$\mathbb{E}\Big[\frac{\alpha_k\,\beta_i}{\hbar},\ \frac{\eta_i\,\xi_k}{\hbar},\ 1+\frac{\gamma\,\eta_i^2\,\xi_k^2\,\epsilon}{4\,\hbar}+\frac{\left(36\,\gamma^2\,\hbar^2\,\eta_i^2\,\xi_k^2+40\,\gamma^2\,\hbar\,\eta_i^3\,\xi_k^3+9\,\gamma^2\,\eta_i^4\,\xi_k^4\right)\epsilon^2}{288\,\hbar^2}+\frac{1}{1152\,\hbar^3}\left(48\,\gamma^3\,\hbar^4\,\eta_i^2\,\xi_k^2+192\,\gamma^3\,\hbar^3\,\eta_i^3\,\xi_k^3+156\,\gamma^3\,\hbar^2\,\eta_i^4\,\xi_k^4+40\,\gamma^3\,\hbar\,\eta_i^5\,\xi_k^5+3\,\gamma^3\,\eta_i^6\,\xi_k^6\right)\epsilon^3+O[\epsilon]^4\Big],\ \text{True}\Big\}$$

aS

In[∘]:=
```
aS_{i_} := 𝔼[α_i a_i, ξ_i x_i, 1] ~ B_i ~ tS_i;
aS_i
```

Out[∘]= $\mathbb{E}\Big[-a_i\,\alpha_i,\ -x_i\,\mathcal{A}_i\,\xi_i,\ 1+\frac{1}{2}\left(-2\hbar\,a_i\,x_i\,\mathcal{A}_i\,\xi_i-\gamma\hbar\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2\right)\in+\frac{1}{8}\left(-4\hbar^2\,a_i^2\,x_i\,\mathcal{A}_i\,\xi_i+2\gamma^2\hbar^2\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2-8\gamma\hbar^2\,a_i\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2+4\hbar^2\,a_i^2\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2-4\gamma^2\hbar^2\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3+4\gamma\hbar^2\,a_i\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3+\gamma^2\hbar^2\,x_i^4\,\mathcal{A}_i^4\,\xi_i^4\right)\in^2+O[\in]^3\Big]$

Comparison with 2018-05/ybax.nb:

In[∘]:= $\mathrm{HL}\left[\mathrm{aS}_i\equiv\mathbb{E}\left[-\alpha_i\,a_i,\ -e^{\alpha_i}\,\xi_i\,x_i,\ 1-\epsilon\,e^{\alpha_i}\,\xi_i\,x_i\left(a_i+\frac{1}{2}e^{\alpha_i}\,\xi_i\,x_i\right)+O[\epsilon]^2\right]\ /.\ \mathrm{l2U}\ /.\ \hbar\ |\ \gamma\to 1\right]$

Out[∘]= True

In[∘]:= $\mathbb{E}[\alpha_i\,a_i,\ 0,\ 1]\sim B_i\sim \mathrm{aS}_i$

Out[∘]= $\mathbb{E}\left[-a_i\,\alpha_i,\ 0,\ 1+O[\in]^3\right]$

In[∘]:= $\mathbb{E}[0,\ \xi_i\,x_i,\ 1]\sim B_i\sim \mathrm{aS}_i$

Out[∘]= $\mathbb{E}\Big[0,\ -x_i\,\xi_i,\ 1+\frac{1}{2}\left(-2\hbar\,a_i\,x_i\,\xi_i-\gamma\hbar\,x_i^2\,\xi_i^2\right)\in+\frac{1}{8}\left(-4\hbar^2\,a_i^2\,x_i\,\xi_i+2\gamma^2\hbar^2\,x_i^2\,\xi_i^2-8\gamma\hbar^2\,a_i\,x_i^2\,\xi_i^2+4\hbar^2\,a_i^2\,x_i^2\,\xi_i^2-4\gamma^2\hbar^2\,x_i^3\,\xi_i^3+4\gamma\hbar^2\,a_i\,x_i^3\,\xi_i^3+\gamma^2\hbar^2\,x_i^4\,\xi_i^4\right)\in^2+O[\in]^3\Big]$

