

In[*]:=

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MyCollect[ε_, vs_List] := MyCollect[ε, vs, Identity];
MyCollect[ε_, vs_List, simp_] :=
  Total[CoefficientRules[ε, vs] /. ((ps_ -> c_) => simp[c] Times @@ (vs^ps))];
MyCollect[εs_List, vs_List] := MyCollect[#, vs] & /@ εs;
MyCollect[εs_List, vs_List, simp_] := MyCollect[#, vs, simp] & /@ εs;
MyCollect[sd_SeriesData, vs_List] := MapAt[MyCollect[#, vs] &, sd, 3];
MyCollect[sd_SeriesData, vs_List, simp_] := MapAt[MyCollect[#, vs, simp] &, sd, 3];
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In[*]:=

$$r = \omega^{-1} + \left(\frac{-2 T \omega d\omega}{(T-1)} x y + 2 T \omega d\omega a + \left(\omega T d\omega - \frac{(T-1)^2}{T} p1 \right) \right) \frac{\epsilon}{\omega^3} +$$

$$\left(2 T \omega^2 (2 d\omega^2 T - d\omega \omega - dd\omega T \omega) a^2 + \left(\frac{2 (-1+T) p1 ((1+T) \omega - 3 (-1+T) T d\omega)}{T} + \right. \right.$$

$$\left. \left. 2 \omega ((-1+T)^2 dp1 + 2 T^2 (d\omega)^2 - T \omega (d\omega + T dd\omega)) \right) \omega a + \right.$$

$$\frac{T \omega^2 (4 d\omega^2 (-1+T) T - d\omega (-3+T) \omega - 2 dd\omega (-1+T) T \omega)}{(-1+T)^3} x^2 y^2 -$$

$$\frac{2 \omega (-3 d\omega p1 (-1+T) T + dp1 (-1+T) T \omega + p1 (1+T) \omega)}{T} x y +$$

$$\frac{4 T \omega^2 (2 d\omega^2 (1-T) T - d\omega \omega - dd\omega (1-T) T \omega)}{(-1+T)^2} a x y - \left(\frac{p2}{2} + \omega \left(3 d\omega p1 (-1+T)^2 - \right. \right.$$

$$\left. \left. \frac{(p1 (-1+T^2) + T (dp1 (-1+T)^2 + 2 d\omega^2 T^2)) \omega}{T} + T (d\omega + dd\omega T) \omega^2 \right) \right) \frac{\epsilon^2}{\omega^5}$$

Out[*]:=

$$\frac{1}{\omega} + \frac{\epsilon \left(-\frac{p1 (-1+T)^2}{T} + d\omega T \omega + 2 a d\omega T \omega - \frac{2 d\omega T x y \omega}{-1+T} \right)}{\omega^3} +$$

$$\frac{1}{\omega^5} \epsilon^2 \left(-\frac{p2}{2} + 2 a^2 T \omega^2 (2 d\omega^2 T - d\omega \omega - dd\omega T \omega) + \frac{4 a T x y \omega^2 (2 d\omega^2 (1-T) T - d\omega \omega - dd\omega (1-T) T \omega)}{(-1+T)^2} + \right.$$

$$\frac{T x^2 y^2 \omega^2 (4 d\omega^2 (-1+T) T - d\omega (-3+T) \omega - 2 dd\omega (-1+T) T \omega)}{(-1+T)^3} -$$

$$\frac{2 x y \omega (-3 d\omega p1 (-1+T) T + dp1 (-1+T) T \omega + p1 (1+T) \omega)}{T} -$$

$$\omega \left(3 d\omega p1 (-1+T)^2 - \frac{(p1 (-1+T^2) + T (dp1 (-1+T)^2 + 2 d\omega^2 T^2)) \omega}{T} + T (d\omega + dd\omega T) \omega^2 \right) +$$

$$a \omega \left(\frac{2 p1 (-1+T) (-3 d\omega (-1+T) T + (1+T) \omega)}{T} + \right.$$

$$\left. \left. 2 \omega (dp1 (-1+T)^2 + 2 d\omega^2 T^2 - T (d\omega + dd\omega T) \omega) \right) \right)$$

