Pensieve header: A unified verification notebook for the sl_2-portfolio project; continues pensieve://Projects/PPSA/nb/Verification.pdf.

Continues pensieve://2017-06/ and pensieve://2017-08/.

Prolog

```
Go;
```

```
wdir = SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio"];
NotebookOpen[wdir <> "\\MakeSnips.nb"];
```

```
HL[ℰ_] := Style[ℰ, Background → Yellow];
```

Initialization / Utilities

The “degree carrier / filtration parameter” is ℏ, and all “coupling constants” are proportional to it.

TD

```
$p = 2; $k = 0; $E := {$k, $p};
$trim := {ℏ^p_. /; p > $p → 0, ϵ^k_. /; k > $k → 0};
SetAttributes[{SS, SST}, HoldAll];
TRule = {T_i_ → e^(ℏ t_i), T → e^(ℏ t)}; q_ℏ = e^(γ ϵ ℏ);
SS[ℰ_, op_] := Collect[
   Normal@Series[If[$p > 0, ℰ, ℰ /. TRule], {ℏ, 0, $p}],
   ℏ, op];
SS[ℰ_] := SS[ℰ, Together];
SST[ℰ_, op___] := SS[ℰ /. TRule, op];
Simp[ℰ_, op_] := Collect[ℰ, _CU | _QU, op];
Simp[ℰ_] := Simp[ℰ, SS[#, Expand] &];
SimpT[ℰ_] := Collect[ℰ, _CU | _QU, SST[#, Expand] &];
```

Differential polynomials (DP):

Utils

```
DP_{α_→D_x_, β_→D_y_}[P_][λ_] :=
 Total[CoefficientRules[P, {α, β}] /. ({m_, n_} → c_) :→ c ∂_{{x,m},{y,n}} λ]
```

```
HL[DP_{x→D_ξ, y→D_η}[x^2 y^3][e^(δ ξ η)] == 6 e^(δ η ξ) δ^3 ξ + 6 e^(δ η ξ) δ^4 η ξ^2 + e^(δ η ξ) δ^5 η^2 ξ^3]
```

True

Self-Pair (SP):

SP

```
SP_{}[P_] := P; SP_{ℰ_→x_, ps___}[P_] := Expand[P // SP_{ps}] /. f_. ℰ^d_. :→ ∂_{x,d} f
```

```
SP_{ξ→x}[(ξ^2 + ξ + 3) (x^5 e^x + 7 x) + 99 a]
```

$7 + 99 a + 21 x + 20 e^x x^3 + 15 e^x x^4 + 5 e^x x^5$

$\text{SP}_{\{\xi\to x,\eta\to y\}}\left[\left(\xi^2+\xi+3+2\,\xi\,\eta\right)\left(x^5\,e^x+7\,x\right)+99\,a+e^{\delta\,x\,y}\,\xi\,\eta\right]$

$7+99\,a+21\,x+20\,e^x\,x^3+15\,e^x\,x^4+5\,e^x\,x^5+e^{x\,y\,\delta}\,\delta+e^{x\,y\,\delta}\,x\,y\,\delta^2$

DeclareAlgebra

QLImplementation

```
Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z__] := (x ** y) ** z;
0 ** _ = _ ** 0 = 0;
(x_Plus) ** y_ := (# ** y) & /@ x; x_ ** (y_Plus) := (x ** #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ** y - y ** x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

QLImplementation

```
DeclareAlgebra[U_Symbol, opts__Rule] := Module[{gp, sr, g, cp, M, CE, pow, k = 0,
    gs = Generators /. {opts},
    cs = Centrals /. {opts} /. Centrals → {} },
  (#_U = U@#) & /@ gs;
  gp = Alternatives @@ gs; gp = gp | gp_; (* gens *)
  sr = Flatten@Table[{g → ++k, g_i_ → {i, k}}, {g, gs}]; (* sorting → *)
  cp = Alternatives @@ cs; (* cents *)
  SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
  CE[ℰ_] := Collect[ℰ, _U, Expand] /. $trim;
  U_i_[ℰ_] := ℰ /. {t : cp :> t_i, u_U :> Replace[u, x_ :> x_i, 1]};
  U_i_[NCM[]] = pow[ℰ_, 0] = U@{} = 1_U = U[];
  B[U@(x_)_i_, U@(y_)_i_] := U_i@B[U@x, U@y];
  B[U@(x_)_i_, U@(y_)_j_] /; i =!= j := 0;
  B[U@y_, U@x_] := CE[-B[U@x, U@y]];
  x_ ** (c_. 1_U) := CE[c x]; (c_. 1_U) ** x_ := CE[c x];
  (a_. U[xx___, x_]) ** (b_. U[y_, yy___]) := If[OrderedQ[{x, y} /. sr],
    CE@M[a b /. $trim, U[xx, x, y, yy]],
    U@xx ** CE@M[a b /. $trim, U@y ** U@x + B[U@x, U@y, $E]] ** U@yy ];
  U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ** U@{r}];
  U@{c_. * l : gp, r___} := CE[c U[l] ** U@{r}];
  U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
  U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
  U@{l_, r___} := U@{Expand[l], r};
  U[ℰ_NonCommutativeMultiply] := U /@ ℰ;
  𝕆_U[specs___, poly_] := Module[{sp, null, vs, us},
    sp = Replace[{specs}, l_List :> l_null, {1}];
    vs = Join @@ (First /@ sp);
    us = Join @@ (sp /. l_s_ :> (l /. x_i_ :> x_s));
    CE[Total[
      CoefficientRules[poly, vs] /. (p_ → c_) :> c U@(us^p)
     ]] /. x_null :> x];
  pow[ℰ_, n_] := pow[ℰ, n - 1] ** ℰ;
  𝕊_U[ℰ_, ss___Rule] := CE@Total[
    CoefficientRules[ℰ, First /@ {ss}] /.
     (p_ → c_) :> c NCM @@ MapThread[pow, {Last /@ {ss}, p}]];
  m_j_→k_[c_. * u_U] := CE[((c /. (t : cp)_j → t_k) DeleteCases[u, _j|k]) **
    U @@ Cases[u, w_j :> w_k] ** U @@ Cases[u, _k]];
  S_i_[c_. * u_U] := CE[((c /. S_i[U, Centrals]) DeleteCases[u, _i]) **
    U_i[NCM @@ Reverse@Cases[u, x_i :> S@U@x]]] ]
```

DeclareMorphism

QLImplementation

```
DeclareMorphism[m_, U_ → V_, ongs_List, oncs_List: {}] := (
  Replace[ongs,
   {(g_ → img_) ⧴ (m[U[g]] = img), (g_ ⧴ img_) ⧴ (m[U[g]] := img /. $trim)}, {1}];
  m[1_U] = 1_V;
  m[U[g_i_]] := V_i[m[U@g]];
  m[U[vs__]] := NCM @@ (m /@ U /@ {vs});
  m[ℰ_] := Simp[ℰ /. oncs /. u_U ⧴ m[u]] /. $trim; )
```

Meta-Operations

QLImplementation

```
m_{j_→j_} = Identity;
m_{j_→k_}[ℰ_Plus] := Simp[m_{j→k} /@ ℰ];
m_{is___,i_,j_→k_}[ℰ_] := m_{j→k}@m_{is,i→j}@ℰ;
S_i_[ℰ_Plus] := Simp[S_i /@ ℰ];
```

Implementing CU = $\mathcal{U}(sl_2^{\gamma\epsilon})$

CU

```
DeclareAlgebra[CU, Generators → {y, a, x}, Centrals → {t}];
B[a_CU, y_CU] = -γ y_CU; B[x_CU, a_CU] = -γ x_CU;
B[x_CU, y_CU] = 2 ϵ a_CU - t 1_CU;
(S@CU@y = -y_CU; S@a_CU = -a_CU; S@x_CU = -x_CU;)
S_i_[CU, Centrals] = {t_i → -t_i};
```

Verifying associativity on triples of generators:

```
With[{bas = CU /@ {y, a, x}},
 Table[z1 ** (z2 ** z3) - (z1 ** z2) ** z3 // Simp // HL,
  {z1, bas}, {z2, bas}, {z3, bas}] ]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

Verifying associativity on a “random” triple:

```
With[{z1 = CU[y, y, a, a, x, x], z2 = CU[y, a, x], z3 = CU[y, y, a, x]}, {
   (rhs = (z1 ** z2) ** z3 // Simp) // Short,
   z1 ** (z2 ** z3) - rhs // Simp // HL
  }] // Timing
```

```
{1.01563, {28 t^2 γ^4 CU[y, y, y, x, x] + <<23>> + CU[y, y, y, y, y, a, a, a, a, x, x, x, x], 0}}
```

Verifying that *S* is an anti-homomorphism on CU:

```
With[{bas = CU /@ {y₁, a₁, x₁}},
 Table[HL@Simp[S₁[z1 ** z2] - S₁[z2] ** S₁[z1]],
  {z1, bas}, {z2, bas} ] ]
```

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

Verifying the involutivity of S on products of triples:

```
With[{bas = CU /@ {y₁, a₁, x₁}},
 Table[HL@Simp[z1 ** z2 ** z3 - S₁@S₁[z1 ** z2 ** z3]],
  {z1, bas}, {z2, bas}, {z3, bas} ] ]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

Verifying relabeling:

```
t₁ t₃ CU[y₁, a₁, x₂] + t₁ t₁ CU[y₁, a₂, x₂] // m₁→₃
```

$CU[a_2, x_2, y_3]\, t_3^2 + CU[x_2, y_3, a_3]\, t_3^2$

Verifying meta-associativity:

```
Module[{z, u},
 Table[u = CU[z[[1]]₁, z[[2]]₂, z[[3]]₃]; z → HL[m₁,₃→₃@m₂,₃→₃@u == m₂,₃→₃@m₁,₂→₂@u],
  {z, Tuples[{y, a, x}, 3]}]]
```

```
{{y, y, y} → True, {y, y, a} → True, {y, y, x} → True, {y, a, y} → True,
 {y, a, a} → True, {y, a, x} → True, {y, x, y} → True, {y, x, a} → True,
 {y, x, x} → True, {a, y, y} → True, {a, y, a} → True, {a, y, x} → True, {a, a, y} → True,
 {a, a, a} → True, {a, a, x} → True, {a, x, y} → True, {a, x, a} → True, {a, x, x} → True,
 {x, y, y} → True, {x, y, a} → True, {x, y, x} → True, {x, a, y} → True, {x, a, a} → True,
 {x, a, x} → True, {x, x, y} → True, {x, x, a} → True, {x, x, x} → True}
```

Implementing QU = $\mathcal{U}_q(sl_2^{\gamma\epsilon})$

Aside

```
Series[(1 - T ⅇ^(-2 ϵ a ℏ)) / ℏ, {a, 0, 3}]
```

Aside

$$\frac{1-T}{\hbar} + 2\,T\,\epsilon\,a - 2\,(T\,\epsilon^2\,\hbar)\,a^2 + \frac{4}{3}\,T\,\epsilon^3\,\hbar^2\,a^3 + O[a]^4$$

QU

```
DeclareAlgebra[QU, Generators → {y, a, x}, Centrals → {t, T}];
B[a_QU, y_QU] = -γ y_QU; B[x_QU, a_QU] = -γ QU@x;
B[x_QU, y_QU] := SS[q_ℏ - 1] QU@{y, x} + 𝕆_QU[{a}, SS[(1 - T ⅇ^(-2 ϵ a ℏ)) / ℏ]];
(S@y_QU := 𝕆_QU[{a, y}, SS[-T^-1 ⅇ^(ℏ ϵ a) y]]; S@a_QU = -a_QU; S@x_QU := 𝕆_QU[{a, x}, SS[-ⅇ^(ℏ ϵ a) x]];)
S_i_[QU, Centrals] = {t_i → -t_i, T_i → T_i^-1};
```

```
With[{bas = QU /@ {y, a, x}}, Table[{z1, z2} → Simp[z1 ** z2 - z2 ** z1], {z1, bas}, {z2, bas}] ]
```

$$\left\{\left\{\{QU[y], QU[y]\} \to 0, \{QU[y], QU[a]\} \to \gamma\, QU[y], \{QU[y], QU[x]\} \to \frac{(-1+T)\, QU[\,]}{\hbar}\right\}, \{\{QU[a], QU[y]\} \to -\gamma\, QU[y], \{QU[a], QU[a]\} \to 0, \{QU[a], QU[x]\} \to \gamma\, QU[x]\}, \left\{\{QU[x], QU[y]\} \to \frac{(1-T)\, QU[\,]}{\hbar}, \{QU[x], QU[a]\} \to -\gamma\, QU[x], \{QU[x], QU[x]\} \to 0\right\}\right\}$$

Verifying associativity on triples of generators:

```
With[{bas = QU /@ {y, a, x}},
 Table[HL[z1 ** (z2 ** z3) - (z1 ** z2) ** z3] // Simp,
  {z1, bas}, {z2, bas}, {z3, bas} ] ]
```

{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}

Verifying associativity on a “random” triple (~34 secs @ $p=5, $k=2):

```
With[{z1 = QU[y, y, a, a, x, x], z2 = QU[y, a, x], z3 = QU[y, y, a, x]}, {
   (rhs = (z1 ** z2) ** z3 // Simp) // Short,
   HL[z1 ** (z2 ** z3) - rhs // Simp ]
  }] // Timing
```

$$\left\{1.60938, \left\{\frac{(28\gamma^4 - 56\, T\gamma^4 + 28\, T^2\gamma^4)\, QU[y, y, y, x, x]}{\hbar^2} + \ll 17 \gg + QU[y, y, y, y, y, a, a, a, a, x, x, x, x], 0\right\}\right\}$$

Verifying that *S* is an anti-homomorphism on QU:

```
With[{bas = QU /@ {y₁, a₁, x₁}},
 Table[{z1, z2} → HL@Simp[S₁[z1 ** z2] - S₁[z2] ** S₁[z1]],
  {z1, bas}, {z2, bas} ] ]
```

$$\{\{\{QU[y_1], QU[y_1]\} \to 0, \{QU[y_1], QU[a_1]\} \to 0, \{QU[y_1], QU[x_1]\} \to 0\}, \{\{QU[a_1], QU[y_1]\} \to 0, \{QU[a_1], QU[a_1]\} \to 0, \{QU[a_1], QU[x_1]\} \to 0\}, \{\{QU[x_1], QU[y_1]\} \to 0, \{QU[x_1], QU[a_1]\} \to 0, \{QU[x_1], QU[x_1]\} \to 0\}\}$$

Verifying that $\mathrm{Lim}_{\hbar\to 0}$ QU = CU using a “random” product (~23 secs @ $p=5, $k=2):

```
With[{z1 = CU[y, y, a, a, x, x], z2 = CU[y, a, x], z3 = CU[y, y, a, x]}, {
   Short[lhs = z1 ** (z2 ** z3)],
   Short[rhs = (QU @@ z1) ** ((QU @@ z2) ** (QU @@ z3))],
   Expand[Limit[rhs /. TRule ⋃ {QU → CU}, ℏ → 0] - lhs] // HL
  }] // Timing
```

$$\left\{3.71875, \left\{28\, t^2\gamma^4\, CU[y, y, y, x, x] + \ll 23 \gg + CU[y, y, y, y, y, a, a, a, a, x, x, x, x], 7\left(\frac{4\gamma^4}{\hbar^2} - \frac{8\, T\gamma^4}{\hbar^2} + \frac{4\, T^2\gamma^4}{\hbar^2}\right) QU[y, y, y, x, x] + \ll 41 \gg + QU[y, y, y, y, y, a, a, a, a, x, x, x, x], 0\right\}\right\}$$

Implementing θ

theta