In[∘]:= $\mathrm{HL}\Big[\mathbb{E}[0,\ \xi_i\,x_i,\ 1]\sim B_i\sim \mathrm{aS}_i\equiv\mathbb{E}\Big[0,\ -\xi_i\,x_i,\ \mathrm{Series}\Big[e^{\xi_i\,x_i}\,\mathrm{Sum}\Big[\mathrm{Expand}\Big[\frac{(-\hbar\,\gamma\,\epsilon)^k}{2^k\,k\,!}\,\mathrm{Nest}\big[\mathrm{Expand}\big[x_i^2\,\partial_{\{x_i,2\}}\#\big]\ \&,\ e^{-\xi_i\,e^{\hbar\,\epsilon\,a_i}\,x_i},\ k\big]\Big],\ \{k,0,\$k\}\Big],\ \{\epsilon,0,\$k\}\Big]\Big]\Big]$

Out[∘]= True

In[∘]:= $\mathrm{HL}\Big[\mathrm{aS}_i\equiv\mathbb{E}\Big[-\alpha_i\,a_j,\ -\xi_i\,x_i,\ \mathrm{Series}\Big[e^{\xi_i\,x_i}\,\mathrm{Sum}\Big[\mathrm{Expand}\Big[\frac{(-\hbar\,\gamma\,\epsilon)^k}{2^k\,k\,!}\,\mathrm{Nest}\big[\mathrm{Expand}\big[x_i^2\,\partial_{\{x_i,2\}}\#\big]\ \&,\ e^{-\xi_i\,e^{\hbar\,\epsilon\,a_i}\,x_i},\ k\big]\Big],\ \{k,0,\$k\}\Big],\ \{\epsilon,0,\$k\}\Big]\Big]\sim B_{i,j}\sim \mathrm{am}_{i,j\to i}\Big]$

Out[∘]= True

bSi

In[∘]:=
```
bSi_i_ := 𝔼[β_i b_i, η_i y_i, 1] ~ B_i ~ b2t_i ~ B_i ~ tS_i ~ B_i ~ t2b_i;
bSi_i
```

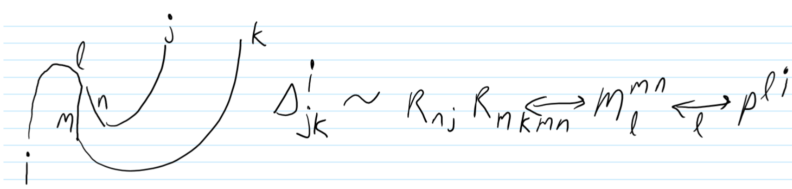
Out[∘]=
$$\mathbb{E}\Big[-b_i\,\beta_i,\; -\frac{y_i\,\eta_i}{B_i},\; 1+\frac{\left(2\,\gamma\,\hbar\,B_i\,y_i\,\eta_i-2\,B_i\,y_i\,\beta_i\,\eta_i-\gamma\,\hbar\,y_i^2\,\eta_i^2\right)\,\epsilon}{2\,B_i^2}+\frac{1}{8\,B_i^4}\left(-4\,\gamma^2\,\hbar^2\,B_i^3\,y_i\,\eta_i+8\,\gamma\,\hbar\,B_i^3\,y_i\,\beta_i\,\eta_i-4\,B_i^3\,y_i\,\beta_i^2\,\eta_i+14\,\gamma^2\,\hbar^2\,B_i^2\,y_i^2\,\eta_i^2-16\,\gamma\,\hbar\,B_i^2\,y_i^2\,\beta_i\,\eta_i^2+4\,B_i^2\,y_i^2\,\beta_i^2\,\eta_i^2-8\,\gamma^2\,\hbar^2\,B_i\,y_i^3\,\eta_i^3+4\,\gamma\,\hbar\,B_i\,y_i^3\,\beta_i\,\eta_i^3+\gamma^2\,\hbar^2\,y_i^4\,\eta_i^4\right)\,\epsilon^2+O[\epsilon]^3\Big]$$

Comparison with 2018-05/ybax.nb:

In[∘]:=
$$\text{HL}\left[\text{bSi}_i \equiv \mathbb{E}\left[-\beta_i\,b_i,\,-e^{b_i}\,\eta_i\,y_i,\,1-\epsilon\,e^{b_i}\,\eta_i\,y_i\left(\beta_i-1+\frac{1}{2}\,e^{b_i}\,\eta_i\,y_i\right)+O[\epsilon]^2\right]\;/.\;\text{l2U}\;/.\;\hbar\,|\,\gamma\to 1\right]$$

Out[∘]= True

aΔ



In[∘]:=
```
Block[{i, j, k, l, m, n}, aΔ_i_→j_,k_ = (nR_n,j nR_m,k) ~ B_n,m ~ bm_n,m→l ~ B_l ~ nP_l,i];
aΔ_i→j,k
```