In[*]:= Res1 = FullSimplify@Log[ω r + 0[ε]³]

$$\text{Out[*]} = \frac{\left(-p1 (-1 + T)^3 + d\omega T^2 (-1 + 2 a (-1 + T) + T - 2 x y) \omega\right) \epsilon}{(-1 + T) T \omega^2} - \frac{1}{2 \omega^4} \left(p2 + \frac{p1^2 (-1 + T)^4}{T^2} - \frac{1}{(-1 + T)^2} \left(2 dp1 (-1 + T)^3 ((1 + 2 a) (-1 + T) - 2 x y) + d\omega^2 T^2 \left((3 + 4 a (1 + a)) (-1 + T)^2 - 4 (-1 + 2 a) (-1 + T) x y + 4 x^2 y^2 \right) \right) \omega^2 + \frac{1}{(-1 + T)^3} 2 T \left((1 + 2 a (1 + a)) (-1 + T)^3 (d\omega + dd\omega T) + 4 a (-1 + T) (d\omega - dd\omega (-1 + T) T) x y + (d\omega (-3 + T) + 2 dd\omega (-1 + T) T) x^2 y^2 \right) \omega^3 + \frac{2 p1 (-1 + 2 a (-1 + T) + T - 2 x y) \omega (2 d\omega (-1 + T) T - (1 + T) \omega)}{T} \right) \epsilon^2 + 0[\epsilon]^3$$

In[*]:= TeXForm[Res1 /. {dω → ω', ddω → ω'', p1 → ρ1, dp1 → ρ1', p2 → ρ2}]

$$\text{Out[*]} // \text{TeXForm} = \frac{\epsilon \left((T^2 \omega \omega' (2 a (T-1) + T - 2 x y - 1) - \rho_1 (T-1)^3 \right) \omega^2 \left(-\frac{\omega^2 \left(T^2 \left(\omega \right)^2 \left(-4 (2 a - 1) (T-1) : y^2 \right) \right) + 2 (T-1)^3 \rho_1' \left((2 a + 1) (T-1) - 2 x y \right) \right) \{ (T-1)^2 \} + \frac{2 T \omega' - (T-1) T \omega'' \right) + (2 a (a+1) (T-1)^3 \left(T \omega \omega' + \omega \omega'' \right) + (T-3) \omega \omega'' \right) \{ (T-1)^3 \} + \frac{2 \rho_1 \omega \left(2 (T-1) T \left((T-1) + T - 2 x y - 1 \right) \right) \{ T \} + \rho_2 \frac{\rho_1^2 (T-1)^4 \{ T^2 \}}{T^2} \right) \{ 2 \omega^4 \} + 0 \omega^4}{(-1 + T) T \omega^2}$$

In[*]:= FullSimplify[Res1 /. T → 1 + T]

$$\text{Out[*]} = \frac{\left(-p1 T^3 + d\omega (1 + T)^2 (T + 2 a T - 2 x y) \omega\right) \epsilon}{T (1 + T) \omega^2} - \frac{1}{2 \omega^4} \left(p2 + \frac{p1^2 T^4}{(1 + T)^2} + 4 d\omega p1 T (T + 2 a T - 2 x y) \omega - \frac{1}{T^2 (1 + T)} \left(2 T^2 (dp1 T (1 + T) + p1 (2 + T)) (T + 2 a T - 2 x y) + d\omega^2 (1 + T)^3 \left((3 + 4 a (1 + a)) T^2 + 4 (1 - 2 a) T x y + 4 x^2 y^2 \right) \right) \omega^2 + \frac{1}{T^3} 2 (1 + T) \left((1 + 2 a (1 + a)) T^3 (dd\omega + d\omega + dd\omega T) - 4 a T (-d\omega + dd\omega T (1 + T)) x y + (d\omega (-2 + T) + 2 dd\omega T (1 + T)) x^2 y^2 \right) \omega^3 \right) \epsilon^2 + 0[\epsilon]^3$$

In[*]:= Res2 = MyCollect[Log[ω r + 0[ε]^3], {a, x, y}, FullSimplify]

$$\begin{aligned}
 \text{Out[*]} = & \left(\frac{2 a d\omega T}{\omega} + \frac{2 d\omega T x y}{\omega - T \omega} + \frac{-p1 (-1 + T)^2 + d\omega T^2 \omega}{T \omega^2} \right) \epsilon + \\
 & \left(- \frac{2 a^2 T (-d\omega^2 T + d\omega \omega + dd\omega T \omega)}{\omega^2} + \frac{T x^2 y^2 (2 d\omega^2 (-1 + T) T - d\omega (-3 + T) \omega - 2 dd\omega (-1 + T) T \omega)}{(-1 + T)^3 \omega^2} - \right. \\
 & \frac{4 a T x y (d\omega^2 (-1 + T) T + d\omega \omega - dd\omega (-1 + T) T \omega)}{(-1 + T)^2 \omega^2} + \\
 & \frac{2 x y (2 d\omega p1 (-1 + T) + \frac{d\omega^2 T^2 \omega}{-1 + T} - \frac{(dp1 (-1 + T) T + p1 (1 + T)) \omega}{T})}{\omega^3} + \\
 & \frac{a (-4 d\omega p1 (-1 + T)^2 T + 2 (p1 (-1 + T)^2) + T (dp1 (-1 + T)^2 + d\omega^2 T^2)) \omega - 2 T^2 (d\omega + dd\omega T) \omega^2}{T \omega^3} + \\
 & \left. \frac{1}{2 T^2 \omega^4} (-p1^2 (-1 + T)^4 - 2 p1 (-1 + T) T \omega (2 d\omega (-1 + T) T - (1 + T) \omega) + \right. \\
 & \left. T^2 (-p2 + \omega^2 (2 dp1 (-1 + T)^2 + T (3 d\omega^2 T - 2 (d\omega + dd\omega T) \omega))) \right) \epsilon^2 + 0[\epsilon]^3
 \end{aligned}$$

