

Pensieve header: km with up to 15 inputs; CF based on CoefficientRules.

```
In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
<< SL2Invariant.m

Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: June 2018. Original version: July 1994.

In[2]:= CCF[ε_] := PPCCF@Expand[PPExp[
  Expand[ε] //. ex_. ey_. :> ex+y /. ex_ :> eCCF[x]];
CF[ε_List] := CF/@ε;
CF[ε_] := PPCF@Module[
  {vs = Cases[ε, (y | b | t | a | x | η | β | τ | α | ε)_, ∞] ∪ {y, b, t, a, x, η, β, τ, α, ε}, 
   Total[CoefficientRules[Expand[ε], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vsps)]
  }];
In[3]:= $k = 1; Clear[km];

In[4]:= km[1] = IE{1}→{1}[a1 α1 + t1 τ1, x1 ε1 + y1 η1, 1];
km[n_Integer] /; n > 1 := km[n] = km[n - 1] // km1,n→1
```

$\ln[f^{\#}] := \text{km}[4]$

$$\begin{aligned}
& \text{Out}[f^{\#}] = \mathbb{E}_{\{1, 2, 3, 4\} \rightarrow \{1\}} \left[ \mathbf{a}_1 \alpha_1 + \mathbf{a}_1 \alpha_2 + \mathbf{a}_1 \alpha_3 + \mathbf{a}_1 \alpha_4 + \mathbf{t} \tau_1 + \mathbf{t} \tau_2 + \mathbf{t} \tau_3 + \mathbf{t} \tau_4, \mathbf{y}_1 \eta_1 + \frac{\mathbf{y}_1 \eta_2}{\mathcal{A}_1} + \frac{\mathbf{y}_1 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} + \right. \\
& \quad \frac{\mathbf{y}_1 \eta_4}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \xi_1}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \left( \frac{1}{\hbar} - \frac{T}{\hbar} \right) \eta_2 \xi_1 + \left( \frac{1}{\hbar \mathcal{A}_2} - \frac{T}{\hbar \mathcal{A}_2} \right) \eta_3 \xi_1 + \left( \frac{1}{\hbar \mathcal{A}_2 \mathcal{A}_3} - \frac{T}{\hbar \mathcal{A}_2 \mathcal{A}_3} \right) \eta_4 \xi_1 + \\
& \quad \frac{\mathbf{x}_1 \xi_2}{\mathcal{A}_3 \mathcal{A}_4} + \left( \frac{1}{\hbar} - \frac{T}{\hbar} \right) \eta_3 \xi_2 + \left( \frac{1}{\hbar \mathcal{A}_3} - \frac{T}{\hbar \mathcal{A}_3} \right) \eta_4 \xi_2 + \frac{\mathbf{x}_1 \xi_3}{\mathcal{A}_4} + \left( \frac{1}{\hbar} - \frac{T}{\hbar} \right) \eta_4 \xi_3 + \mathbf{x}_1 \xi_4, \\
& \quad 1 + \left( 2 T \mathbf{a}_1 \eta_2 \xi_1 + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_2 \xi_1}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \mathbf{y}_1 \left( \frac{\gamma}{2 \mathcal{A}_1} - \frac{3 T \gamma}{2 \mathcal{A}_1} \right) \eta_2^2 \xi_1 + \frac{2 T \mathbf{a}_1 \eta_3 \xi_1}{\mathcal{A}_2} + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} + \right. \\
& \quad \mathbf{y}_1 \left( \frac{\gamma}{\mathcal{A}_1 \mathcal{A}_2} - \frac{3 T \gamma}{\mathcal{A}_1 \mathcal{A}_2} \right) \eta_2 \eta_3 \xi_1 + \mathbf{y}_1 \left( \frac{\gamma}{2 \mathcal{A}_1 \mathcal{A}_2^2} - \frac{3 T \gamma}{2 \mathcal{A}_1 \mathcal{A}_2^2} \right) \eta_3^2 \xi_1 + \frac{2 T \mathbf{a}_1 \eta_4 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \\
& \quad \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_4 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3^2 \mathcal{A}_4} + \mathbf{y}_1 \left( \frac{\gamma}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} - \frac{3 T \gamma}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} \right) \eta_2 \eta_4 \xi_1 + \mathbf{y}_1 \left( \frac{\gamma}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3} - \frac{3 T \gamma}{\mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3} \right) \eta_3 \eta_4 \xi_1 + \\
& \quad \mathbf{y}_1 \left( \frac{\gamma}{2 \mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3^2} - \frac{3 T \gamma}{2 \mathcal{A}_1 \mathcal{A}_2^2 \mathcal{A}_3^2} \right) \eta_4^2 \xi_1 + \mathbf{x}_1 \left( \frac{\gamma}{2 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{2 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} \right) \eta_2 \xi_1^2 + \left( \frac{\gamma}{4 \hbar} - \frac{T \gamma}{\hbar} + \frac{3 T^2 \gamma}{4 \hbar} \right) \eta_2^2 \xi_1^2 + \\
& \quad \mathbf{x}_1 \left( \frac{\gamma}{2 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{2 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} \right) \eta_3 \xi_1^2 + \left( \frac{\gamma}{2 \hbar \mathcal{A}_2} - \frac{2 T \gamma}{\hbar \mathcal{A}_2} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_2} \right) \eta_2 \eta_3 \xi_1^2 + \\