```
DeclareMorphism[Cθ, CU → CU, {y → -x_CU, a → -a_CU, x → -y_CU}, {t → -t, T → T^-1}];
DeclareMorphism[Qθ, QU → QU, {y :→ 𝕆_QU[{a, x}, SS[-T^(-1/2) ℯ^(ħ ϵ a) x]],
  a → -a_QU, x :→ 𝕆_QU[{a, y}, SS[-T^(-1/2) ℯ^(ħ ϵ a) y]]}, {t → -t, T → T^-1}]
```

Verifying involutivity on CU:

```
With[{bas = CU /@ {y, a, x}},
 Table[z → Cθ[z] → HL[Cθ[Cθ[z]]], {z, bas}] ]
```

{CU[y] → -CU[x] → CU[y], CU[a] → -CU[a] → CU[a], CU[x] → -CU[y] → CU[x]}

Verifying that θ is a multiplicative homomorphism on CU:

```
With[{bas = CU /@ {y, a, x}},
 Table[Cθ[z1 ** z2] - Cθ[z1] ** Cθ[z2] // HL, {z1, bas}, {z2, bas}] ]
```

{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}

Verifying involutivity on QU:

```
With[{bas = QU /@ {y, a, x}},
 Table[z → Qθ[z] → HL[Simp[Qθ[Qθ[z]], PowerExpand]], {z, bas}] ]
```

$\left\{QU[y] \to -\frac{QU[x]}{\sqrt{T}} \to QU[y],\ QU[a] \to -QU[a] \to QU[a],\ QU[x] \to -\frac{QU[y]}{\sqrt{T}} \to QU[x]\right\}$

Verifying that θ is a multiplicative homomorphism on QU:

```
With[{bas = QU /@ {y, a, x}},
 Table[{z1, z2} → HL[Simp[Qθ[z1 ** z2] - Qθ[z1] ** Qθ[z2]]], {z1, bas}, {z2, bas}] ]
```

{{{QU[y], QU[y]} → 0, {QU[y], QU[a]} → 0, {QU[y], QU[x]} → 0},
 {{QU[a], QU[y]} → 0, {QU[a], QU[a]} → 0, {QU[a], QU[x]} → 0},
 {{QU[x], QU[y]} → 0, {QU[x], QU[a]} → 0, {QU[x], QU[x]} → 0}}

The Asymmetric Dequantizator

Following pensieve://People/VanDerVeen/Dequant1.pdf.

ADeq

$$A\mathbb{D}\$f = \gamma\left(\left(\mathrm{Cosh}\left[\hbar\left(a\,\epsilon + \frac{\gamma\,\epsilon}{2} - \frac{t}{2}\right)\right] - \mathrm{Cosh}\left[\hbar\sqrt{\left(\frac{t-\gamma\,\epsilon}{2}\right)^2 + \epsilon\,\omega}\right]\right)\Big/\left(\hbar\,e^{\hbar\,((a+\gamma)\,\epsilon - t/2)}\,\mathrm{Sinh}\left[\frac{\gamma\,\epsilon\,\hbar}{2}\right]\left(a^2\,\epsilon + a\,\gamma\,\epsilon - a\,t - \omega\right)\right)\right);$$

Scaling behaviour of A𝔻$f:

```
HL@Simplify[A𝔻$f == ((A𝔻$f /. γ → 1) /. {ϵ → γ ϵ, a → γ^-1 a, ω → γ^-1 ω})]
```

True

```
HL@FullSimplify[
  A𝔻$f == ((A𝔻$f /. γ → 1) /. {ℏ → γ^2 ℏ, ϵ → ϵ / γ, a → a / γ, t → γ^-2 t, ω → γ^-3 ω})]
```

True

ADeq

```
A𝔻$ω = γ CU[y, x] + ϵ CU[a, a] - (t - γ ϵ) CU[a];
```

ADeq

```
DeclareMorphism[A𝔻, QU → CU,
  {a → a_CU, x → CU@x, y :→ 𝕊_CU[SS[A𝔻$f], a → a_CU, ω → A𝔻$ω] ** y_CU}]
```

Verifying that the asymmetric dequantizator is a homomorphism:

```
With[{bas = QU /@ {y, a, x}},
 Table[{z1, z2} → HL[SimpT[A𝔻[z1 ** z2] - A𝔻[z1] ** A𝔻[z2]]], {z1, bas}, {z2, bas}] ]
```

```
{{{QU[y], QU[y]} → 0, {QU[y], QU[a]} → 0, {QU[y], QU[x]} → 0},
 {{QU[a], QU[y]} → 0, {QU[a], QU[a]} → 0, {QU[a], QU[x]} → 0},
 {{QU[x], QU[y]} → 0, {QU[x], QU[a]} → 0, {QU[x], QU[x]} → 0}}
```

The Symmetric Dequantizator

Following pensieve://People/VanDerVeen/Dequant1.pdf.

SDeq

$$S\mathbb{D}\$g = \sqrt{\left(\left(2\gamma\left(\mathrm{Cosh}\left[\frac{\hbar}{2}\sqrt{t^2+\gamma^2\epsilon^2+4\epsilon\varpi}\right]-\mathrm{Cosh}\left[\frac{t-\epsilon\gamma-2\epsilon a}{2/\hbar}\right]\right)\right)\Big/\left(\mathrm{Sinh}\left[\frac{\gamma\epsilon\hbar}{2}\right]\left(t\,(2a+\gamma)-2a\,(a+\gamma)\,\epsilon+2\varpi\right)\hbar\right)\right)};$$

Verify agreement with the formulas in pensieve://People/VanDerVeen/Dequant1.pdf:

```
{SD$P = (Cosh[ħ ((ϵ-t)/2 + ϵ a)] - Cosh[ħ Sqrt[(t^2+ϵ^2)/4 + ϵ ϖ]]) / (ħ Sinh[-ϵ ħ/2] (ϖ - ϵ a^2 + (t - ϵ) a + t/2)),
 Simplify[SD$P == (SD$P /. {a → -a - 1, t → -t})] // HL,
 PowerExpand@Simplify[(SD$P /. {ħ → γ^2 ħ, ϵ → ϵ/γ, a → a/γ, t → γ^-2 t, ϖ → γ^-3 ϖ}) ==
    SD$g (SD$g /. {a → -a - γ, t → -t})] // HL,
 SD$Q = Simplify[SD$P /. {a → c - 1/2}],
 Simplify[SD$Q == (SD$Q /. {c → -c, t → -t})] // HL,
 FullSimplify[SD$g == FullSimplify[
    Sqrt[SD$Q] /. c → a + 1/2 /. {ħ → γ^2 ħ, ϵ → ϵ/γ, a → a/γ, t → γ^-2 t, ϖ → γ^-3 ϖ}]] // HL
}
```

$$\left\{-\left(\left(\left(\mathrm{Cosh}\left[\left(a\,\epsilon+\frac{1}{2}(-t+\epsilon)\right)\hbar\right]-\mathrm{Cosh}\left[\sqrt{\frac{1}{4}(t^2+\epsilon^2)+\epsilon\,\varpi}\;\hbar\right]\right)\mathrm{Csch}\left[\frac{\epsilon\,\hbar}{2}\right]\right)\Big/\left(\left(\frac{t}{2}+a(t-\epsilon)-a^2\epsilon+\varpi\right)\hbar\right)\right),\ \text{True},\ \text{True},\right.$$

$$\left.-\left(\left(4\left(\mathrm{Cosh}\left[\frac{1}{2}(t-2c\,\epsilon)\hbar\right]-\mathrm{Cosh}\left[\frac{1}{2}\sqrt{t^2+\epsilon^2+4\,\epsilon\,\varpi}\;\hbar\right]\right)\mathrm{Csch}\left[\frac{\epsilon\,\hbar}{2}\right]\right)\Big/\left(\left(4\,c\,t+\epsilon-4\,c^2\epsilon+4\,\varpi\right)\hbar\right)\right),\ \text{True},\ \text{True}\right\}$$

SDeq

```
SD$f = Simplify[e^(ħ (t/2-ϵ a)) (SD$g /. {a → -a, t → -t})];
```

SDeq

```
SD$ϖ = γ CU[y, x] + ϵ CU[a, a] - (t - γ ϵ) CU[a] - t γ 1_CU/2;
```

SDeq

```
DeclareMorphism[SD, QU → CU, {a → a_CU,
  x :→ S_CU[SS[SD$f], a → a_CU, ϖ → SD$ϖ] ** x_CU,
  y :→ S_CU[SS[SD$g], a → a_CU, ϖ → SD$ϖ] ** y_CU }]
```

Verifying the θ-symmetry:

```
Table[HL@SimpT[Cθ[SD[z]] == SD[Qθ[z]]], {z, QU /@ {y, a, x}}]
```

{True, True, True}

Verifying that the symmetric dequantizator is a homomorphism:

```
With[{bas = QU /@ {y, a, x}},
 Table[{z1, z2} → HL@SimpT[SD[z1 ** z2] - SD[z1] ** SD[z2]], {z1, bas}, {z2, bas}] ]
```

```
{{{QU[y], QU[y]} → 0, {QU[y], QU[a]} → 0, {QU[y], QU[x]} → 0},
 {{QU[a], QU[y]} → 0, {QU[a], QU[a]} → 0, {QU[a], QU[x]} → 0},
 {{QU[x], QU[y]} → 0, {QU[x], QU[a]} → 0, {QU[x], QU[x]} → 0}}
```

The representation ρ

rho

```
ρ@y_CU = ρ@y_QU = (0 0; ϵ 0); ρ@a_CU = ρ@a_QU = (γ 0; 0 0);
ρ@x_CU = (0 γ; 0 0); ρ@x_QU = (0 (1 - e^(-γ ϵ ħ)) / (ϵ ħ); 0 0);
ρ[e^ℰ_] := MatrixExp[ρ[ℰ]];
ρ[ℰ_] := (ℰ /. TRule /. t → γ ϵ /. (U : CU | QU)[u___] ⧴ Fold[Dot, (1 0; 0 1), ρ /@ U /@ {u}])
```

Verifying that ρ represents CU and QU:

```
Table[HL[SS[ρ[z1 ** z2] == ρ[z1].ρ[z2]] /. ϵ^k_· /; k > $k → 0],
 {U, {CU, QU}}, {z1, U /@ {y, a, x}}, {z2, U /@ {y, a, x}} ]
```

```
{{{True, True, True}, {True, True, True}, {True, True, True}},
 {{True, True, True}, {True, True, True}, {True, True, True}}}
```

Commuting $e^{\alpha a}$ with $e^{\xi x}$:

```
Table[HL[ρ[e^(ξ U@x)].ρ[e^(α U@a)] == ρ[e^(α U@a)].ρ[e^(e^(-γ α) ξ U@x)]], {U, {CU, QU}}]
```

```
{True, True}
```

Œ and the logoi Λ

Logoi from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

CdsO

```
CU@Œ_CU[specs___, Q_, P_] := 𝕆_CU[specs, SS[e^Q P]];
QU@Œ_QU[specs___, Q_, P_] := 𝕆_QU[specs, SS[e^Q P]];
```

Logos

```
c_Integer_{k_Integer} := c + O[ϵ]^(k+1);
Λ_{U_,k_}[{α_, β_}, {x_, x_}] := Œ_U[{x}, (α + β) x, 1_k];
Λ_{U_,k_}[{ξ_, α_}, {x, a}] := Œ_U[{a, x}, α a + e^(-γ α) ξ x, 1_k];
Λ_{U_,k_}[{α_, η_}, {a, y}] := Œ_U[{y, a}, α a + e^(-γ α) η y, 1_k];
```

```
Table[
 {Λ_{U,1}[{α, β}, {u, u}],
  lhs = U@Œ_U[{u_1, u_2}, ℏ (α u_1 + β u_2), 1], HL[lhs == U@Λ_{U,1}[ℏ {α, β}, {u, u}]]},
 {U, {CU, QU}}, {u, {y, a, x}}]
```

$\{\{\{Œ_{CU}[\{y\}, y(\alpha+\beta), 1+O[\epsilon]^2],$
$CU[] + (\alpha\hbar+\beta\hbar)\,CU[y] + \left(\frac{\alpha^2\hbar^2}{2}+\alpha\beta\hbar^2+\frac{\beta^2\hbar^2}{2}\right)CU[y,y], \text{True}\},$
$\{Œ_{CU}[\{a\}, a(\alpha+\beta), 1+O[\epsilon]^2], CU[] + (\alpha\hbar+\beta\hbar)\,CU[a] + \left(\frac{\alpha^2\hbar^2}{2}+\alpha\beta\hbar^2+\frac{\beta^2\hbar^2}{2}\right)CU[a,a],$
$\text{True}\}, \{Œ_{CU}[\{x\}, x(\alpha+\beta), 1+O[\epsilon]^2],$
$CU[] + (\alpha\hbar+\beta\hbar)\,CU[x] + \left(\frac{\alpha^2\hbar^2}{2}+\alpha\beta\hbar^2+\frac{\beta^2\hbar^2}{2}\right)CU[x,x], \text{True}\}\},$
$\{\{Œ_{QU}[\{y\}, y(\alpha+\beta), 1+O[\epsilon]^2], QU[] + (\alpha\hbar+\beta\hbar)\,QU[y] + \left(\frac{\alpha^2\hbar^2}{2}+\alpha\beta\hbar^2+\frac{\beta^2\hbar^2}{2}\right)QU[y,y],$
$\text{True}\}, \{Œ_{QU}[\{a\}, a(\alpha+\beta), 1+O[\epsilon]^2], QU[] + (\alpha\hbar+\beta\hbar)\,QU[a] +$
$\left(\frac{\alpha^2\hbar^2}{2}+\alpha\beta\hbar^2+\frac{\beta^2\hbar^2}{2}\right)QU[a,a], \text{True}\}, \{Œ_{QU}[\{x\}, x(\alpha+\beta), 1+O[\epsilon]^2],$
$QU[] + (\alpha\hbar+\beta\hbar)\,QU[x] + \left(\frac{\alpha^2\hbar^2}{2}+\alpha\beta\hbar^2+\frac{\beta^2\hbar^2}{2}\right)QU[x,x], \text{True}\}\}\}$

```
{Λ_{#,1}[{ξ, α}, {x, a}], lhs = #@Œ_#[{x, a}, ℏ (ξ x + α a), 1],
   HL[lhs == #@Λ_{#,1}[ℏ {ξ, α}, {x, a}]]} & /@ {CU, QU}
```

$\{\{Œ_{CU}[\{a, x\}, a\alpha + e^{-\alpha\gamma} x\xi, 1+O[\epsilon]^2],$
$CU[] + \alpha\hbar\,CU[a] + (\xi\hbar - \alpha\gamma\xi\hbar^2)\,CU[x] + \frac{1}{2}\alpha^2\hbar^2\,CU[a,a] + \alpha\xi\hbar^2\,CU[a,x] + \frac{1}{2}\xi^2\hbar^2\,CU[x,x],$
$\text{True}\}, \{Œ_{QU}[\{a, x\}, a\alpha + e^{-\alpha\gamma} x\xi, 1+O[\epsilon]^2], QU[] + \alpha\hbar\,QU[a] +$
$(\xi\hbar - \alpha\gamma\xi\hbar^2)\,QU[x] + \frac{1}{2}\alpha^2\hbar^2\,QU[a,a] + \alpha\xi\hbar^2\,QU[a,x] + \frac{1}{2}\xi^2\hbar^2\,QU[x,x], \text{True}\}\}$