Out[∘]=
$$\mathbb{E}\Big[a_j\,\alpha_i+a_k\,\alpha_i,\; x_j\,\xi_i+x_k\,\xi_i,\; 1+\frac{1}{2}\left(-2\,\hbar\,a_j\,x_k\,\xi_i+\gamma\,\hbar\,x_j\,x_k\,\xi_i^2\right)\,\epsilon+\frac{1}{24}\left(12\,\hbar^2\,a_j^2\,x_k\,\xi_i+6\,\gamma^2\,\hbar^2\,x_j\,x_k\,\xi_i^2-12\,\gamma\,\hbar^2\,a_j\,x_j\,x_k\,\xi_i^2+12\,\hbar^2\,a_j^2\,x_k^2\,\xi_i^2+4\,\gamma^2\,\hbar^2\,x_j^2\,x_k\,\xi_i^3+4\,\gamma^2\,\hbar^2\,x_j\,x_k^2\,\xi_i^3-12\,\gamma\,\hbar^2\,a_j\,x_j\,x_k^2\,\xi_i^3+3\,\gamma^2\,\hbar^2\,x_j^2\,x_k^2\,\xi_i^4\right)\,\epsilon^2+O[\epsilon]^3\Big]$$

Comparison with 2018-05/ybax.nb:

In[∘]:=
$$\text{HL}\left[a\Delta_{i\to j,k} \equiv \mathbb{E}\left[\alpha_i\,(a_j+a_k),\,\xi_i\,(x_j+x_k),\,1+\epsilon\,\xi_i\,x_k\left(-a_j+\frac{1}{2}\,\xi_i\,x_j\right)+O[\epsilon]^2\right]\;/.\;\text{l2U}\;/.\;\hbar\,|\,\gamma\to 1\right]$$

Out[∘]= True

Comparison with tΔ:

In[∘]:=
```
HL[aΔ_i→j,k ≡ 𝔼[α_i a_i, ξ_i x_i, 1] ~ B_i ~ tΔ_i→j,k]
```

Out[∘]= True

bΔ

In[∘]:=
```
Block[{i, j, k, l, m, n}, bΔ_{i_→j_,k_} = (nR_{j,n} nR_{k,m}) ~ B_{n,m} ~ am_{n,m→l} ~ B_l ~ nP_{i,l}];
bΔ_{i→j,k}
```

Out[∘]=
$$\mathbb{E}\Big[b_j \beta_i + b_k \beta_i,\ B_k y_j \eta_i + y_k \eta_i,\ 1 + \frac{1}{2}\gamma\hbar B_k y_j y_k \eta_i^2 \epsilon + \frac{1}{24}\left(6\gamma^2\hbar^2 B_k y_j y_k \eta_i^2 + 4\gamma^2\hbar^2 B_k^2 y_j^2 y_k \eta_i^3 + 4\gamma^2\hbar^2 B_k y_j y_k^2 \eta_i^3 + 3\gamma^2\hbar^2 B_k^2 y_j^2 y_k^2 \eta_i^4\right)\epsilon^2 + O[\epsilon]^3\Big]$$

Comparison with 2018-05/ybax.nb:

In[∘]:=
$$\text{HL@Simplify}\Big[\text{b}\Delta_{i\to j,k} \equiv \mathbb{E}\Big[\beta_i\left(b_j + b_k\right),\ \eta_i\left(e^{-b_k} y_j + y_k\right),\ 1 + \frac{1}{2}\epsilon\,\eta_i{}^2 y_j y_k e^{-b_k} + O[\epsilon]^2\Big]\ /.\ \text{l2U}\ /.\ \hbar \mid \gamma \to 1\Big]$$

Out[∘]= True

Comparison with tΔ:

In[∘]:=
$$\text{HL}\Big[\text{b}\Delta_{i\to k,j} \equiv \left(\mathbb{E}[\beta_i b_i,\ \eta_i y_i,\ 1] \sim B_i \sim \text{b2t}_i \sim B_i \sim \text{t}\Delta_{i\to j,k} \sim B_{j,k} \sim \left(\text{t2b}_j\ \text{t2b}_k\right)\right)\Big]$$

Out[∘]= True

Log Logoi

In[∘]:=
```
𝔼 /: Simplify[ℰ_𝔼] := Simplify /@ (ℰ /. l2U)
Column@(Portfolio = Simplify /@ {nR_{i,j}, nP_{i,j}, am_{i,j→k}, bm_{i,j→k}, aS_i, bS_i, aΔ_{i→j,k}, bΔ_{i→j,k}})
```

