$In[\bullet]:=$ **{U, kk, υ, u} = {QU, 1, ξ, x}**

$Out[\bullet]=$ {QU, 1, ξ, x}

$In[\bullet]:=$ **fs = Flatten@Table[$f_{i,j}[\upsilon]$, {i, 0, 2 kk}, {j, 0, 2 kk – i}]**

$Out[\bullet]=$ {$f_{0,0}[\xi]$, $f_{0,1}[\xi]$, $f_{0,2}[\xi]$, $f_{1,0}[\xi]$, $f_{1,1}[\xi]$, $f_{2,0}[\xi]$}

$In[\bullet]:=$ **F = 1 + ϵ^{kk} fs. (fs /. $f_{i\_,j\_}[\upsilon] :\!\to a^i u^j$)**

$Out[\bullet]=$ $1 + \epsilon \left(f_{0,0}[\xi] + x\, f_{0,1}[\xi] + x^2\, f_{0,2}[\xi] + a\, f_{1,0}[\xi] + a\, x\, f_{1,1}[\xi] + a^2\, f_{2,0}[\xi] \right)$

$In[\bullet]:=$ **S[u_U]**

$Out[\bullet]=$ $-QU[x] - \epsilon\, \hbar\, QU[a, x]$

$In[\bullet]:=$ **qq = Coefficient[$(s = S[u_U])$ /. $\epsilon \to 0$, u_U]**

$Out[\bullet]=$ -1

$In[\bullet]:=$ **Expand[s – qq u_U /. U $\to$ Times]**

$Out[\bullet]=$ $-a\, x\, \epsilon\, \hbar$

$In[\bullet]:=$ **rhs = Normal@Last@Cord$\big[$ Œ$_U\big[$ {y_1, a_1, x_1, y_2, a_2, x_2}$_1$, q υ u_1,**
(F /. (w : y | a | x) $:\!\to w_1$) (s – qq u_U /. {U $\to$ Times, (w : y | a | x) $:\!\to w_2$}) + 0$_{kk}$ $\big]$ /.
$\upsilon \to \hbar\, \upsilon\big]$ /. $\{\upsilon \to \hbar^{-1}\, \upsilon$, ($w$: y | a | x)$_1 :\!\to w\}$

» {x_1, y_2}

» Œ$_{QU}\left[\{y_1, a_1, y_{\$18}, a_{\$18}, x_{\$18}, a_2, x_2\}_1, q\, \xi\, \hbar\, x_{\$18}, -\hbar\, a_2\, x_2\, \epsilon + O[\epsilon]^2 \right]$

» {a_1, $y_{\$18}$}

» Œ$_{QU}\left[\{y_1, y_{\$19}, a_{\$19}, a_{\$18}, x_{\$18}, a_2, x_2\}_1, q\, \xi\, \hbar\, x_{\$18}, -\hbar\, a_2\, x_2\, \epsilon + O[\epsilon]^2 \right]$

» {y_1, $y_{\$19}$}

» Œ$_{QU}\left[\{y_{\$20}, a_{\$19}, a_{\$18}, x_{\$18}, a_2, x_2\}_1, q\, \xi\, \hbar\, x_{\$18}, -\hbar\, a_2\, x_2\, \epsilon + O[\epsilon]^2 \right]$

» {$a_{\$19}$, $a_{\$18}$}

» Œ$_{QU}\left[\{y_{\$20}, a_{\$21}, x_{\$18}, a_2, x_2\}_1, q\, \xi\, \hbar\, x_{\$18}, -\hbar\, a_2\, x_2\, \epsilon + O[\epsilon]^2 \right]$

» {$x_{\$18}$, a_2}

» Œ$_{QU}\left[\{y_{\$20}, a_{\$21}, a_{\$22}, x_{\$22}, x_2\}_1, q\, \xi\, \hbar\, x_{\$22}, \left(-\hbar\, a_{\$22}\, x_2 + q\, \gamma\, \xi\, \hbar^2\, x_2\, x_{\$22}\right) \epsilon + O[\epsilon]^2 \right]$

» {$a_{\$21}$, $a_{\$22}$}

» Œ$_{QU}\left[\{y_{\$20}, a_{\$23}, x_{\$22}, x_2\}_1, q\, \xi\, \hbar\, x_{\$22}, \left(-\hbar\, a_{\$23}\, x_2 + q\, \gamma\, \xi\, \hbar^2\, x_2\, x_{\$22}\right) \epsilon + O[\epsilon]^2 \right]$

» {$x_{\$22}$, x_2}

» Œ$_{QU}\left[\{y_{\$20}, a_{\$23}, x_{\$24}\}_1, q\, \xi\, \hbar\, x_{\$24}, \left(-\hbar\, a_{\$23}\, x_{\$24} + q\, \gamma\, \xi\, \hbar^2\, x_{\$24}^2\right) \epsilon + O[\epsilon]^2 \right]$

$Out[\bullet]=$ $\epsilon \left(-a\, x\, \hbar + q\, x^2\, \gamma\, \xi\, \hbar \right)$

$In[\bullet]:=$ **at0 = $\big(\# == 0\big)$ & /@ Flatten@CoefficientList[F – 1 /. $\upsilon \to 0$, {a, u}]**

$Out[\bullet]=$ {$\epsilon\, f_{0,0}[0] == 0$, $\epsilon\, f_{0,1}[0] == 0$, $\epsilon\, f_{0,2}[0] == 0$,
$\epsilon\, f_{1,0}[0] == 0$, $\epsilon\, f_{1,1}[0] == 0$, True, $\epsilon\, f_{2,0}[0] == 0$, True, True}

In[◦]:= $\mathbf{at}\upsilon$ = $\big(\#$ == $\mathbf{0}\big)$ & /@ Flatten@CoefficientList[$(\partial_\upsilon F)$ + qq F − rhs, {a, u}]

Out[◦]= $\{-\mathbf{1} - \in f_{0,0}[\xi] + \in f_{0,0}{}'[\xi]$ == $\mathbf{0},\ -\in f_{0,1}[\xi] + \in f_{0,1}{}'[\xi]$ == $\mathbf{0},$
  $-q\,\gamma \in \xi\,\hbar - \in f_{0,2}[\xi] + \in f_{0,2}{}'[\xi]$ == $\mathbf{0},\ -\in f_{1,0}[\xi] + \in f_{1,0}{}'[\xi]$ == $\mathbf{0},$
  $\in \hbar - \in f_{1,1}[\xi] + \in f_{1,1}{}'[\xi]$ == $\mathbf{0},\ \text{True},\ -\in f_{2,0}[\xi] + \in f_{2,0}{}'[\xi]$ == $\mathbf{0},\ \text{True},\ \text{True}\}$

In[◦]:= Expand$\big[$F /. DSolve$\big[$And @@ $\big($at0 $\bigcup$ at$\upsilon\big)$, fs, $\upsilon\big]$ ⟦1⟧$\big]$

Out[◦]= $\mathrm{e}^{\xi} + a\,x \in \hbar - a\,\mathrm{e}^{\xi}\,x \in \hbar - q\,x^2\,\gamma \in \hbar + \mathrm{e}^{\xi}\,q\,x^2\,\gamma \in \hbar - q\,x^2\,\gamma \in \xi\,\hbar$