In[*]:= MyCollect[Log[(ω r / . ε → ω^2 ε) + 0[ε]^3], {ω, dω, ddω, p1, dp1, p2}, Simplify]

$$\begin{aligned}
 \text{Out[*]} = & \left(- \frac{p1 (-1 + T)^2}{T} + d\omega T \left(1 + 2 a - \frac{2 x y}{-1 + T} \right) \omega \right) \epsilon + \\
 & \left(- \frac{p2}{2} - \frac{p1^2 (-1 + T)^4}{2 T^2} - 2 d\omega p1 (-1 + T) (-1 + 2 a (-1 + T) + T - 2 x y) \omega + \right. \\
 & dp1 (-1 + T) (-1 + 2 a (-1 + T) + T - 2 x y) \omega^2 + \frac{p1 (1 + T) (-1 + 2 a (-1 + T) + T - 2 x y) \omega^2}{T} + \\
 & \frac{d\omega^2 T^2 (3 + 4 a^2 (-1 + T)^2 + 3 T^2 - 4 x y + 4 x^2 y^2 + 4 a (-1 + T) (-1 + T - 2 x y) + T (-6 + 4 x y)) \omega^2}{2 (-1 + T)^2} - \\
 & \frac{dd\omega T^2 (1 + 2 a^2 (-1 + T)^2 - 2 T + T^2 + 2 x^2 y^2 + 2 a (-1 + T) (-1 + T - 2 x y)) \omega^3}{(-1 + T)^2} + \\
 & \left. d\omega T \left(-1 - 2 a^2 - \frac{(-3 + T) x^2 y^2}{(-1 + T)^3} + a \left(-2 - \frac{4 x y}{(-1 + T)^2} \right) \right) \omega^3 \right) \epsilon^2 + 0[\epsilon]^3
 \end{aligned}$$

In[]:= **MyCollect**[**Log**[($\omega r / \epsilon \rightarrow \omega^2 \epsilon$) + **O**[ϵ^3]], {**a**, **x**, **y**}, **FullSimplify**]

$$\begin{aligned} \text{Out[]} = & \left(-\frac{p1 (-1 + \tau)^2}{\tau} + d\omega \tau \omega + 2 a d\omega \tau \omega - \frac{2 d\omega \tau x y \omega}{-1 + \tau} \right) \epsilon + \left(-2 a^2 \tau \omega^2 (-d\omega^2 \tau + d\omega \omega + dd\omega \tau \omega) + \right. \\ & \frac{\tau x^2 y^2 \omega^2 (2 d\omega^2 (-1 + \tau) \tau - d\omega (-3 + \tau) \omega - 2 dd\omega (-1 + \tau) \tau \omega)}{(-1 + \tau)^3} - \\ & \frac{4 a \tau x y \omega^2 (d\omega^2 (-1 + \tau) \tau + d\omega \omega - dd\omega (-1 + \tau) \tau \omega)}{(-1 + \tau)^2} + \\ & \left. 2 x y \omega \left(2 d\omega p1 (-1 + \tau) + \frac{d\omega^2 \tau^2 \omega}{-1 + \tau} - \frac{(dp1 (-1 + \tau) \tau + p1 (1 + \tau)) \omega}{\tau} \right) + \right. \\ & \left. \frac{2 a \omega (-2 d\omega p1 (-1 + \tau)^2 \tau + p1 (-1 + \tau^2) \omega + \tau (dp1 (-1 + \tau)^2 + d\omega^2 \tau^2) \omega - \tau^2 (d\omega + dd\omega \tau) \omega^2)}{\tau} + \right. \\ & \left. \frac{1}{2 \tau^2} (-p1^2 (-1 + \tau)^4 - 2 p1 (-1 + \tau) \tau \omega (2 d\omega (-1 + \tau) \tau - (1 + \tau) \omega) + \right. \\ & \left. \left. \tau^2 (-p2 + \omega^2 (2 dp1 (-1 + \tau)^2 + \tau (3 d\omega^2 \tau - 2 (d\omega + dd\omega \tau) \omega))) \right) \right) \epsilon^2 + O[\epsilon]^3 \end{aligned}$$