Out[∘]=

$$\mathbb{E}\Big[\hbar a_j b_i,\ \hbar x_j y_i,\ 1 - \frac{1}{4}\left(\gamma\hbar^3 x_j^2 y_i^2\right)\epsilon + \frac{1}{288}\gamma^2\hbar^5 x_j^3 y_i^3\left(32 + 9\hbar x_j y_i\right)\epsilon^2 + O[\epsilon]^3\Big]$$

$$\mathbb{E}\Big[\frac{\alpha_j\beta_i}{\hbar},\ \frac{\eta_i\xi_j}{\hbar},\ 1 + \frac{\gamma\eta_i^2\xi_j^2\epsilon}{4\hbar} + \frac{\gamma^2\eta_i^2\xi_j^2\left(36\hbar^2 + 40\hbar\eta_i\xi_j + 9\eta_i^2\xi_j^2\right)\epsilon^2}{288\hbar^2} + O[\epsilon]^3\Big]$$

$$\mathbb{E}\Big[a_k\left(\alpha_i + \alpha_j\right),\ x_k\left(\frac{\xi_i}{\mathcal{A}_j} + \xi_j\right),\ 1\Big]$$

$$\mathbb{E}\Big[b_k\left(\beta_i + \beta_j\right),\ y_k\left(\eta_i + \eta_j\right),\ 1 - y_k\beta_i\eta_j\epsilon + \frac{1}{2}y_k\beta_i^2\eta_j\left(1 + y_k\eta_j\right)\epsilon^2 + O[\epsilon]^3\Big]$$

$$\mathbb{E}\Big[-a_i\alpha_i,\ -x_i\mathcal{A}_i\xi_i,\ 1 - \frac{1}{2}\left(\hbar x_i\mathcal{A}_i\xi_i\left(2a_i + \gamma x_i\mathcal{A}_i\xi_i\right)\right)\epsilon + \frac{1}{8}\hbar^2 x_i\mathcal{A}_i\xi_i\left(4\gamma a_i x_i\mathcal{A}_i\xi_i\left(-2 + x_i\mathcal{A}_i\xi_i\right) + 4a_i^2\left(-1 + x_i\mathcal{A}_i\xi_i\right) + \gamma^2 x_i\mathcal{A}_i\xi_i\left(2 - 4x_i\mathcal{A}_i\xi_i + x_i^2\mathcal{A}_i^2\xi_i^2\right)\right)\epsilon^2 + O[\epsilon]^3\Big]$$

$$\mathbb{E}\Big[-b_i\beta_i,\ -\frac{y_i\eta_i}{B_i},\ 1 - \frac{\left(y_i\eta_i\left(B_i\left(-2\gamma\hbar + 2\beta_i\right) + \gamma\hbar y_i\eta_i\right)\right)\epsilon}{2B_i^2} + \frac{1}{8B_i^4}y_i\eta_i\left(-4B_i^3\left(-\gamma\hbar + \beta_i\right)^2 + 2B_i^2 y_i\left(7\gamma^2\hbar^2 - 8\gamma\hbar\beta_i + 2\beta_i^2\right)\eta_i + 4\gamma\hbar B_i y_i^2\left(-2\gamma\hbar + \beta_i\right)\eta_i^2 + \gamma^2\hbar^2 y_i^3\eta_i^3\right)\epsilon^2 + O[\epsilon]^3\Big]$$

$$\mathbb{E}\Big[\left(a_j + a_k\right)\alpha_i,\ \left(x_j + x_k\right)\xi_i,\ 1 + \frac{1}{2}\hbar x_k\xi_i\left(-2a_j + \gamma x_j\xi_i\right)\epsilon + \frac{1}{24}\hbar^2 x_k\xi_i\left(12a_j^2\left(1 + x_k\xi_i\right) - 12\gamma a_j x_j\xi_i\left(1 + x_k\xi_i\right) + \gamma^2 x_j\xi_i\left(6 + 4x_k\xi_i + x_j\xi_i\left(4 + 3x_k\xi_i\right)\right)\right)\epsilon^2 + O[\epsilon]^3\Big]$$

$$\mathbb{E}\Big[\left(b_j + b_k\right)\beta_i,\ \left(B_k y_j + y_k\right)\eta_i,\ 1 + \frac{1}{2}\gamma\hbar B_k y_j y_k\eta_i^2\epsilon + \frac{1}{24}\gamma^2\hbar^2 B_k y_j y_k\eta_i^2\left(6 + 4y_k\eta_i + B_k y_j\eta_i\left(4 + 3y_k\eta_i\right)\right)\epsilon^2 + O[\epsilon]^3\Big]$$