```
{Λ_{#,2}[{α, η}, {a, y}], lhs = #@Œ_#[{a, y}, ℏ (η y + α a), 1],
   HL[lhs == #@Λ_{#,2}[ℏ {α, η}, {a, y}]]} & /@ {CU, QU}
```

$\{\{Œ_{CU}[\{y, a\}, a\alpha + e^{-\alpha\gamma} y\eta, 1+O[\epsilon]^3],$
$CU[] + \alpha\hbar\,CU[a] + (\eta\hbar - \alpha\gamma\eta\hbar^2)\,CU[y] + \frac{1}{2}\alpha^2\hbar^2\,CU[a,a] + \alpha\eta\hbar^2\,CU[y,a] + \frac{1}{2}\eta^2\hbar^2\,CU[y,y],$
$\text{True}\}, \{Œ_{QU}[\{y, a\}, a\alpha + e^{-\alpha\gamma} y\eta, 1+O[\epsilon]^3], QU[] + \alpha\hbar\,QU[a] +$
$(\eta\hbar - \alpha\gamma\eta\hbar^2)\,QU[y] + \frac{1}{2}\alpha^2\hbar^2\,QU[a,a] + \alpha\eta\hbar^2\,QU[y,a] + \frac{1}{2}\eta^2\hbar^2\,QU[y,y], \text{True}\}\}$

Goal. In either U, compute $F = e^{-\eta y} e^{\xi x} e^{\eta y} e^{-\xi x}$. First compute $G = e^{\xi x}\, y\, e^{-\xi x}$, a finite sum. Now F satisfies the ODE $\partial_\eta F = \partial_\eta\left(e^{-\eta y} e^{\eta G}\right) = -yF + FG$ with initial conditions $F(\eta=0) = 1$. So we set it up and solve:

```
If[$k > 0, With[{U = CU},
  Module[{G, F, fs, bs, e, b, es, sol},
   G = Echo@Simp[Table[ξ^k/k!, {k, 0, $k + 1}].NestList[Simp[B[x_U, #]] &, y_U, $k + 1]];
   fs = Echo@Flatten@Table[f_{l,i,j,k}[η], {l, 0, $k}, {i, 0, 1}, {j, 0, 1}, {k, 0, 1}];
   F = Echo[fs.(bs = fs /. f_{l_,i_,j_,k_}[η] :> ϵ^l U@{y^i, a^j, x^k})];
   es = Flatten[
     Table[Coefficient[e, b] == 0, {e, {F - 1_U /. η → 0, F ** G - y_U ** F - ∂_η F}}, {b, bs}]];
   sol = Echo@First[F /. DSolve[es, fs, η]];
   Echo[sol /. {ℯ- → 1, U → Times}];
   Collect[sol /. {ℯ- → 1, U → Times}, ϵ, Simplify]
  ]]]
```

Logos

```
Λ_{U_,kk_}[{ξ1_, η1_}, {x, y}] :=
  Λ_U[{ξ1, η1}, {x, y}] = Block[{$k = kk}, Module[{ξ, η, G, F, fs, f, bs, e, b, es},
     G = Simp[Table[ξ^k/k!, {k, 0, $k + 1}].NestList[Simp[B[x_U, #]] &, y_U, $k + 1]];
     fs = Flatten@Table[f_{l,i,j,k}[η], {l, 0, $k}, {i, 0, 1}, {j, 0, 1}, {k, 0, 1}];
     F = fs.(bs = fs /. f_{l_,i_,j_,k_}[η] :> ϵ^l U@{y^i, a^j, x^k});
     es = Flatten[
       Table[Coefficient[e, b] == 0, {e, {F - 1_U /. η → 0, F ** G - y_U ** F - ∂_η F}}, {b, bs}]];
     F = F /. DSolve[es, fs, η][[1]];
     Œ_U[{y, a, x},
       ξ x + η y + (U /. {CU → -t η ξ, QU → η ξ (1 - T) / ℏ}),
       F + O_$k /. {ℯ- → 1, U → Times}
      ] /. {ξ → ξ1, η → η1}]];
```

```
{Λ_{CU,1}[{ξ, η}, {x, y}], lhs = CU@Œ_CU[{x, y}, ℏ (ξ x + η y), 1],
 HL[lhs == CU@Λ_{CU,1}[ℏ {ξ, η}, {x, y}]]}
```

$$\left\{\text{Œ}_{CU}\left[\{y,a,x\},\ y\eta+x\xi-t\eta\xi,\ 1+\frac{1}{2}\eta\xi\left(4a-2y\gamma\eta-2x\gamma\xi+t\gamma\eta\xi\right)\epsilon+O[\epsilon]^2\right],\right.$$
$$\left(1-t\eta\xi\hbar^2\right)CU[\,]+\xi\hbar\,CU[x]+\eta\hbar\,CU[y]+$$
$$\left.\frac{1}{2}\xi^2\hbar^2CU[x,x]+\eta\xi\hbar^2CU[y,x]+\frac{1}{2}\eta^2\hbar^2CU[y,y],\ \text{True}\right\}$$

```
{Λ_{QU,1}[{ξ, η}, {x, y}], lhs = QU@Œ_QU[{x, y}, ℏ (ξ x + η y), 1],
 HL@SimpT[lhs == QU@Λ_{QU,1}[ℏ {ξ, η}, {x, y}]]}
```

$$\left\{\text{Œ}_{QU}\left[\{y,a,x\},\ y\eta+x\xi+\frac{(1-T)\eta\xi}{\hbar},\ 1+\frac{1}{4\hbar}\right.\right.$$
$$\eta\xi\left(\gamma\eta\xi-4T\gamma\eta\xi+3T^2\gamma\eta\xi+8aT\hbar+2y\gamma\eta\hbar-6Ty\gamma\eta\hbar+2x\gamma\xi\hbar-6Tx\gamma\xi\hbar+4xy\gamma\hbar^2\right)\epsilon+$$
$$\left.O[\epsilon]^2\right],\ \left(1+\eta\xi\hbar-T\eta\xi\hbar\right)QU[\,]+\xi\hbar\,QU[x]+\eta\hbar\,QU[y]+$$
$$\left.\frac{1}{2}\xi^2\hbar^2QU[x,x]+\eta\xi\hbar^2QU[y,x]+\frac{1}{2}\eta^2\hbar^2QU[y,y],\ \text{True}\right\}$$

```
{tt = Last[Λ_CU[{ξ, η}, {x, y}]], Normal@Series[Log[tt], {ϵ, 0, $k}]}
```

$$\left\{1+\frac{1}{2}\eta\xi\left(4a-2y\gamma\eta-2x\gamma\xi+t\gamma\eta\xi\right)\epsilon+O[\epsilon]^2,\ \epsilon\left(2a\eta\xi-y\gamma\eta^2\xi-x\gamma\eta\xi^2+\frac{1}{2}t\gamma\eta^2\xi^2\right)\right\}$$

```
{tt = Last[Λ_QU[{ξ, η}, {x, y}]], Normal@Series[Log[tt], {ϵ, 0, $k}]}
```

$$\left\{1+\frac{1}{4\hbar}\eta\,\xi\left(\gamma\,\eta\,\xi-4\,T\,\gamma\,\eta\,\xi+3\,T^2\,\gamma\,\eta\,\xi+8\,a\,T\,\hbar+2\,y\,\gamma\,\eta\,\hbar-6\,T\,y\,\gamma\,\eta\,\hbar+2\,x\,\gamma\,\xi\,\hbar-6\,T\,x\,\gamma\,\xi\,\hbar+4\,x\,y\,\gamma\,\hbar^2\right)\epsilon+O[\epsilon]^2,\ \frac{1}{4\hbar}\epsilon\left(\gamma\,\eta^2\,\xi^2-4\,T\,\gamma\,\eta^2\,\xi^2+3\,T^2\,\gamma\,\eta^2\,\xi^2+8\,a\,T\,\eta\,\xi\,\hbar+2\,y\,\gamma\,\eta^2\,\xi\,\hbar-6\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar+2\,x\,\gamma\,\eta\,\xi^2\,\hbar-6\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar+4\,x\,y\,\gamma\,\eta\,\xi\,\hbar^2\right)\right\}$$

Logos

```
Simp[Œ_U_[specs___, Q_, P_]] := Œ_U[specs, ExpandNumerator@Together[Q /. ⅇ^δ_ :> ⅇ^Simplify[δ]],
    MapAt[ExpandNumerator@*Together, P, 3]];
```

Logos

```
Λ_U_,k_[{υ1_, ω1_, δ_}, {u_, w_}] := Simp@Module[{υ, ω, yax, q, p, Q, d},
    {yax, q, p} = List @@ Λ_U,k[{υ, ω}, {u, w}];
    Œ_U[yax, Q = (υ u + ω w + δ u w + d υ ω) / (1 - d δ),
      Expand[(1 - d δ)^-1 ⅇ^-Q DP_{υ→D_u, ω→D_w}[p][ⅇ^Q]] + 0_k] /. {d → ∂_{υ,ω} q} /. {υ → υ1, ω → ω1}];
```

```
{Λ_CU,2[{ξ, η, δ}, {x, y}], lhs = CU@Œ_CU[{x, y}, ℏ (ξ x + η y + δ x y), 1],
 HL[lhs == CU@Λ_CU,1[ℏ {ξ, η, δ}, {x, y}]]}
```

$$\left\{Œ_{CU}\left[\{y,a,x\},\ \frac{x\,y\,\delta+y\,\eta+x\,\xi-t\,\eta\,\xi}{1+t\,\delta},\ \frac{1}{1+t\,\delta}+\frac{\left(4\,a\,\eta\,\xi-2\,y\,\gamma\,\eta^2\,\xi-2\,x\,\gamma\,\eta\,\xi^2+t\,\gamma\,\eta^2\,\xi^2\right)\epsilon}{2\,(1+t\,\delta)}+\frac{1}{24\,(1+t\,\delta)}\left(48\,a^2\,\eta^2\,\xi^2-24\,a\,\gamma\,\eta^2\,\xi^2-48\,a\,y\,\gamma\,\eta^3\,\xi^2+24\,y\,\gamma^2\,\eta^3\,\xi^2+12\,y^2\,\gamma^2\,\eta^4\,\xi^2-48\,a\,x\,\gamma\,\eta^2\,\xi^3+24\,x\,\gamma^2\,\eta^2\,\xi^3+24\,a\,t\,\gamma\,\eta^3\,\xi^3-8\,t\,\gamma^2\,\eta^3\,\xi^3+24\,x\,y\,\gamma^2\,\eta^3\,\xi^3-12\,t\,y\,\gamma^2\,\eta^4\,\xi^3+12\,x^2\,\gamma^2\,\eta^2\,\xi^4-12\,t\,x\,\gamma^2\,\eta^3\,\xi^4+3\,t^2\,\gamma^2\,\eta^4\,\xi^4\right)\epsilon^2+O[\epsilon]^3\right],\right.$$

$$\left(1-t\,\delta\,\hbar+t^2\,\delta^2\,\hbar^2-t\,\eta\,\xi\,\hbar^2\right)CU[\,]+\left(\xi\,\hbar-2\,t\,\delta\,\xi\,\hbar^2\right)CU[x]+\left(\eta\,\hbar-2\,t\,\delta\,\eta\,\hbar^2\right)CU[y]+\frac{1}{2}\xi^2\,\hbar^2\,CU[x,x]+\left(\delta\,\hbar-2\,t\,\delta^2\,\hbar^2+\eta\,\xi\,\hbar^2\right)CU[y,x]+\frac{1}{2}\eta^2\,\hbar^2\,CU[y,y]+\delta\,\xi\,\hbar^2\,CU[y,x,x]+\delta\,\eta\,\hbar^2\,CU[y,y,x]+\frac{1}{2}\delta^2\,\hbar^2\,CU[y,y,x,x],\ \text{True}\Big\}$$

```
{ΛQU,2[{ξ, η, δ}, {x, y}], lhs = QU@ŒQU[{x, y}, ℏ (ξ x + η y + δ x y), 1],
 HL@SimpT[lhs == QU@ΛQU,1[ℏ {ξ, η, δ}, {x, y}]]}
```

$$\Big\{\text{Œ}_{QU}\big[\{y,a,x\},\ \frac{\eta\,\xi-T\,\eta\,\xi+x\,y\,\delta\,\hbar+y\,\eta\,\hbar+x\,\xi\,\hbar}{-\delta+T\,\delta+\hbar},$$

$$\frac{\hbar}{-\delta+T\,\delta+\hbar}+\Big(\big(\gamma\,\eta^2\,\xi^2-4\,T\,\gamma\,\eta^2\,\xi^2+3\,T^2\,\gamma\,\eta^2\,\xi^2+8\,a\,T\,\eta\,\xi\,\hbar+2\,y\,\gamma\,\eta^2\,\xi\,\hbar-$$
$$6\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar+2\,x\,\gamma\,\eta\,\xi^2\,\hbar-6\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar+4\,x\,y\,\gamma\,\eta\,\xi\,\hbar^2\big)\,\epsilon\Big)\Big/\big(4\,(-\delta+T\,\delta+\hbar)\big)+$$

$$\frac{1}{288\,\hbar\,(-\delta+T\,\delta+\hbar)}\Big(9\,\gamma^2\,\eta^4\,\xi^4-72\,T\,\gamma^2\,\eta^4\,\xi^4+198\,T^2\,\gamma^2\,\eta^4\,\xi^4-216\,T^3\,\gamma^2\,\eta^4\,\xi^4+$$
$$81\,T^4\,\gamma^2\,\eta^4\,\xi^4+144\,a\,T\,\gamma\,\eta^3\,\xi^3\,\hbar-576\,a\,T^2\,\gamma\,\eta^3\,\xi^3\,\hbar+432\,a\,T^3\,\gamma\,\eta^3\,\xi^3\,\hbar+40\,\gamma^2\,\eta^3\,\xi^3\,\hbar-$$
$$312\,T\,\gamma^2\,\eta^3\,\xi^3\,\hbar+600\,T^2\,\gamma^2\,\eta^3\,\xi^3\,\hbar-328\,T^3\,\gamma^2\,\eta^3\,\xi^3\,\hbar+36\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar-252\,T\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar+$$
$$540\,T^2\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar-324\,T^3\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar+36\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar-252\,T\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar+$$
$$540\,T^2\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar-324\,T^3\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar+576\,a^2\,T^2\,\eta^2\,\xi^2\,\hbar^2+576\,a\,T\,\gamma\,\eta^2\,\xi^2\,\hbar^2-$$
$$864\,a\,T^2\,\gamma\,\eta^2\,\xi^2\,\hbar^2+36\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2-216\,T\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2+180\,T^2\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2+288\,a\,T\,y\,\gamma\,\eta^3\,\xi^2\,\hbar^2-$$
$$864\,a\,T^2\,y\,\gamma\,\eta^3\,\xi^2\,\hbar^2+120\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar^2-816\,T\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar^2+984\,T^2\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar^2+$$
$$36\,y^2\,\gamma^2\,\eta^4\,\xi^2\,\hbar^2-216\,T\,y^2\,\gamma^2\,\eta^4\,\xi^2\,\hbar^2+324\,T^2\,y^2\,\gamma^2\,\eta^4\,\xi^2\,\hbar^2+288\,a\,T\,x\,\gamma\,\eta^2\,\xi^3\,\hbar^2-$$
$$864\,a\,T^2\,x\,\gamma\,\eta^2\,\xi^3\,\hbar^2+120\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar^2-816\,T\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar^2+984\,T^2\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar^2+$$
$$144\,x\,y\,\gamma^2\,\eta^3\,\xi^3\,\hbar^2-720\,T\,x\,y\,\gamma^2\,\eta^3\,\xi^3\,\hbar^2+864\,T^2\,x\,y\,\gamma^2\,\eta^3\,\xi^3\,\hbar^2+36\,x^2\,\gamma^2\,\eta^2\,\xi^4\,\hbar^2-$$
$$216\,T\,x^2\,\gamma^2\,\eta^2\,\xi^4\,\hbar^2+324\,T^2\,x^2\,\gamma^2\,\eta^2\,\xi^4\,\hbar^2-576\,a^2\,T\,\eta\,\xi\,\hbar^3+864\,a\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar^3+$$
$$72\,y\,\gamma^2\,\eta^2\,\xi\,\hbar^3-360\,T\,y\,\gamma^2\,\eta^2\,\xi\,\hbar^3+48\,y^2\,\gamma^2\,\eta^3\,\xi\,\hbar^3-336\,T\,y^2\,\gamma^2\,\eta^3\,\xi\,\hbar^3+864\,a\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar^3+$$
$$72\,x\,\gamma^2\,\eta\,\xi^2\,\hbar^3-360\,T\,x\,\gamma^2\,\eta\,\xi^2\,\hbar^3+576\,a\,T\,x\,y\,\gamma\,\eta^2\,\xi^2\,\hbar^3+360\,x\,y\,\gamma^2\,\eta^2\,\xi^2\,\hbar^3-$$
$$1512\,T\,x\,y\,\gamma^2\,\eta^2\,\xi^2\,\hbar^3+144\,x\,y^2\,\gamma^2\,\eta^3\,\xi^2\,\hbar^3-432\,T\,x\,y^2\,\gamma^2\,\eta^3\,\xi^2\,\hbar^3+48\,x^2\,\gamma^2\,\eta\,\xi^3\,\hbar^3-$$
$$336\,T\,x^2\,\gamma^2\,\eta\,\xi^3\,\hbar^3+144\,x^2\,y\,\gamma^2\,\eta^2\,\xi^3\,\hbar^3-432\,T\,x^2\,y\,\gamma^2\,\eta^2\,\xi^3\,\hbar^3+144\,x\,y\,\gamma^2\,\eta\,\xi\,\hbar^4+$$
$$144\,x\,y^2\,\gamma^2\,\eta^2\,\xi\,\hbar^4+144\,x^2\,y\,\gamma^2\,\eta\,\xi^2\,\hbar^4+144\,x^2\,y^2\,\gamma^2\,\eta^2\,\xi^2\,\hbar^4\Big)\,\epsilon^2+O[\epsilon]^3\big],$$

$$\big(1+\delta-T\,\delta+\delta^2-2\,T\,\delta^2+T^2\,\delta^2+\eta\,\xi\,\hbar-T\,\eta\,\xi\,\hbar\big)\,QU[\,]+$$
$$\big(\xi\,\hbar+2\,\delta\,\xi\,\hbar-2\,T\,\delta\,\xi\,\hbar\big)\,QU[x]+$$
$$\big(\eta\,\hbar+2\,\delta\,\eta\,\hbar-2\,T\,\delta\,\eta\,\hbar\big)\,QU[y]+\frac{1}{2}\,\xi^2\,\hbar^2\,QU[x,x]+$$
$$\big(\delta\,\hbar+2\,\delta^2\,\hbar-2\,T\,\delta^2\,\hbar+\eta\,\xi\,\hbar^2\big)\,QU[y,x]+$$
$$\frac{1}{2}\,\eta^2\,\hbar^2\,QU[y,y]+$$
$$\delta\,\xi\,\hbar^2\,QU[y,x,x]+\delta\,\eta\,\hbar^2\,QU[y,y,x]+$$
$$\frac{1}{2}\,\delta^2\,\hbar^2\,QU[y,y,x,x],\ \text{True}\Big\}$$