In[]:= **MyCollect**[$\omega^5 r + O[\epsilon]^3$, { ω , **d** ω , **dd** ω , **p1**, **dp1**, **p2**}, **Simplify**]

$$\begin{aligned} \text{Out[]} = & \omega^4 + \left(-\frac{p1 (-1 + \tau)^2 \omega^2}{\tau} + d\omega \tau \left(1 + 2 a - \frac{2 x y}{-1 + \tau} \right) \omega^3 \right) \epsilon + \\ & \left(-\frac{p2}{2} - 3 d\omega p1 (-1 + \tau) (-1 + 2 a (-1 + \tau) + \tau - 2 x y) \omega + \right. \\ & dp1 (-1 + \tau) (-1 + 2 a (-1 + \tau) + \tau - 2 x y) \omega^2 + \frac{p1 (1 + \tau) (-1 + 2 a (-1 + \tau) + \tau - 2 x y) \omega^2}{\tau} + \\ & \frac{2 d\omega^2 \tau^2 (1 + 2 a^2 (-1 + \tau)^2 - 2 \tau + \tau^2 + 2 x^2 y^2 + 2 a (-1 + \tau) (-1 + \tau - 2 x y)) \omega^2}{(-1 + \tau)^2} - \\ & \left. \frac{dd\omega \tau^2 (1 + 2 a^2 (-1 + \tau)^2 - 2 \tau + \tau^2 + 2 x^2 y^2 + 2 a (-1 + \tau) (-1 + \tau - 2 x y)) \omega^3}{(-1 + \tau)^2} + \right. \\ & \left. d\omega \tau \left(-1 - 2 a^2 - \frac{(-3 + \tau) x^2 y^2}{(-1 + \tau)^3} + a \left(-2 - \frac{4 x y}{(-1 + \tau)^2} \right) \right) \omega^3 \right) \epsilon^2 + O[\epsilon]^3 \end{aligned}$$

`In[]:= MyCollect[Log[(ω r /. ε → ω² ε) + 0[ε]³] /. {ρ1 → $\frac{\tau \rho 1}{(\tau - 1)^2}$, dp1 → $\frac{-(1 + \tau) \rho 1 + (-1 + \tau) \tau d\rho 1}{(-1 + \tau)^3}$ },`
`{a, x, y}, FullSimplify]`

$$\begin{aligned} \text{Out[]:= } & \left(-\rho 1 + d\omega \tau \omega + 2 a d\omega \tau \omega - \frac{2 d\omega \tau x y \omega}{-1 + \tau} \right) \epsilon + \\ & \left(-2 a^2 \tau \omega^2 (-d\omega^2 \tau + d\omega \omega + dd\omega \tau \omega) + \frac{2 \tau x y \omega (2 d\omega \rho 1 - d\rho 1 \omega + d\omega^2 \tau \omega)}{-1 + \tau} + \right. \\ & \frac{\tau x^2 y^2 \omega^2 (2 d\omega^2 (-1 + \tau) \tau - d\omega (-3 + \tau) \omega - 2 dd\omega (-1 + \tau) \tau \omega)}{(-1 + \tau)^3} - \\ & \frac{4 a \tau x y \omega^2 (d\omega^2 (-1 + \tau) \tau + d\omega \omega - dd\omega (-1 + \tau) \tau \omega)}{(-1 + \tau)^2} + \\ & \left. 2 a \tau \omega (-2 d\omega \rho 1 + (d\rho 1 + d\omega^2 \tau) \omega - (d\omega + dd\omega \tau) \omega^2) + \right. \\ & \left. \frac{1}{2} (-p2 - \rho 1^2 - 4 d\omega \tau \rho 1 \omega + \tau \omega^2 (2 d\rho 1 + 3 d\omega^2 \tau - 2 (d\omega + dd\omega \tau) \omega)) \right) \epsilon^2 + 0[\epsilon]^3 \end{aligned}$$

`In[]:= FullSimplify[∂τ $\left(\frac{\tau \rho 1[\tau]}{(\tau - 1)^2} \right)$]`

$$\text{Out[]:= } \frac{-(1 + \tau) \rho 1[\tau] + (-1 + \tau) \tau \rho 1'[\tau]}{(-1 + \tau)^3}$$