In[∘]:=
```
𝔼 /: Log[𝔼[L_, Q_, P_]] := 𝔼L[L, Q /. l2U, Simplify@Log[P /. l2U]];
Column[Log /@ Portfolio]
```

Out[∘]=

$$\mathbb{E}L\left[\hbar\, a_j\, b_i,\ \hbar\, x_j\, y_i,\ -\frac{1}{4}\left(\gamma\,\hbar^3\, x_j^2\, y_i^2\right)\epsilon + \frac{1}{9}\gamma^2\,\hbar^5\, x_j^3\, y_i^3\,\epsilon^2 + O[\epsilon]^3\right]$$

$$\mathbb{E}L\left[\frac{\alpha_j\,\beta_i}{\hbar},\ \frac{\eta_i\,\xi_j}{\hbar},\ \frac{\gamma\,\eta_i^2\,\xi_j^2\,\epsilon}{4\,\hbar} + \frac{\gamma^2\,\eta_i^2\,\xi_j^2\,(9\,\hbar+10\,\eta_i\,\xi_j)\,\epsilon^2}{72\,\hbar} + O[\epsilon]^3\right]$$

$$\mathbb{E}L\left[a_k\,(\alpha_i+\alpha_j),\ x_k\left(\frac{\xi_i}{\mathcal{A}_j}+\xi_j\right),\ 0\right]$$

$$\mathbb{E}L\left[b_k\,(\beta_i+\beta_j),\ y_k\,(\eta_i+\eta_j),\ -y_k\,\beta_i\,\eta_j\,\epsilon + \frac{1}{2}y_k\,\beta_i^2\,\eta_j\,\epsilon^2 + O[\epsilon]^3\right]$$

$$\mathbb{E}L\left[-a_i\,\alpha_i,\ -x_i\,\mathcal{A}_i\,\xi_i,\ -\frac{1}{2}\left(\hbar\, x_i\,\mathcal{A}_i\,\xi_i\left(2\,a_i+\gamma\, x_i\,\mathcal{A}_i\,\xi_i\right)\right)\epsilon - \frac{1}{4}\left(\hbar^2\, x_i\,\mathcal{A}_i\,\xi_i\left(2\,a_i^2+4\,\gamma\, a_i\, x_i\,\mathcal{A}_i\,\xi_i+\gamma^2\, x_i\,\mathcal{A}_i\,\xi_i\left(-1+2\,x_i\,\mathcal{A}_i\,\xi_i\right)\right)\right)\epsilon^2 + O[\epsilon]^3\right]$$

$$\mathbb{E}L\left[-b_i\,\beta_i,\ -\frac{y_i\,\eta_i}{B_i},\ -\frac{(y_i\,\eta_i\,(B_i\,(-2\,\gamma\,\hbar+2\,\beta_i)+\gamma\,\hbar\, y_i\,\eta_i))\,\epsilon}{2\,B_i^2} - \frac{1}{4\,B_i^3}\left(y_i\,\eta_i\left(2\,B_i^2\,(-\gamma\,\hbar+\beta_i)^2+\gamma\,\hbar\, B_i\, y_i\,(-5\,\gamma\,\hbar+4\,\beta_i)\,\eta_i+2\,\gamma^2\,\hbar^2\, y_i^2\,\eta_i^2\right)\right)\epsilon^2 + O[\epsilon]^3\right]$$

$$\mathbb{E}L\left[a_j\,\alpha_i+a_k\,\alpha_i,\ x_j\,\xi_i+x_k\,\xi_i,\ \frac{1}{2}\hbar\, x_k\,\xi_i\left(-2\,a_j+\gamma\, x_j\,\xi_i\right)\epsilon + \frac{1}{12}\hbar^2\, x_k\,\xi_i\left(6\,a_j^2-6\,\gamma\, a_j\, x_j\,\xi_i+\gamma^2\, x_j\,\xi_i\left(3+2\,x_j\,\xi_i+2\,x_k\,\xi_i\right)\right)\epsilon^2 + O[\epsilon]^3\right]$$

$$\mathbb{E}L\left[b_j\,\beta_i+b_k\,\beta_i,\ B_k\, y_j\,\eta_i+y_k\,\eta_i,\ \frac{1}{2}\gamma\,\hbar\, B_k\, y_j\, y_k\,\eta_i^2\,\epsilon + \frac{1}{12}\gamma^2\,\hbar^2\, B_k\, y_j\, y_k\,\eta_i^2\left(3+2\,B_k\, y_j\,\eta_i+2\,y_k\,\eta_i\right)\epsilon^2 + O[\epsilon]^3\right]$$