```
{tt = Last[ΛCU,2[{ξ, η, δ}, {x, y}]], Normal@Series[Log[tt], {ϵ, 0, 2}]}
```

$$\left\{\frac{1}{1+t\,\delta}+\frac{\left(4\,a\,\eta\,\xi-2\,y\,\gamma\,\eta^2\,\xi-2\,x\,\gamma\,\eta\,\xi^2+t\,\gamma\,\eta^2\,\xi^2\right)\epsilon}{2\,\left(1+t\,\delta\right)}+\frac{1}{24\,\left(1+t\,\delta\right)}\left(48\,a^2\,\eta^2\,\xi^2-24\,a\,\gamma\,\eta^2\,\xi^2-48\,a\,y\,\gamma\,\eta^3\,\xi^2+24\,y\,\gamma^2\,\eta^3\,\xi^2+12\,y^2\,\gamma^2\,\eta^4\,\xi^2-48\,a\,x\,\gamma\,\eta^2\,\xi^3+24\,x\,\gamma^2\,\eta^2\,\xi^3+24\,a\,t\,\gamma\,\eta^3\,\xi^3-8\,t\,\gamma^2\,\eta^3\,\xi^3+24\,x\,y\,\gamma^2\,\eta^3\,\xi^3-12\,t\,y\,\gamma^2\,\eta^4\,\xi^3+12\,x^2\,\gamma^2\,\eta^2\,\xi^4-12\,t\,x\,\gamma^2\,\eta^3\,\xi^4+3\,t^2\,\gamma^2\,\eta^4\,\xi^4\right)\epsilon^2+O[\epsilon]^3,\right.$$

$$\left.\epsilon\left(2\,a\,\eta\,\xi-y\,\gamma\,\eta^2\,\xi-x\,\gamma\,\eta\,\xi^2+\frac{1}{2}\,t\,\gamma\,\eta^2\,\xi^2\right)+\frac{1}{3}\,\epsilon^2\left(-3\,a\,\gamma\,\eta^2\,\xi^2+3\,y\,\gamma^2\,\eta^3\,\xi^2+3\,x\,\gamma^2\,\eta^2\,\xi^3-t\,\gamma^2\,\eta^3\,\xi^3\right)+\mathrm{Log}\left[\frac{1}{1+t\,\delta}\right]\right\}$$

```
{tt = Last[ΛQU,2[{ξ, η, δ}, {x, y}]], Normal@Series[Log[tt], {ϵ, 0, 2}]}
```

$$\left\{\frac{\hbar}{-\delta+T\,\delta+\hbar}+\left(\left(\gamma\,\eta^2\,\xi^2-4\,T\,\gamma\,\eta^2\,\xi^2+3\,T^2\,\gamma\,\eta^2\,\xi^2+8\,a\,T\,\eta\,\xi\,\hbar+2\,y\,\gamma\,\eta^2\,\xi\,\hbar-6\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar+2\,x\,\gamma\,\eta\,\xi^2\,\hbar-6\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar+4\,x\,y\,\gamma\,\eta\,\xi\,\hbar^2\right)\epsilon\right)\Big/\left(4\,\left(-\delta+T\,\delta+\hbar\right)\right)+\frac{1}{288\,\hbar\,\left(-\delta+T\,\delta+\hbar\right)}\right.$$

$$\left(9\,\gamma^2\,\eta^4\,\xi^4-72\,T\,\gamma^2\,\eta^4\,\xi^4+198\,T^2\,\gamma^2\,\eta^4\,\xi^4-216\,T^3\,\gamma^2\,\eta^4\,\xi^4+81\,T^4\,\gamma^2\,\eta^4\,\xi^4+144\,a\,T\,\gamma\,\eta^3\,\xi^3\,\hbar-576\,a\,T^2\,\gamma\,\eta^3\,\xi^3\,\hbar+432\,a\,T^3\,\gamma\,\eta^3\,\xi^3\,\hbar+40\,\gamma^2\,\eta^3\,\xi^3\,\hbar-312\,T\,\gamma^2\,\eta^3\,\xi^3\,\hbar+600\,T^2\,\gamma^2\,\eta^3\,\xi^3\,\hbar-328\,T^3\,\gamma^2\,\eta^3\,\xi^3\,\hbar+36\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar-252\,T\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar+540\,T^2\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar-324\,T^3\,y\,\gamma^2\,\eta^4\,\xi^3\,\hbar+36\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar-252\,T\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar+540\,T^2\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar-324\,T^3\,x\,\gamma^2\,\eta^3\,\xi^4\,\hbar+576\,a^2\,T^2\,\eta^2\,\xi^2\,\hbar^2+576\,a\,T\,\gamma\,\eta^2\,\xi^2\,\hbar^2-864\,a\,T^2\,\gamma\,\eta^2\,\xi^2\,\hbar^2+36\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2-216\,T\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2+180\,T^2\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2+288\,a\,T\,y\,\gamma\,\eta^3\,\xi^2\,\hbar^2-864\,a\,T^2\,y\,\gamma\,\eta^3\,\xi^2\,\hbar^2+120\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar^2-816\,T\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar^2+984\,T^2\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar^2+36\,y^2\,\gamma^2\,\eta^4\,\xi^2\,\hbar^2-216\,T\,y^2\,\gamma^2\,\eta^4\,\xi^2\,\hbar^2+324\,T^2\,y^2\,\gamma^2\,\eta^4\,\xi^2\,\hbar^2+288\,a\,T\,x\,\gamma\,\eta^2\,\xi^3\,\hbar^2-864\,a\,T^2\,x\,\gamma\,\eta^2\,\xi^3\,\hbar^2+120\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar^2-816\,T\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar^2+984\,T^2\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar^2+144\,x\,y\,\gamma^2\,\eta^3\,\xi^3\,\hbar^2-720\,T\,x\,y\,\gamma^2\,\eta^3\,\xi^3\,\hbar^2+864\,T^2\,x\,y\,\gamma^2\,\eta^3\,\xi^3\,\hbar^2+36\,x^2\,\gamma^2\,\eta^2\,\xi^4\,\hbar^2-216\,T\,x^2\,\gamma^2\,\eta^2\,\xi^4\,\hbar^2+324\,T^2\,x^2\,\gamma^2\,\eta^2\,\xi^4\,\hbar^2-576\,a^2\,T\,\eta\,\xi\,\hbar^3+864\,a\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar^3+72\,y\,\gamma^2\,\eta^2\,\xi\,\hbar^3-360\,T\,y\,\gamma^2\,\eta^2\,\xi\,\hbar^3+48\,y^2\,\gamma^2\,\eta^3\,\xi\,\hbar^3-336\,T\,y^2\,\gamma^2\,\eta^3\,\xi\,\hbar^3+864\,a\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar^3+72\,x\,\gamma^2\,\eta\,\xi^2\,\hbar^3-360\,T\,x\,\gamma^2\,\eta\,\xi^2\,\hbar^3+576\,a\,T\,x\,y\,\gamma\,\eta^2\,\xi^2\,\hbar^3+360\,x\,y\,\gamma^2\,\eta^2\,\xi^2\,\hbar^3-1512\,T\,x\,y\,\gamma^2\,\eta^2\,\xi^2\,\hbar^3+144\,x\,y^2\,\gamma^2\,\eta^3\,\xi^2\,\hbar^3-432\,T\,x\,y^2\,\gamma^2\,\eta^3\,\xi^2\,\hbar^3+48\,x^2\,\gamma^2\,\eta\,\xi^3\,\hbar^3-336\,T\,x^2\,\gamma^2\,\eta\,\xi^3\,\hbar^3+144\,x^2\,y\,\gamma^2\,\eta^2\,\xi^3\,\hbar^3-432\,T\,x^2\,y\,\gamma^2\,\eta^2\,\xi^3\,\hbar^3+144\,x\,y\,\gamma^2\,\eta\,\xi\,\hbar^4+144\,x\,y^2\,\gamma^2\,\eta^2\,\xi\,\hbar^4+144\,x^2\,y\,\gamma^2\,\eta\,\xi^2\,\hbar^4+144\,x^2\,y^2\,\gamma^2\,\eta^2\,\xi^2\,\hbar^4\right)\epsilon^2+O[\epsilon]^3,$$

$$\frac{1}{4\,\hbar}\epsilon\left(\gamma\,\eta^2\,\xi^2-4\,T\,\gamma\,\eta^2\,\xi^2+3\,T^2\,\gamma\,\eta^2\,\xi^2+8\,a\,T\,\eta\,\xi\,\hbar+2\,y\,\gamma\,\eta^2\,\xi\,\hbar-6\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar+2\,x\,\gamma\,\eta\,\xi^2\,\hbar-6\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar+4\,x\,y\,\gamma\,\eta\,\xi\,\hbar^2\right)+$$

$$\frac{1}{72\,\hbar}\,\epsilon^2\left(10\,\gamma^2\,\eta^3\,\xi^3-78\,T\,\gamma^2\,\eta^3\,\xi^3+150\,T^2\,\gamma^2\,\eta^3\,\xi^3-82\,T^3\,\gamma^2\,\eta^3\,\xi^3+144\,a\,T\,\gamma\,\eta^2\,\xi^2\,\hbar-216\,a\,T^2\,\gamma\,\eta^2\,\xi^2\,\hbar+9\,\gamma^2\,\eta^2\,\xi^2\,\hbar-54\,T\,\gamma^2\,\eta^2\,\xi^2\,\hbar+45\,T^2\,\gamma^2\,\eta^2\,\xi^2\,\hbar+30\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar-204\,T\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar+246\,T^2\,y\,\gamma^2\,\eta^3\,\xi^2\,\hbar+30\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar-204\,T\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar+246\,T^2\,x\,\gamma^2\,\eta^2\,\xi^3\,\hbar-144\,a^2\,T\,\eta\,\xi\,\hbar^2+216\,a\,T\,y\,\gamma\,\eta^2\,\xi\,\hbar^2+18\,y\,\gamma^2\,\eta^2\,\xi\,\hbar^2-90\,T\,y\,\gamma^2\,\eta^2\,\xi\,\hbar^2+12\,y^2\,\gamma^2\,\eta^3\,\xi\,\hbar^2-84\,T\,y^2\,\gamma^2\,\eta^3\,\xi\,\hbar^2+216\,a\,T\,x\,\gamma\,\eta\,\xi^2\,\hbar^2+18\,x\,\gamma^2\,\eta\,\xi^2\,\hbar^2-90\,T\,x\,\gamma^2\,\eta\,\xi^2\,\hbar^2+90\,x\,y\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2-378\,T\,x\,y\,\gamma^2\,\eta^2\,\xi^2\,\hbar^2+12\,x^2\,\gamma^2\,\eta\,\xi^3\,\hbar^2-84\,T\,x^2\,\gamma^2\,\eta\,\xi^3\,\hbar^2+36\,x\,y\,\gamma^2\,\eta\,\xi\,\hbar^3+36\,x\,y^2\,\gamma^2\,\eta^2\,\xi\,\hbar^3+36\,x^2\,y\,\gamma^2\,\eta\,\xi^2\,\hbar^3\right)+\mathrm{Log}\left[\frac{\hbar}{-\delta+T\,\delta+\hbar}\right]\Big\}$$

Reorderings with Rord

Rord

```
Rord_{u_i_,w_j_→k_}[Œ_U_[L___, {l___, u_i_, w_j_, r___}_s_, R___, Q_, P_]] :=
  Simp@Module[{υ, ω, δ, Λ1, yax, q, p, n, kk = P[[5]], δ1 = ∂_{u_i,w_j} Q},
    {yax, q, p} = List @@ If[δ1 === 0, Λ_{U,kk}[{υ, ω}, {u, w}], Λ_{U,kk}[{υ, ω, δ}, {u, w}]] /.
      {y → y_n, a → a_n, x → x_n, t → t_s, T → T_s};
    Œ_U[L, {l, Sequence @@ yax, r}_s, R, q + (Q /. u_i | w_j → 0), e^-q SP_{u_i→υ, w_j→ω}[P p e^q]] /.
     {n → k, υ → ∂_{u_i} Q /. w_j → 0, ω → ∂_{w_j} Q /. u_i → 0, δ → δ1}];
```

```
With[{cΦ = Œ_CU[{y1, x1}_1, {x2, a2, y2}_2, ħ t1 a2 + ħ t1^-1 (e^t1 - 1) y1 x2, 1_2 + ϵ x1 y2]},
 {Short[rhs = cΦ // Rord_{x2,a2→3}, 3], HL[CU[cΦ] ≡ CU[rhs]]}]
```

$$\left\{Œ_{CU}\left[\{y_1, x_1\}_1, \{a_3, x_3, y_2\}_2, \frac{e^{-\gamma\hbar t_1}\left(e^{\gamma\hbar t_1}\hbar a_3 t_1^2 - \hbar x_3 y_1 + e^{t_1}\hbar x_3 y_1\right)}{t_1}, 1 + x_1 y_2 \epsilon + O[\epsilon]^3\right], \text{True}\right\}$$

```
With[{cΦ = Œ_CU[{y1, a1, a2}_1, {x2, x1, y2}_2,
    ħ (l11 t1 a1 + l12 t1 a2 + l21 t2 a1 + l22 t2 a2 + γ11 x1 y1 + γ12 x1 y2 + γ21 x2 y1 + γ22 x2 y2),
    1_2 + ϵ (l1 a1 + l2 a2 + p11 x1 y1 + p12 x1 y2 + p21 x2 y1 + p22 x2 y2)]},
 {Short[rhs = cΦ // Rord_{a1,a2→3} // Rord_{x2,x1→4}, 3], HL[CU[cΦ] ≡ CU[rhs]]}]
```

$$\{Œ_{CU}[\{y_1, a_3\}_1, \{x_4, y_2\}_2, \hbar a_3 l_{11} t_1 + \hbar a_3 l_{12} t_1 + \hbar a_3 l_{21} t_2 + \hbar a_3 l_{22} t_2 + \hbar x_4 y_1 \gamma_{11} + \hbar x_4 y_2 \gamma_{12} + \hbar x_4 y_1 \gamma_{21} + \hbar x_4 y_2 \gamma_{22}, 1 + O[\epsilon]^3], \text{True}\}$$

```
ħ a3 l11 t1 + ħ a3 l12 t1 + ħ a3 l21 t2 + ħ a3 l22 t2 +
  ħ x4 y1 γ11 + ħ x4 y2 γ12 + ħ x4 y1 γ21 + ħ x4 y2 γ22 // Simplify
```

$$\hbar\left(a_3\left(l_{11} t_1 + l_{12} t_1 + \left(l_{21} + l_{22}\right) t_2\right) + x_4\left(y_1(\gamma_{11} + \gamma_{21}) + y_2(\gamma_{12} + \gamma_{22})\right)\right)$$

$$\text{With}\Big[\Big\{c\oplus = \mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1,\ \{x_2, a_2, y_2\}_2,\ \hbar\left(l_{11}\,t_1\,a_1 + l_{12}\,t_1\,a_2 + l_{21}\,t_2\,a_1 + l_{22}\,t_2\,a_2 + \gamma_{11}\,x_1\,y_1 + \gamma_{12}\,x_1\,y_2 + \gamma_{21}\,x_2\,y_1 + \gamma_{22}\,x_2\,y_2\right),\ 1_2 + \epsilon\left(l_1\,a_1 + l_2\,a_2 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2 + p_{21}\,x_2\,y_1 + p_{22}\,x_2\,y_2\right)\Big]\Big\},\ \{\text{Short}[rhs = c\oplus\ //\ \text{Rord}_{x_2,a_2\to 3},\ 3],\ \text{HL}[\text{CU}[c\oplus] == \text{CU}[rhs]]\}\Big]$$

$$\Big\{\mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1,\ \{a_3, x_3, y_2\}_2,\ e^{-\gamma\hbar(l_{12}t_1+l_{22}t_2)}\left(e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_3\,l_{12}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{21}\,t_2 + e^{\ll 1\gg}\,\hbar\,a_3\,l_{22}\,t_2 + \ll 1\gg + e^{\gamma\hbar(\ll 1\gg)}\,\hbar\,x_1\,y_2\,\gamma_{12} + \hbar\,x_3\,y_1\,\gamma_{21} + \hbar\,x_3\,y_2\,\gamma_{22}\right),\ 1 + \left(a_1\,l_1 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2\right)\epsilon + O[\epsilon]^3\Big],\ \text{True}\Big\}$$

$$\Big\{\mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1,\ \{a_3, x_3, y_2\}_2,\ e^{-\gamma\hbar(l_{12}t_1+l_{22}t_2)}\left(e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_3\,l_{12}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{21}\,t_2 + e^{\ll 1\gg}\,\hbar\,a_3\,l_{22}\,t_2 + \ll 1\gg + e^{\gamma\hbar(\ll 1\gg)}\,\hbar\,x_1\,y_2\,\gamma_{12} + \hbar\,x_3\,y_1\,\gamma_{21} + \hbar\,x_3\,y_2\,\gamma_{22}\right),\ 1 + \left(a_1\,l_1 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2\right)\epsilon + O[\epsilon]^3\Big],\ \text{True}\Big\}$$

$$\Big\{\mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1,\ \{a_3, x_3, y_2\}_2,\ e^{-\gamma\hbar(l_{12}t_1+l_{22}t_2)}\left(e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_3\,l_{12}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{\ll 2\gg}\ll 1\gg)}\,\hbar\,a_1\,l_{21}\,t_2 + \ll 1\gg + \ll 1\gg + e^{\ll 1\gg}\,\hbar\,x_1\,y_2\,\gamma_{12} + \hbar\,x_3\,y_1\,\gamma_{21} + \hbar\,x_3\,y_2\,\gamma_{22}\right),\ 1 + \left(a_1\,l_1 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2\right)\epsilon + O[\epsilon]^3\Big],\ \text{True}\Big\}$$

$$\Big\{\mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1,\ \{a_3, x_3, y_2\}_2,\ e^{-\gamma\hbar(l_{12}t_1+l_{22}t_2)}\left(e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_3\,l_{12}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{\ll 2\gg}\ll 1\gg)}\,\hbar\,a_1\,l_{21}\,t_2 + \ll 1\gg + \ll 1\gg + e^{\ll 1\gg}\,\hbar\,x_1\,y_2\,\gamma_{12} + \hbar\,x_3\,y_1\,\gamma_{21} + \hbar\,x_3\,y_2\,\gamma_{22}\right),\ 1 + \left(a_1\,l_1 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2\right)\epsilon + O[\epsilon]^2\Big],\ \text{True}\Big\}$$

$$\text{With}\Big[\Big\{q\oplus = \mathbb{E}_{QU}\Big[\{y_1, a_1, x_1\}_1,\ \{x_2, a_2, y_2\}_2,\ \hbar\left(l_{11}\,t_1\,a_1 + l_{12}\,t_1\,a_2 + l_{21}\,t_2\,a_1 + l_{22}\,t_2\,a_2 + \gamma_{11}\,x_1\,y_1 + \gamma_{12}\,x_1\,y_2 + \gamma_{21}\,x_2\,y_1 + \gamma_{22}\,x_2\,y_2\right),\ 1_2 + \epsilon\left(l_1\,a_1 + l_2\,a_2 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2 + p_{21}\,x_2\,y_1 + p_{22}\,x_2\,y_2\right)\Big]\Big\},\ \{\text{Short}[rhs = q\oplus\ //\ \text{Rord}_{x_2,a_2\to 3},\ 3],\ \text{HL}[\text{QU}[q\oplus] == \text{QU}[rhs]]\}\Big]$$

$$\Big\{\mathbb{E}_{QU}\Big[\{y_1, a_1, x_1\}_1,\ \{a_3, x_3, y_2\}_2,\ e^{-\gamma\hbar(l_{12}t_1+l_{22}t_2)}\left(e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_3\,l_{12}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{21}\,t_2 + e^{\ll 1\gg}\,\hbar\,a_3\,l_{22}\,t_2 + \ll 1\gg + e^{\gamma\hbar(\ll 1\gg)}\,\hbar\,x_1\,y_2\,\gamma_{12} + \hbar\,x_3\,y_1\,\gamma_{21} + \hbar\,x_3\,y_2\,\gamma_{22}\right),\ 1 + \left(a_1\,l_1 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2\right)\epsilon + O[\epsilon]^3\Big],\ \text{True}\Big\}$$

$$\text{With}\Big[\Big\{q\oplus = \mathbb{E}_{QU}\Big[\{y_1, a_1, x_1\}_1,\ \{x_2, a_2, y_2\}_2,\ \hbar\left(l_{11}\,t_1\,a_1 + l_{12}\,t_1\,a_2 + l_{21}\,t_2\,a_1 + l_{22}\,t_2\,a_2 + \gamma_{11}\,x_1\,y_1 + \gamma_{12}\,x_1\,y_2 + \gamma_{21}\,x_2\,y_1 + \gamma_{22}\,x_2\,y_2\right),\ 1_2 + \epsilon\left(l_1\,a_1 + l_2\,a_2 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2 + p_{21}\,x_2\,y_1 + p_{22}\,x_2\,y_2\right)\Big]\Big\},\ \{\text{Short}[rhs = q\oplus\ //\ \text{Rord}_{a_2,y_2\to 3},\ 3],\ \text{HL}[\text{QU}[q\oplus] == \text{QU}[rhs]]\}\Big]$$

$$\Big\{\mathbb{E}_{QU}\Big[\{y_1, a_1, x_1\}_1,\ \{x_2, y_3, a_3\}_2,\ e^{-\gamma\hbar(l_{12}t_1+l_{22}t_2)}\left(e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,a_3\,l_{12}\,t_1 + e^{\gamma\hbar(l_{12}t_1+l_{22}\ll 1\gg)}\,\hbar\,a_1\,l_{21}\,t_2 + e^{\ll 1\gg}\ll 3\gg t_2 + \ll 1\gg + \hbar\,x_1\,y_3\,\gamma_{12} + e^{\gamma\hbar(l_{12}t_1+l_{22}t_2)}\,\hbar\,x_2\,y_1\,\gamma_{21} + \hbar\,x_2\,y_3\,\gamma_{22}\right),\ 1 + \left(a_1\,l_1 + p_{11}\,x_1\,y_1 + p_{21}\,x_2\,y_1\right)\epsilon + O[\epsilon]^3\Big],\ \text{True}\Big\}$$

```
Timing@With[{q𝔼 = 𝔼QU[{x1, y1}1, {x2, a2, y2}2,
      ℏ (l12 t1 a2 + l22 t2 a2 + γ11 x1 y1 + γ12 x1 y2 + γ21 x2 y1 + γ22 x2 y2),
      12 + ϵ (l2 a2 + p11 x1 y1 + p12 x1 y2 + p21 x2 y1 + p22 x2 y2)]},
  {Short[rhs = q𝔼 // Rord_{x1,y1→3}, 5], HL@SimpT[QU[q𝔼] == QU[rhs]]}]
```

$\{53.1875, \{\mathbb{E}_{QU}[\{y_3, a_3, x_3\}_1, \{x_2, a_2, y_2\}_2, \frac{\hbar\, a_2\, l_{12}\, t_1 + \ll 16 \gg + \hbar\, T_1\, x_2\, y_2\, \gamma_{11}\, \gamma_{22}}{1 - \gamma_{11} + T_1\, \gamma_{11}},$
$\frac{1}{1 - \gamma_{11} + T_1 \gamma_{11}} + ((4\, a_2\, l_2 + 4\, p_{22}\, x_2\, y_2 + 8\, \hbar^2\, a_3\, T_1\, x_2\, y_2\, \gamma_{12}\, \gamma_{21} + 4\, \gamma\, \hbar^3\, x_2\, x_3\, y_2\, y_3\, \gamma_{12}\, \gamma_{21} + \ll 6 \gg +$
$\gamma\, \hbar^3\, x_2^2\, y_2^2\, \gamma_{12}^2\, \gamma_{21}^2 - 4\, \gamma\, \hbar^3\, T_1\, x_2^2\, y_2^2\, \gamma_{12}^2\, \gamma_{21}^2 + 3\, \gamma\, \hbar^3\, T_1^2\, x_2^2\, y_2^2\, \gamma_{12}^2\, \gamma_{21}^2)\, \epsilon) / (4\, (1 - \gamma_{11} + T_1\, \gamma_{11})) +$
$((-576\, \hbar^3\, a_3^2\, T_1\, x_2\, y_2\, \gamma_{12}\, \gamma_{21} + 576\, \hbar^2\, a_2\, a_3\, l_2\, T_1\, x_2\, y_2\, \gamma_{12}\, \gamma_{21} + \ll 113 \gg + 81\, \gamma^2\, \hbar^6\, T_1^4\, x_2^4\, y_2^4\, \gamma_{12}^4\, \gamma_{21}^4)$
$\epsilon^2) / (288\, (1 - \gamma_{11} + T_1\, \gamma_{11})) + O[\epsilon]^3], \text{True}\}\}$

```
Timing@With[{q𝔼 = 𝔼QU[{x1, y1}1, {x2, a2, y2}2,
      ℏ (l12 t1 a2 + l22 t2 a2 + γ11 x1 y1 + γ12 x1 y2 + γ21 x2 y1 + γ22 x2 y2),
      12 + ϵ (l2 a2 + p11 x1 y1 + p12 x1 y2 + p21 x2 y1 + p22 x2 y2)]},
  {Short[rhs = q𝔼 // Rord_{x1,y1→1}, 5], HL@SimpT[QU[q𝔼] == QU[rhs]]}]
```

$\{52.8594, \{\mathbb{E}_{QU}[\{y_1, a_1, x_1\}_1, \{x_2, a_2, y_2\}_2, \frac{\hbar\, a_2\, l_{12}\, t_1 + \ll 16 \gg + \hbar\, T_1\, x_2\, y_2\, \gamma_{11}\, \gamma_{22}}{1 - \gamma_{11} + T_1\, \gamma_{11}},$
$\frac{1}{1 - \gamma_{11} + T_1 \gamma_{11}} + ((4\, a_2\, l_2 + 4\, p_{22}\, x_2\, y_2 + 8\, \hbar^2\, a_1\, T_1\, x_2\, y_2\, \gamma_{12}\, \gamma_{21} + 4\, \gamma\, \hbar^3\, x_1\, x_2\, y_1\, y_2\, \gamma_{12}\, \gamma_{21} + \ll 6 \gg +$
$\gamma\, \hbar^3\, x_2^2\, y_2^2\, \gamma_{12}^2\, \gamma_{21}^2 - 4\, \gamma\, \hbar^3\, T_1\, x_2^2\, y_2^2\, \gamma_{12}^2\, \gamma_{21}^2 + 3\, \gamma\, \hbar^3\, T_1^2\, x_2^2\, y_2^2\, \gamma_{12}^2\, \gamma_{21}^2)\, \epsilon) / (4\, (1 - \gamma_{11} + T_1\, \gamma_{11})) +$
$((-576\, \hbar^3\, a_1^2\, T_1\, x_2\, y_2\, \gamma_{12}\, \gamma_{21} + 576\, \hbar^2\, a_1\, a_2\, l_2\, T_1\, x_2\, y_2\, \gamma_{12}\, \gamma_{21} + \ll 113 \gg + 81\, \gamma^2\, \hbar^6\, T_1^4\, x_2^4\, y_2^4\, \gamma_{12}^4\, \gamma_{21}^4)$
$\epsilon^2) / (288\, (1 - \gamma_{11} + T_1\, \gamma_{11})) + O[\epsilon]^3], \text{True}\}\}$

Canonical ordering with Cord

Cord

```
Cord[𝔼U_[L___, {l___, u_i_, w_j_, r___}s_, R___, Q_, P_]] /;
   OrderedQ[{w, u} /. {y → 1, a → 2, x → 3}] :=
  Cord[Rord_{ui,wj→Unique[]}[𝔼U[L, {l, ui, wj, r}s, R, Q, P]]];
Cord[𝔼U_[specs__, Q_, P_]] := 𝔼U[specs, Q, P] /.
  Flatten[{specs} /. {yax__}s_ :> ({yax} /. u_i_ :> (ui → us))]
```

With$\Big[\Big\{c\Phi = \mathbb{E}_{CU}\big[\{y_1, a_1, x_1, x_2, a_2, y_2\}_1,$
$\quad \hbar\,(l_{11}\,t_1\,a_1 + l_{12}\,t_1\,a_2 + l_{21}\,t_2\,a_1 + l_{22}\,t_2\,a_2 + \gamma_{11}\,x_1\,y_1 + \gamma_{12}\,x_1\,y_2 + \gamma_{21}\,x_2\,y_1 + \gamma_{22}\,x_2\,y_2),$
$\quad 1_0 + \epsilon\,(l_1\,a_1 + l_2\,a_2 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2 + p_{21}\,x_2\,y_1 + p_{22}\,x_2\,y_2)\big]\Big\},$
Cord$[$
$c\Phi]\Big]$

$$\mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1, \frac{1}{e^{\gamma\,(\hbar\,l_{12}\,t_1+\hbar\,l_{22}\,t_2)} + \hbar\,t_1\,\gamma_{12} + \hbar\,t_1\,\gamma_{22}}$$
$$e^{-\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\big(e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{11}\,t_1 +$$
$$e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{12}\,t_1 + e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{21}\,t_2 +$$
$$e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{22}\,t_2 + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,x_1\,y_1\,\gamma_{11} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{11}\,t_1^2\,\gamma_{12} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{12}\,t_1^2\,\gamma_{12} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{21}\,t_1\,t_2\,\gamma_{12} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{22}\,t_1\,t_2\,\gamma_{12} +$$
$$\hbar\,x_1\,y_1\,\gamma_{12} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,x_1\,y_1\,\gamma_{21} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{11}\,t_1^2\,\gamma_{22} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{12}\,t_1^2\,\gamma_{22} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{21}\,t_1\,t_2\,\gamma_{22} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar^2\,a_1\,l_{22}\,t_1\,t_2\,\gamma_{22} + \hbar\,x_1\,y_1\,\gamma_{22}\big),$$
$$\frac{e^{\gamma\,(\hbar\,l_{12}\,t_1+\hbar\,l_{22}\,t_2)}}{e^{\gamma\,(\hbar\,l_{12}\,t_1+\hbar\,l_{22}\,t_2)} + \hbar\,t_1\,\gamma_{12} + \hbar\,t_1\,\gamma_{22}} + O[\epsilon]^1\Big]$$

With$\Big[\Big\{q\Phi = \mathbb{E}_{QU}\big[\{y_1, a_1, x_1, x_2, a_2, y_2\}_1,$
$\quad \hbar\,(l_{11}\,t_1\,a_1 + l_{12}\,t_1\,a_2 + l_{21}\,t_2\,a_1 + l_{22}\,t_2\,a_2 + \gamma_{11}\,x_1\,y_1 + \gamma_{12}\,x_1\,y_2 + \gamma_{21}\,x_2\,y_1 + \gamma_{22}\,x_2\,y_2),$
$\quad 1_0 + \epsilon\,(l_1\,a_1 + l_2\,a_2 + p_{11}\,x_1\,y_1 + p_{12}\,x_1\,y_2 + p_{21}\,x_2\,y_1 + p_{22}\,x_2\,y_2)\big]\Big\},$
Cord$[$
$q\Phi]\Big]$

$$\mathbb{E}_{QU}\Big[\{y_1, a_1, x_1\}_1, \Big(e^{-\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}$$
$$\big(e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{11}\,t_1 + e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{12}\,t_1 +$$
$$e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{21}\,t_2 + e^{\gamma\,\hbar\,l_{11}\,t_1+2\,\gamma\,\hbar\,l_{12}\,t_1+\gamma\,\hbar\,l_{21}\,t_2+2\,\gamma\,\hbar\,l_{22}\,t_2}\,\hbar\,a_1\,l_{22}\,t_2 +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,x_1\,y_1\,\gamma_{11} - e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{11}\,t_1\,\gamma_{12} -$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{12}\,t_1\,\gamma_{12} - e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{21}\,t_2\,\gamma_{12} -$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{22}\,t_2\,\gamma_{12} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{11}\,t_1\,T_1\,\gamma_{12} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{12}\,t_1\,T_1\,\gamma_{12} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{21}\,t_2\,T_1\,\gamma_{12} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{22}\,t_2\,T_1\,\gamma_{12} + \hbar\,x_1\,y_1\,\gamma_{12} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,x_1\,y_1\,\gamma_{21} - e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{11}\,t_1\,\gamma_{22} -$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{12}\,t_1\,\gamma_{22} - e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{21}\,t_2\,\gamma_{22} -$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{22}\,t_2\,\gamma_{22} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{11}\,t_1\,T_1\,\gamma_{22} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{12}\,t_1\,T_1\,\gamma_{22} + e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{21}\,t_2\,T_1\,\gamma_{22} +$$
$$e^{\gamma\,(\hbar\,l_{11}\,t_1+\hbar\,l_{12}\,t_1+\hbar\,l_{21}\,t_2+\hbar\,l_{22}\,t_2)}\,\hbar\,a_1\,l_{22}\,t_2\,T_1\,\gamma_{22} + \hbar\,x_1\,y_1\,\gamma_{22}\big)\Big)\Big/$$
$$\big(e^{\gamma\,(\hbar\,l_{12}\,t_1+\hbar\,l_{22}\,t_2)} - \gamma_{12} + T_1\,\gamma_{12} - \gamma_{22} + T_1\,\gamma_{22}\big),$$
$$\frac{e^{\gamma\,(\hbar\,l_{12}\,t_1+\hbar\,l_{22}\,t_2)}}{e^{\gamma\,(\hbar\,l_{12}\,t_1+\hbar\,l_{22}\,t_2)} - \gamma_{12} + T_1\,\gamma_{12} - \gamma_{22} + T_1\,\gamma_{22}} +$$
$$O[\epsilon]^1\Big]$$

Stitching Œ's.

StitchingOEs

```
m_{j_→k_}[Œ_{U_}[specs__, Q_, P_]] := Cord[Œ_U[Sequence @@ Append[DeleteCases[{specs}, {__}_{j|k}],
       Flatten[{Cases[{specs}, {us__}_j :> {us}], Cases[{specs}, {us__}_k :> {us}]}]_k],
    Q, P] /. {t_j → t_k, T_j → T_k}]
```

```
cœ = Œ_CU[{y1, a1, x1}_1, {y2, a2, x2}_2,
   {y3, a3, x3}_3, ℏ Sum[l_{10 i+j} t_i a_j + γ_{10 i+j} y_i x_j, {i, 3}, {j, 3}], 1_2];
{cœ // m_{3→4}, HL@Simp[CU[m_{3→4}[cœ]] - m_{3→4}[CU[cœ]]]}
```

$\{Œ_{CU}[\{y_1, a_1, x_1\}_1, \{y_2, a_2, x_2\}_2, \{y_4, a_4, x_4\}_4,$
$\hbar\,(a_1 l_{11} t_1 + a_2 l_{12} t_1 + a_4 l_{13} t_1 + a_1 l_{21} t_2 + a_2 l_{22} t_2 + a_4 l_{23} t_2 +$
$a_1 l_{31} t_4 + a_2 l_{32} t_4 + a_4 l_{33} t_4 + x_1 y_1 \gamma_{11} + x_2 y_1 \gamma_{12} + x_4 y_1 \gamma_{13} + x_1 y_2 \gamma_{21} +$
$x_2 y_2 \gamma_{22} + x_4 y_2 \gamma_{23} + x_1 y_4 \gamma_{31} + x_2 y_4 \gamma_{32} + x_4 y_4 \gamma_{33}),\ 1 + O[\epsilon]^3],\ 0\}$

Verifying that *m* commutes with evaluation, in CU:

$c\oplus = \mathbb{E}_{CU}[\{y_1, a_1, x_1\}_1, \{y_2, a_2, x_2\}_2,$
$\{y_3, a_3, x_3\}_3, \hbar\, \text{Sum}[l_{10\,i+j}\, t_i\, a_j + \gamma_{10\,i+j}\, y_i\, x_j,\ \{i, 3\}, \{j, 3\}], 1_2];$
$\text{Timing}@\{c\oplus\ //\ m_{2\to3},\ \text{HL}@\text{Simp}[\text{CU}[m_{2\to3}[c\oplus]] - m_{2\to3}[\text{CU}[c\oplus]]]\}$

$$\Big\{27.5938,$$

$$\Big\{\mathbb{E}_{CU}\Big[\{y_1, a_1, x_1\}_1, \{y_3, a_3, x_3\}_3, \frac{1}{1+\hbar t_3 \gamma_{32}}\, e^{-2\gamma\hbar l_{12} t_1-\gamma\hbar l_{13} t_1-2\gamma\hbar l_{22} t_3-\gamma\hbar l_{23} t_3-2\gamma\hbar l_{32} t_3-\gamma\hbar l_{33} t_3}$$

$$\Big(e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_1 l_{11} t_1 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_3 l_{12} t_1 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_3 l_{13} t_1 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_1 l_{21} t_3 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_3 l_{22} t_3 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_3 l_{23} t_3 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_1 l_{31} t_3 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_3 l_{32} t_3 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, a_3 l_{33} t_3 +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, x_1 y_1 \gamma_{11} +$$
$$e^{2\gamma(\hbar l_{12} t_1+\hbar l_{22} t_3+\hbar l_{32} t_3)}\, \hbar\, x_3 y_1 \gamma_{12} + e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}$$
$$\hbar\, x_3 y_1 \gamma_{13} + e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, x_1 y_3 \gamma_{21} +$$
$$e^{2\gamma(\hbar l_{12} t_1+\hbar l_{22} t_3+\hbar l_{32} t_3)}\, \hbar\, x_3 y_3 \gamma_{22} + e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}$$
$$\hbar\, x_3 y_3 \gamma_{23} + e^{\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, x_1 y_3 \gamma_{31} -$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_1 y_1 \gamma_{12} \gamma_{31} -$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_1 y_3 \gamma_{22} \gamma_{31} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_1 l_{11} t_1 t_3 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_3 l_{12} t_1 t_3 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_3 l_{13} t_1 t_3 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_1 l_{21} t_3^2 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_3 l_{22} t_3^2 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_3 l_{23} t_3^2 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_1 l_{31} t_3^2 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_3 l_{32} t_3^2 \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, a_3 l_{33} t_3^2 \gamma_{32} +$$
$$e^{\gamma(\hbar l_{12} t_1+\hbar l_{22} t_3+\hbar l_{32} t_3)}\, \hbar\, x_3 y_3 \gamma_{32} + e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2$$
$$t_3 x_1 y_1 \gamma_{11} \gamma_{32} + e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_3 y_1 \gamma_{13} \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_1 y_3 \gamma_{21} \gamma_{32} +$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_3 y_3 \gamma_{23} \gamma_{32} +$$
$$e^{\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar\, x_3 y_3 \gamma_{33} -$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_3 y_1 \gamma_{12} \gamma_{33} -$$
$$e^{2\gamma\hbar l_{12} t_1+\gamma\hbar l_{13} t_1+2\gamma\hbar l_{22} t_3+\gamma\hbar l_{23} t_3+2\gamma\hbar l_{32} t_3+\gamma\hbar l_{33} t_3}\, \hbar^2\, t_3 x_3 y_3 \gamma_{22} \gamma_{33}\Big),$$

$$\frac{1}{1+\hbar t_3 \gamma_{32}} + \frac{\gamma \hbar^4 t_3 x_1^2 y_1^2 \gamma_{12}^2 \gamma_{31}^2 \epsilon}{2\left(1+\hbar t_3 \gamma_{32}\right)} + \left(\left(-8\gamma^2 \hbar^6 t_3 x_1^3 y_1^3 \gamma_{12}^3 \gamma_{31}^3 + 3\gamma^2 \hbar^8 t_3^2 x_1^4 y_1^4 \gamma_{12}^4 \gamma_{31}^4\right) \epsilon^2\right) /$$
$$\left(24\left(1+\hbar t_3 \gamma_{32}\right)\right) + O[\epsilon]^3\Big], 0\Big\}\Big\}$$

Verifying that *m* commutes with evaluation, in QU:

$q\oplus = \mathbb{E}_{QU}[\{y_1, a_1, x_1\}_1, \{y_2, a_2, x_2\}_2,$
$\{y_3, a_3, x_3\}_3, \hbar\, \text{Sum}[l_{10\,i+j}\, t_i\, a_j + \gamma_{10\,i+j}\, y_i\, x_j,\ \{i, 3\}, \{j, 3\}], 1_2];$
$\text{Timing}@\{q\oplus\ //\ m_{2\to3},\ \text{HL}@\text{SimpT}[\text{QU}[m_{2\to3}[q\oplus]] - m_{2\to3}[\text{QU}[q\oplus]]]\}$

$$\Big\{62.5313,$$

$$\Big\{\mathbb{E}_{QU}\Big[\{y_1, a_1, x_1\}_1, \{y_3, a_3, x_3\}_3, \frac{1}{1-\gamma_{32}+T_3 \gamma_{32}}\, e^{-2\gamma\hbar l_{12} t_1-\gamma\hbar l_{13} t_1-2\gamma\hbar l_{22} t_3-\gamma\hbar l_{23} t_3-2\gamma\hbar l_{32} t_3-\gamma\hbar l_{33} t_3}$$

$$
\begin{array}{l}
\left(e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{11} t_1+\right.\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{12} t_1+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{13} t_1+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{21} t_3+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{22} t_3+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{23} t_3+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{31} t_3+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{32} t_3+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{33} t_3+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_1 y_1 \gamma_{11}+\\
e^{2\gamma(\hbar l_{12}t_1+\hbar l_{22}t_3+\hbar l_{32}t_3)}\hbar x_3 y_1 \gamma_{12}+e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\\
\quad\hbar x_3 y_1 \gamma_{13}+e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_1 y_3 \gamma_{21}+\\
e^{2\gamma(\hbar l_{12}t_1+\hbar l_{22}t_3+\hbar l_{32}t_3)}\hbar x_3 y_3 \gamma_{22}+e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\\
\quad\hbar x_3 y_3 \gamma_{23}+e^{\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_1 y_3 \gamma_{31}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_1 y_1 \gamma_{12}\gamma_{31}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_1 y_1 \gamma_{12}\gamma_{31}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_1 y_3 \gamma_{22}\gamma_{31}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_1 y_3 \gamma_{22}\gamma_{31}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{11} t_1 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{12} t_1 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{13} t_1 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{21} t_3 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{22} t_3 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{23} t_3 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{31} t_3 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{32} t_3 \gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{33} t_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{11} t_1 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{12} t_1 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{13} t_1 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{21} t_3 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{22} t_3 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{23} t_3 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_1 l_{31} t_3 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{32} t_3 T_3 \gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar a_3 l_{33} t_3 T_3 \gamma_{32}+\\
e^{\gamma(\hbar l_{12}t_1+\hbar l_{22}t_3+\hbar l_{32}t_3)}\hbar x_3 y_3 \gamma_{32}-e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\\
\quad\hbar x_1 y_1 \gamma_{11}\gamma_{32}+e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_1 y_1 \gamma_{11}\gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_3 y_1 \gamma_{13}\gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_3 y_1 \gamma_{13}\gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_1 y_3 \gamma_{21}\gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_1 y_3 \gamma_{21}\gamma_{32}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_3 y_3 \gamma_{23}\gamma_{32}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_3 y_3 \gamma_{23}\gamma_{32}+\\
e^{\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_3 y_3 \gamma_{33}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_3 y_1 \gamma_{12}\gamma_{33}-\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_3 y_1 \gamma_{12}\gamma_{33}+\\
e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar x_3 y_3 \gamma_{22}\gamma_{33}-\\
\left.e^{2\gamma\hbar l_{12}t_1+\gamma\hbar l_{13}t_1+2\gamma\hbar l_{22}t_3+\gamma\hbar l_{23}t_3+2\gamma\hbar l_{32}t_3+\gamma\hbar l_{33}t_3}\hbar T_3 x_3 y_3 \gamma_{22}\gamma_{33}\right),
\end{array}
$$

$$
\frac{1}{1-\gamma_{32}+T_3\gamma_{32}}+\left(\left(\gamma\hbar^3 x_1^2 y_1^2 \gamma_{12}^2\gamma_{31}^2-4\gamma\hbar^3 T_3 x_1^2 y_1^2\gamma_{12}^2\gamma_{31}^2+3\gamma\hbar^3 T_3^2 x_1^2 y_1^2\gamma_{12}^2\gamma_{31}^2\right)\epsilon\right)\Big/
$$

$$\left(4\left(1-\gamma_{32}+T_3\gamma_{32}\right)\right)+$$
$$\frac{1}{288\left(1-\gamma_{32}+T_3\gamma_{32}\right)}\left(36\gamma^2\hbar^4 x_1^2 y_1^2\gamma_{12}^2\gamma_{31}^2-216\gamma^2\hbar^4 T_3 x_1^2 y_1^2\gamma_{12}^2\gamma_{31}^2+180\gamma^2\hbar^4 T_3^2 x_1^2 y_1^2\gamma_{12}^2\gamma_{31}^2+\right.$$
$$40\gamma^2\hbar^5 x_1^3 y_1^3\gamma_{12}^3\gamma_{31}^3-312\gamma^2\hbar^5 T_3 x_1^3 y_1^3\gamma_{12}^3\gamma_{31}^3+600\gamma^2\hbar^5 T_3^2 x_1^3 y_1^3\gamma_{12}^3\gamma_{31}^3-$$
$$328\gamma^2\hbar^5 T_3^3 x_1^3 y_1^3\gamma_{12}^3\gamma_{31}^3+9\gamma^2\hbar^6 x_1^4 y_1^4\gamma_{12}^4\gamma_{31}^4-72\gamma^2\hbar^6 T_3 x_1^4 y_1^4\gamma_{12}^4\gamma_{31}^4+$$
$$\left.198\gamma^2\hbar^6 T_3^2 x_1^4 y_1^4\gamma_{12}^4\gamma_{31}^4-216\gamma^2\hbar^6 T_3^3 x_1^4 y_1^4\gamma_{12}^4\gamma_{31}^4+81\gamma^2\hbar^6 T_3^4 x_1^4 y_1^4\gamma_{12}^4\gamma_{31}^4\right)\epsilon^2+O[\epsilon]^3\Big],\ 0\Big\}\Big\}$$

Verifying meta-associativity in CU:

$$\mathbf{c}\oplus = Œ_{CU}[\{y_1, a_1, x_1\}_1, \{y_2, a_2, x_2\}_2,$$
$$\{y_3, a_3, x_3\}_3, \hbar\,\mathrm{Sum}[\lambda_{10\,i+j}\, t_i\, a_j + \gamma_{10\,i+j}\, y_i\, x_j,\ \{i, 3\}, \{j, 3\}],\ 1_0];$$
$$\mathrm{Timing@HL}[(\mathbf{c}\oplus \,//\, m_{1,2\to1} \,//\, m_{1,3\to1}) == (\mathbf{c}\oplus \,//\, m_{2,3\to2} \,//\, m_{1,2\to1})]$$

$$\Big\{21.6875,\ Œ_{CU}\Big[\{y_1, a_1, x_1\}_1,$$
$$\Big(e^{-\gamma\hbar t_1(10\lambda_{11}+6\lambda_{12}+\lambda_{13}+10\lambda_{21}+6\lambda_{22}+\lambda_{23}+10\lambda_{31}+6\lambda_{32}+\lambda_{33})}\Big(e^{6\gamma\hbar t_1(2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22}+2\lambda_{31}+\lambda_{32})}\hbar x_1 y_1\gamma_{11}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+12\lambda_{21}+7\lambda_{22}+12\lambda_{31}+7\lambda_{32})}\hbar x_1 y_1\gamma_{12}+e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar$$
$$x_1 y_1\gamma_{13}+e^{\gamma\hbar t_1(11\lambda_{11}+6\lambda_{12}+11\lambda_{21}+6\lambda_{22}+11\lambda_{31}+6\lambda_{32})}\hbar x_1 y_1\gamma_{21}+e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+12\lambda_{21}+7\lambda_{22}+12\lambda_{31}+7\lambda_{32})}$$
$$\hbar^2 t_1 x_1 y_1\gamma_{12}\gamma_{21}+e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 t_1 x_1 y_1\gamma_{13}\gamma_{21}+$$
$$e^{\gamma\hbar t_1(11\lambda_{11}+7\lambda_{12}+11\lambda_{21}+7\lambda_{22}+11\lambda_{31}+7\lambda_{32})}\hbar x_1 y_1\gamma_{22}-e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+12\lambda_{21}+7\lambda_{22}+12\lambda_{31}+7\lambda_{32})}$$
$$\hbar^2 t_1 x_1 y_1\gamma_{11}\gamma_{22}+e^{\gamma\hbar t_1(11\lambda_{11}+7\lambda_{12}+\lambda_{13}+11\lambda_{21}+7\lambda_{22}+\lambda_{23}+11\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar x_1 y_1\gamma_{23}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 t_1 x_1 y_1\gamma_{11}\gamma_{23}+$$
$$e^{\gamma\hbar t_1(11\lambda_{11}+5\lambda_{12}+11\lambda_{21}+5\lambda_{22}+11\lambda_{31}+5\lambda_{32})}\hbar x_1 y_1\gamma_{31}+e^{\gamma\hbar t_1(12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})}$$
$$\hbar^2 t_1 x_1 y_1\gamma_{13}\gamma_{31}-e^{\gamma\hbar t_1(11\lambda_{11}+6\lambda_{12}+11\lambda_{21}+6\lambda_{22}+11\lambda_{31}+6\lambda_{32})}\hbar^2 t_1 x_1 y_1\gamma_{22}\gamma_{31}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 t_1^2 x_1 y_1\gamma_{13}\gamma_{22}\gamma_{31}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 t_1^2 x_1 y_1\gamma_{12}\gamma_{23}\gamma_{31}+$$
$$e^{\gamma\hbar t_1(11\lambda_{11}+6\lambda_{12}+11\lambda_{21}+6\lambda_{22}+11\lambda_{31}+6\lambda_{32})}\hbar x_1 y_1\gamma_{32}+e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}$$
$$\hbar^2 t_1 x_1 y_1\gamma_{13}\gamma_{32}+e^{\gamma\hbar t_1(11\lambda_{11}+6\lambda_{12}+11\lambda_{21}+6\lambda_{22}+11\lambda_{31}+6\lambda_{32})}\hbar^2 t_1 x_1 y_1\gamma_{21}\gamma_{32}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 t_1^2 x_1 y_1\gamma_{13}\gamma_{21}\gamma_{32}+$$
$$e^{\gamma\hbar t_1(11\lambda_{11}+7\lambda_{12}+\lambda_{13}+11\lambda_{21}+7\lambda_{22}+\lambda_{23}+11\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 t_1 x_1 y_1\gamma_{23}\gamma_{32}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 t_1^2 x_1 y_1\gamma_{11}\gamma_{23}\gamma_{32}+$$
$$e^{\gamma\hbar t_1(11\lambda_{11}+6\lambda_{12}+\lambda_{13}+11\lambda_{21}+6\lambda_{22}+\lambda_{23}+11\lambda_{31}+6\lambda_{32}+\lambda_{33})}\hbar x_1 y_1\gamma_{33}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})}\hbar^2 t_1 x_1 y_1\gamma_{11}\gamma_{33}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 t_1 x_1 y_1\gamma_{12}\gamma_{33}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 t_1^2 x_1 y_1\gamma_{12}\gamma_{21}\gamma_{33}-$$
$$e^{\gamma\hbar t_1(11\lambda_{11}+7\lambda_{12}+\lambda_{13}+11\lambda_{21}+7\lambda_{22}+\lambda_{23}+11\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 t_1 x_1 y_1\gamma_{22}\gamma_{33}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 t_1^2 x_1 y_1\gamma_{11}\gamma_{22}\gamma_{33}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar a_1 t_1\lambda_{11}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{11}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{11}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{11}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{11}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{11}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar a_1 t_1\lambda_{12}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{12}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{12}-$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{12}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{12}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{12}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar a_1 t_1\lambda_{13}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{13}+$$
$$e^{\gamma\hbar t_1(12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{13}-$$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{13} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{13} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{13} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar a_1 t_1 \lambda_{21} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{21} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{21} -$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{21} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{21} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{21} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar a_1 t_1 \lambda_{22} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{22} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{22} -$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{22} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{22} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{22} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar a_1 t_1 \lambda_{23} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{23} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{23} -$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{23} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{23} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{23} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar a_1 t_1 \lambda_{31} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{31} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{31} -$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{31} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{31} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{31} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar a_1 t_1 \lambda_{32} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{32} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{32} -$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{32} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{32} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{32} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar a_1 t_1 \lambda_{33} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{33} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+6\lambda_{12}+\lambda_{13}+12\lambda_{21}+6\lambda_{22}+\lambda_{23}+12\lambda_{31}+6\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{33} -$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{33} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{33} +$

$e^{\gamma \hbar t_1 (12\lambda_{11}+7\lambda_{12}+\lambda_{13}+12\lambda_{21}+7\lambda_{22}+\lambda_{23}+12\lambda_{31}+7\lambda_{32}+\lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{33})\Big) \Big/$

$\Big(e^{\gamma \hbar t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22}+2\lambda_{31}+\lambda_{32})} + e^{\gamma \hbar t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22}+2\lambda_{31}+\lambda_{32})} \hbar t_1 \gamma_{21} +$

$e^{2\gamma \hbar t_1 (\lambda_{11}+\lambda_{21}+\lambda_{31})} \hbar t_1 \gamma_{31} - e^{\gamma \hbar t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22}+2\lambda_{31}+\lambda_{32})} \hbar^2 t_1^2 \gamma_{22} \gamma_{31} +$

$e^{\gamma \hbar t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22}+2\lambda_{31}+\lambda_{32})} \hbar t_1 \gamma_{32} +$

$e^{\gamma \hbar t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22}+2\lambda_{31}+\lambda_{32})} \hbar^2 t_1^2 \gamma_{21} \gamma_{32}\Big),$

$e^{\gamma \hbar (t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22})+t_1 (2\lambda_{31}+\lambda_{32}))} \Big/ \Big(e^{\gamma \hbar (t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22})+t_1 (2\lambda_{31}+\lambda_{32}))} +$

$e^{\gamma \hbar (t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22})+t_1 (2\lambda_{31}+\lambda_{32}))} \hbar t_1 \gamma_{21} + e^{2\gamma \hbar (t_1 (\lambda_{11}+\lambda_{21})+t_1 \lambda_{31})} \hbar t_1 \gamma_{31} -$

$e^{\gamma \hbar (t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22})+t_1 (2\lambda_{31}+\lambda_{32}))} \hbar^2 t_1^2 \gamma_{22} \gamma_{31} + e^{\gamma \hbar (t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22})+t_1 (2\lambda_{31}+\lambda_{32}))} \hbar t_1 \gamma_{32} +$

$e^{\gamma \hbar (t_1 (2\lambda_{11}+\lambda_{12}+2\lambda_{21}+\lambda_{22})+t_1 (2\lambda_{31}+\lambda_{32}))} \hbar^2 t_1^2 \gamma_{21} \gamma_{32}\Big) + O[\epsilon]^1\Big] ==$

$Œ_{CU}\Big[\{y_1, a_1, x_1\}_1, \Big(e^{-\gamma \hbar t_1 (2\lambda_{11}+9\lambda_{12}+5\lambda_{13}+2\lambda_{21}+9\lambda_{22}+5\lambda_{23}+2\lambda_{31}+9\lambda_{32}+5\lambda_{33})}$

$\Big(e^{\gamma \hbar t_1 (2\lambda_{11}+10\lambda_{12}+5\lambda_{13}+2\lambda_{21}+10\lambda_{22}+5\lambda_{23}+2\lambda_{31}+10\lambda_{32}+5\lambda_{33})} \hbar x_1 y_1 \gamma_{11} +$

$e^{\gamma \hbar t_1 (2\lambda_{11}+11\lambda_{12}+5\lambda_{13}+2\lambda_{21}+11\lambda_{22}+5\lambda_{23}+2\lambda_{31}+11\lambda_{32}+5\lambda_{33})} \hbar x_1 y_1 \gamma_{12} +$

$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar x_1 y_1 \gamma_{13} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+10 \lambda_{12}+5 \lambda_{13}+\lambda_{21}+10 \lambda_{22}+5 \lambda_{23}+\lambda_{31}+10 \lambda_{32}+5 \lambda_{33})} \hbar x_1 y_1 \gamma_{21} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+5 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+5 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+5 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{12} \gamma_{21} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{13} \gamma_{21} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+11 \lambda_{12}+5 \lambda_{13}+\lambda_{21}+11 \lambda_{22}+5 \lambda_{23}+\lambda_{31}+11 \lambda_{32}+5 \lambda_{33})} \hbar x_1 y_1 \gamma_{22} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+5 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+5 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+5 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{11} \gamma_{22} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+\lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+\lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar x_1 y_1 \gamma_{23} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{11} \gamma_{23} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+9 \lambda_{12}+5 \lambda_{13}+\lambda_{21}+9 \lambda_{22}+5 \lambda_{23}+\lambda_{31}+9 \lambda_{32}+5 \lambda_{33})} \hbar x_1 y_1 \gamma_{31} +$
$e^{2 \gamma \hbar t_1 (\lambda_{11}+5 \lambda_{12}+3 \lambda_{13}+\lambda_{21}+5 \lambda_{22}+3 \lambda_{23}+\lambda_{31}+5 \lambda_{32}+3 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{13} \gamma_{31} -$
$e^{\gamma \hbar t_1 (\lambda_{11}+10 \lambda_{12}+5 \lambda_{13}+\lambda_{21}+10 \lambda_{22}+5 \lambda_{23}+\lambda_{31}+10 \lambda_{32}+5 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{22} \gamma_{31} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 t_1^2 x_1 y_1 \gamma_{13} \gamma_{22} \gamma_{31} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 t_1^2 x_1 y_1 \gamma_{12} \gamma_{23} \gamma_{31} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+10 \lambda_{12}+5 \lambda_{13}+\lambda_{21}+10 \lambda_{22}+5 \lambda_{23}+\lambda_{31}+10 \lambda_{32}+5 \lambda_{33})} \hbar x_1 y_1 \gamma_{32} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{13} \gamma_{32} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+10 \lambda_{12}+5 \lambda_{13}+\lambda_{21}+10 \lambda_{22}+5 \lambda_{23}+\lambda_{31}+10 \lambda_{32}+5 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{21} \gamma_{32} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 t_1^2 x_1 y_1 \gamma_{13} \gamma_{21} \gamma_{32} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+\lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+\lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{23} \gamma_{32} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 t_1^2 x_1 y_1 \gamma_{11} \gamma_{23} \gamma_{32} +$
$e^{\gamma \hbar t_1 (\lambda_{11}+10 \lambda_{12}+6 \lambda_{13}+\lambda_{21}+10 \lambda_{22}+6 \lambda_{23}+\lambda_{31}+10 \lambda_{32}+6 \lambda_{33})} \hbar x_1 y_1 \gamma_{33} -$
$e^{2 \gamma \hbar t_1 (\lambda_{11}+5 \lambda_{12}+3 \lambda_{13}+\lambda_{21}+5 \lambda_{22}+3 \lambda_{23}+\lambda_{31}+5 \lambda_{32}+3 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{11} \gamma_{33} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{12} \gamma_{33} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 t_1^2 x_1 y_1 \gamma_{12} \gamma_{21} \gamma_{33} -$
$e^{\gamma \hbar t_1 (\lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+\lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+\lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 t_1 x_1 y_1 \gamma_{22} \gamma_{33} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 t_1^2 x_1 y_1 \gamma_{11} \gamma_{22} \gamma_{33} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar a_1 t_1 \lambda_{11} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{11} +$
$e^{2 \gamma \hbar t_1 (\lambda_{11}+5 \lambda_{12}+3 \lambda_{13}+\lambda_{21}+5 \lambda_{22}+3 \lambda_{23}+\lambda_{31}+5 \lambda_{32}+3 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{11} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{11} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{11} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{11} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar a_1 t_1 \lambda_{12} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{12} +$
$e^{2 \gamma \hbar t_1 (\lambda_{11}+5 \lambda_{12}+3 \lambda_{13}+\lambda_{21}+5 \lambda_{22}+3 \lambda_{23}+\lambda_{31}+5 \lambda_{32}+3 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{12} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{12} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{12} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{12} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar a_1 t_1 \lambda_{13} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{13} +$
$e^{2 \gamma \hbar t_1 (\lambda_{11}+5 \lambda_{12}+3 \lambda_{13}+\lambda_{21}+5 \lambda_{22}+3 \lambda_{23}+\lambda_{31}+5 \lambda_{32}+3 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{13} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{13} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{13} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{13} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar a_1 t_1 \lambda_{21} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{21} +$
$e^{2 \gamma \hbar t_1 (\lambda_{11}+5 \lambda_{12}+3 \lambda_{13}+\lambda_{21}+5 \lambda_{22}+3 \lambda_{23}+\lambda_{31}+5 \lambda_{32}+3 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{31} \lambda_{21} -$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{22} \gamma_{31} \lambda_{21} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{32} \lambda_{21} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^3 a_1 t_1^3 \gamma_{21} \gamma_{32} \lambda_{21} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar a_1 t_1 \lambda_{22} +$
$e^{\gamma \hbar t_1 (2 \lambda_{11}+11 \lambda_{12}+6 \lambda_{13}+2 \lambda_{21}+11 \lambda_{22}+6 \lambda_{23}+2 \lambda_{31}+11 \lambda_{32}+6 \lambda_{33})} \hbar^2 a_1 t_1^2 \gamma_{21} \lambda_{22} +$

$$e^{2\gamma\hbar t_1(\lambda_{11}+5\lambda_{12}+3\lambda_{13}+\lambda_{21}+5\lambda_{22}+3\lambda_{23}+\lambda_{31}+5\lambda_{32}+3\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{22}-$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{22}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{22}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{22}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar a_1 t_1\lambda_{23}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{23}+$$
$$e^{2\gamma\hbar t_1(\lambda_{11}+5\lambda_{12}+3\lambda_{13}+\lambda_{21}+5\lambda_{22}+3\lambda_{23}+\lambda_{31}+5\lambda_{32}+3\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{23}-$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{23}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{23}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{23}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar a_1 t_1\lambda_{31}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{31}+$$
$$e^{2\gamma\hbar t_1(\lambda_{11}+5\lambda_{12}+3\lambda_{13}+\lambda_{21}+5\lambda_{22}+3\lambda_{23}+\lambda_{31}+5\lambda_{32}+3\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{31}-$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{31}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{31}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{31}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar a_1 t_1\lambda_{32}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{32}+$$
$$e^{2\gamma\hbar t_1(\lambda_{11}+5\lambda_{12}+3\lambda_{13}+\lambda_{21}+5\lambda_{22}+3\lambda_{23}+\lambda_{31}+5\lambda_{32}+3\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{32}-$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{32}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{32}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{32}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar a_1 t_1\lambda_{33}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{21}\lambda_{33}+$$
$$e^{2\gamma\hbar t_1(\lambda_{11}+5\lambda_{12}+3\lambda_{13}+\lambda_{21}+5\lambda_{22}+3\lambda_{23}+\lambda_{31}+5\lambda_{32}+3\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{31}\lambda_{33}-$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{22}\gamma_{31}\lambda_{33}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^2 a_1 t_1^2\gamma_{32}\lambda_{33}+$$
$$e^{\gamma\hbar t_1(2\lambda_{11}+11\lambda_{12}+6\lambda_{13}+2\lambda_{21}+11\lambda_{22}+6\lambda_{23}+2\lambda_{31}+11\lambda_{32}+6\lambda_{33})}\hbar^3 a_1 t_1^3\gamma_{21}\gamma_{32}\lambda_{33}\Big)\Big)\Big/$$
$$\Big(e^{\gamma\hbar t_1(2\lambda_{12}+\lambda_{13}+2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33})}+e^{\gamma\hbar t_1(2\lambda_{12}+\lambda_{13}+2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33})}\hbar t_1\gamma_{21}+$$
$$e^{\gamma\hbar t_1(\lambda_{12}+\lambda_{13}+\lambda_{22}+\lambda_{23}+\lambda_{32}+\lambda_{33})}\hbar t_1\gamma_{31}-$$
$$e^{\gamma\hbar t_1(2\lambda_{12}+\lambda_{13}+2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33})}\hbar^2 t_1^2\gamma_{22}\gamma_{31}+$$
$$e^{\gamma\hbar t_1(2\lambda_{12}+\lambda_{13}+2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33})}\hbar t_1\gamma_{32}+$$
$$e^{\gamma\hbar t_1(2\lambda_{12}+\lambda_{13}+2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33})}\hbar^2 t_1^2\gamma_{21}\gamma_{32}\Big),$$
$$e^{\gamma\hbar(t_1(2\lambda_{12}+\lambda_{13})+t_1(2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33}))}\Big/\Big(e^{\gamma\hbar(t_1(2\lambda_{12}+\lambda_{13})+t_1(2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33}))}+$$
$$e^{\gamma\hbar(t_1(2\lambda_{12}+\lambda_{13})+t_1(2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33}))}\hbar t_1\gamma_{21}+e^{\gamma\hbar(t_1(\lambda_{12}+\lambda_{13})+t_1(\lambda_{22}+\lambda_{23}+\lambda_{32}+\lambda_{33}))}\hbar t_1\gamma_{31}-$$
$$e^{\gamma\hbar(t_1(2\lambda_{12}+\lambda_{13})+t_1(2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33}))}\hbar^2 t_1^2\gamma_{22}\gamma_{31}+e^{\gamma\hbar(t_1(2\lambda_{12}+\lambda_{13})+t_1(2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33}))}\hbar t_1\gamma_{32}+$$
$$e^{\gamma\hbar(t_1(2\lambda_{12}+\lambda_{13})+t_1(2\lambda_{22}+\lambda_{23}+2\lambda_{32}+\lambda_{33}))}\hbar^2 t_1^2\gamma_{21}\gamma_{32}\Big)+O[\epsilon]^1\Big]\Big\}$$

R in QU.

Faddeev-Quesne's formula:

Faddeev

```
e_{q_,k_}[x_] := e^( Sum_{j=1}^{k+1} (1 - q)^j x^j / (j (1 - q^j)) ); e_{q_}[x_] := e_{q,$k}[x]
```

$$\mathbb{e}_{q\_,k\_}[x\_]:=\mathbb{e}^\wedge\left(\sum_{j=1}^{k+1}\frac{(1-q)^j x^j}{j\,(1-q^j)}\right);\ \mathbb{e}_{q\_}[x\_]:=\mathbb{e}_{q,\$k}[x]$$

```
Table[Series[𝕖_{q_ℏ,k}[x], {ϵ, 0, 4}], {k, 0, 5}] // Column
```

e^x

$e^x - \frac{1}{4}\left(e^x x^2 \gamma \hbar\right) \epsilon + \frac{1}{32} e^x x^4 \gamma^2 \hbar^2 \epsilon^2 - \frac{1}{384}\left(e^x x^2 \left(-8 + x^4\right) \gamma^3 \hbar^3\right) \epsilon^3 + \frac{e^x x^4 \left(-32+x^4\right) \gamma^4 \hbar^4 \epsilon^4}{6144} + O[\epsilon]^5$

$e^x - \frac{1}{4}\left(e^x x^2 \gamma \hbar\right) \epsilon + \frac{1}{288} e^x x^3 \left(32 + 9 x\right) \gamma^2 \hbar^2 \epsilon^2 - \frac{\left(e^x x^2 \left(-24+32 x^3+3 x^4\right) \gamma^3 \hbar^3\right) \epsilon^3}{1152} + \frac{e^x x^3 \left(-4608-864 x+1024 x^3+576 x^4+27 x^5\right) \gamma^4 \hbar^4 \epsilon^4}{165888} + O[\epsilon]^5$

$e^x - \frac{1}{4}\left(e^x x^2 \gamma \hbar\right) \epsilon + \frac{1}{288} e^x x^3 \left(32 + 9 x\right) \gamma^2 \hbar^2 \epsilon^2 - \frac{\left(e^x x^2 \left(-24+72 x^2+32 x^3+3 x^4\right) \gamma^3 \hbar^3\right) \epsilon^3}{1152} + \frac{e^x x^3 \left(-4608-864 x+3616 x^3+576 x^4+27 x^5\right) \gamma^4 \hbar^4 \epsilon^4}{165888} + O[\epsilon]^5$

$e^x - \frac{1}{4}\left(e^x x^2 \gamma \hbar\right) \epsilon + \frac{1}{288} e^x x^3 \left(32 + 9 x\right) \gamma^2 \hbar^2 \epsilon^2 - \frac{\left(e^x x^2 \left(-24+72 x^2+32 x^3+3 x^4\right) \gamma^3 \hbar^3\right) \epsilon^3}{1152} + \frac{1}{4147200} e^x x^3 \left(-115200 - 21600 x + 165888 x^2 + 90400 x^3 + 14400 x^4 + 675 x^5\right) \gamma^4 \hbar^4 \epsilon^4 + O[\epsilon]^5$

$e^x - \frac{1}{4}\left(e^x x^2 \gamma \hbar\right) \epsilon + \frac{1}{288} e^x x^3 \left(32 + 9 x\right) \gamma^2 \hbar^2 \epsilon^2 - \frac{\left(e^x x^2 \left(-24+72 x^2+32 x^3+3 x^4\right) \gamma^3 \hbar^3\right) \epsilon^3}{1152} + \frac{1}{4147200} e^x x^3 \left(-115200 - 21600 x + 165888 x^2 + 90400 x^3 + 14400 x^4 + 675 x^5\right) \gamma^4 \hbar^4 \epsilon^4 + O[\epsilon]^5$

```
Table[Together@SeriesCoefficient[𝕖_{q,5}[x], {x, 0, n}], {n, 0, 5}]
```

$\left\{1, 1, \frac{1}{1+q}, \frac{1}{(1+q)\left(1+q+q^2\right)}, \frac{1}{(1+q)^2\left(1+q^2\right)\left(1+q+q^2\right)}, 1\Big/\left((1+q)^2\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\right\}$

```
Table[HL@FunctionExpand[QFactorial[n, q] SeriesCoefficient[𝕖_{q,5}[x], {x, 0, n}]], {n, 0, 5}]
```

{1, 1, 1, 1, 1, 1}

R

```
QU[R_{i_,j_}] := 𝕆_QU[{y1, a1}_i, {a2, x2}_j, SS[𝕖^{ℏ b1 a2} 𝕖_{q_ℏ}[ℏ y1 x2] /. b1 → γ^-1 (ϵ a1 - t_i)]];
QU[R^-1_{i_,j_}] := S_j@QU[R_{i,j}];
```

```
QU[R_{3,4}] // Short
```

$QU[\,] + \hbar\, QU[y_3, x_4] + \frac{1}{2}\hbar^2 QU[y_3, y_3, x_4, x_4] - \frac{\hbar\, QU[a_4]\, t_3}{\gamma} - \frac{\hbar^2 QU[y_3, a_4, x_4]\, t_3}{\gamma} + \frac{\hbar^2 QU[a_4, a_4]\, t_3^2}{2\gamma^2}$

Verifying R2 (~2 secs @ $p=4, $k=2):

```
QU[R_{1,2} ** R^-1_{1,2}] // Simp // HL // Timing
```

{0.09375, QU[]}

Verifying R3 (~156 secs @ $p=4, $k=2):

```
{Short[lhs = QU[R_{1,2} ** R_{1,3} ** R_{2,3}]], HL@SimpT[lhs - QU[R_{2,3} ** R_{1,3} ** R_{1,2}]]} // Timing
```

$\left\{0.1875, \left\{QU[\,] + \hbar\, QU[y_1, x_2] + \hbar\, QU[y_1, x_3] + \ll 38 \gg + \frac{\hbar^2 QU[a_3, a_3]\, t_2^2}{2\gamma^2} + QU[y_1, x_3]\,(\hbar - \hbar T_2), 0\right\}\right\}$

R in $\mathbb{OE}_{QU}$.

RinOE

```
𝔼QU,k_[Ri_,j_] := 𝔼QU[{yi, ai, xi}i, {yj, aj, xj}j, -ℏ γ^-1 ti aj + ℏ yi xj,
   Series[𝕖^(ℏ γ^-1 ti aj - ℏ yi xj) (𝕖^(ℏ bi aj) 𝕖q_ℏ,k[ℏ yi xj] /. bi → γ^-1 (ϵ ai - ti)), {ϵ, 0, k}]]
```

```
{𝔼QU,1[R1,2], 𝔼QU,2[R1,2]}
```

$\{\mathbb{E}_{QU}[\{y_1, a_1, x_1\}_1, \{y_2, a_2, x_2\}_2, -\frac{\hbar a_2 t_1}{\gamma} + \hbar x_2 y_1, 1 + \left(\frac{\hbar a_1 a_2}{\gamma} - \frac{1}{4}\gamma \hbar^3 x_2^2 y_1^2\right) \epsilon + O[\epsilon]^2],$

$\mathbb{E}_{QU}[\{y_1, a_1, x_1\}_1, \{y_2, a_2, x_2\}_2, -\frac{\hbar a_2 t_1}{\gamma} + \hbar x_2 y_1, 1 + \left(\frac{\hbar a_1 a_2}{\gamma} - \frac{1}{4}\gamma \hbar^3 x_2^2 y_1^2\right) \epsilon +$

$\frac{1}{288 \gamma^2}\left(144 \hbar^2 a_1^2 a_2^2 - 72 \gamma^2 \hbar^4 a_1 a_2 x_2^2 y_1^2 + 32 \gamma^4 \hbar^5 x_2^3 y_1^3 + 9 \gamma^4 \hbar^6 x_2^4 y_1^4\right) \epsilon^2 + O[\epsilon]^3]\}$

Alternative Algorithms

AltLogos

```
λalt,k_[CU] := If[k == 0, 1, Module[{eq, d, b, c, so},
    eq = ρ@𝕖^(ξ xCU).ρ@𝕖^(η yCU) == ρ@𝕖^(d yCU).ρ@𝕖^(c (t 1CU - 2 ϵ aCU)).ρ@𝕖^(b xCU);
    {so} = Solve[Thread[Flatten /@ eq], {d, b, c}] /. C@1 → 0;
    Series[𝕖^(-η y - ξ x + η ξ t + c t + d y - 2 ϵ c a + b x) /. so, {ϵ, 0, k}]]];
```

```
{λalt,2[CU], HL@Simplify@Normal[λalt,2[CU] == Last[ΛCU,2[{ξ, η}, {x, y}]]]}
```

$\{1 + \left(2 a \eta \xi - y \gamma \eta^2 \xi - x \gamma \eta \xi^2 + \frac{1}{2} t \gamma \eta^2 \xi^2\right) \epsilon +$

$\frac{1}{2}\left(\left(2 a \eta \xi - y \gamma \eta^2 \xi - x \gamma \eta \xi^2 + \frac{1}{2} t \gamma \eta^2 \xi^2\right)^2 + 2\left(-a \gamma \eta^2 \xi^2 + y \gamma^2 \eta^3 \xi^2 + x \gamma^2 \eta^2 \xi^3 - \frac{1}{3} t \gamma^2 \eta^3 \xi^3\right)\right)$

$\epsilon^2 + O[\epsilon]^3$, True}