& \quad \left( \frac{\gamma}{4 \hbar \mathcal{A}_2^2} - \frac{T \gamma}{\hbar \mathcal{A}_2^2} + \frac{3 T^2 \gamma}{4 \hbar \mathcal{A}_2^2} \right) \eta_3^2 \xi_1^2 + \mathbf{x}_1 \left( \frac{\gamma}{2 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{2 \mathcal{A}_2^2 \mathcal{A}_3 \mathcal{A}_4} \right) \eta_4 \xi_1^2 + \\
& \quad \left( \frac{\gamma}{2 \hbar \mathcal{A}_2 \mathcal{A}_3} - \frac{2 T \gamma}{\hbar \mathcal{A}_2 \mathcal{A}_3} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_2 \mathcal{A}_3} \right) \eta_2 \eta_4 \xi_1^2 + \left( \frac{\gamma}{2 \hbar \mathcal{A}_2^2 \mathcal{A}_3} - \frac{2 T \gamma}{\hbar \mathcal{A}_2^2 \mathcal{A}_3} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_2^2 \mathcal{A}_3} \right) \eta_3 \eta_4 \xi_1^2 + \\
& \quad \left( \frac{\gamma}{4 \hbar \mathcal{A}_2^2 \mathcal{A}_3^2} - \frac{T \gamma}{\hbar \mathcal{A}_2^2 \mathcal{A}_3^2} + \frac{3 T^2 \gamma}{4 \hbar \mathcal{A}_2^2 \mathcal{A}_3^2} \right) \eta_4^2 \xi_1^2 + 2 T \mathbf{a}_1 \eta_3 \xi_2 + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_3 \xi_2}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \\
& \quad \mathbf{y}_1 \left( \frac{\gamma}{2 \mathcal{A}_1 \mathcal{A}_2} - \frac{3 T \gamma}{2 \mathcal{A}_1 \mathcal{A}_2} \right) \eta_3^2 \xi_2 + \frac{2 T \mathbf{a}_1 \eta_4 \xi_2}{\mathcal{A}_3} + \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_4 \xi_2}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3^2 \mathcal{A}_4} + \mathbf{y}_1 \left( \frac{\gamma}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} - \frac{3 T \gamma}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} \right) \eta_3 \eta_4 \xi_2 + \\
& \quad \mathbf{y}_1 \left( \frac{\gamma}{2 \mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3^2} - \frac{3 T \gamma}{2 \mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3^2} \right) \eta_4^2 \xi_2 + \mathbf{x}_1 \left( \frac{\gamma}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} \right) \eta_3 \xi_1 \xi_2 + \\
& \quad \left( \frac{\gamma}{2 \hbar \mathcal{A}_2} - \frac{2 T \gamma}{\hbar \mathcal{A}_2} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_2} \right) \eta_3^2 \xi_1 \xi_2 + \mathbf{x}_1 \left( \frac{\gamma}{\mathcal{A}_2 \mathcal{A}_3^2 \mathcal{A}_4} - \frac{3 T \gamma}{\mathcal{A}_2 \mathcal{A}_3^2 \mathcal{A}_4} \right) \eta_4 \xi_1 \xi_2 + \\
& \quad \left( \frac{\gamma}{\hbar \mathcal{A}_2 \mathcal{A}_3} - \frac{4 T \gamma}{\hbar \mathcal{A}_2 \mathcal{A}_3} + \frac{3 T^2 \gamma}{\hbar \mathcal{A}_2 \mathcal{A}_3} \right) \eta_3 \eta_4 \xi_1 \xi_2 + \left( \frac{\gamma}{2 \hbar \mathcal{A}_2 \mathcal{A}_3^2} - \frac{2 T \gamma}{\hbar \mathcal{A}_2 \mathcal{A}_3^2} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_2 \mathcal{A}_3^2} \right) \eta_4^2 \xi_1 \xi_2 + \\
& \quad \mathbf{x}_1 \left( \frac{\gamma}{2 \mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{2 \mathcal{A}_3 \mathcal{A}_4} \right) \eta_3 \xi_2^2 + \left( \frac{\gamma}{4 \hbar} - \frac{T \gamma}{\hbar} + \frac{3 T^2 \gamma}{4 \hbar} \right) \eta_3^2 \xi_2^2 + \mathbf{x}_1 \left( \frac{\gamma}{2 \mathcal{A}_3^2 \mathcal{A}_4} - \frac{3 T \gamma}{2 \mathcal{A}_3^2 \mathcal{A}_4} \right) \eta_4 \xi_2^2 + \\
& \quad \left( \frac{\gamma}{2 \hbar \mathcal{A}_3} - \frac{2 T \gamma}{\hbar \mathcal{A}_3} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_3} \right) \eta_3 \eta_4 \xi_2^2 + \left( \frac{\gamma}{4 \hbar \mathcal{A}_3^2} - \frac{T \gamma}{\hbar \mathcal{A}_3^2} + \frac{3 T^2 \gamma}{4 \hbar \mathcal{A}_3^2} \right) \eta_4^2 \xi_2^2 + 2 T \mathbf{a}_1 \eta_4 \xi_3 + \\
& \quad \frac{\gamma \hbar \mathbf{x}_1 \mathbf{y}_1 \eta_4 \xi_3}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \mathbf{y}_1 \left( \frac{\gamma}{2 \mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} - \frac{3 T \gamma}{2 \mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} \right) \eta_4^2 \xi_3 + \mathbf{x}_1 \left( \frac{\gamma}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} \right) \eta_4 \xi_1 \xi_3 + \\
& \quad \left( \frac{\gamma}{2 \hbar \mathcal{A}_2 \mathcal{A}_3} - \frac{2 T \gamma}{\hbar \mathcal{A}_2 \mathcal{A}_3} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_2 \mathcal{A}_3} \right) \eta_4^2 \xi_1 \xi_3 + \mathbf{x}_1 \left( \frac{\gamma}{\mathcal{A}_3 \mathcal{A}_4} - \frac{3 T \gamma}{\mathcal{A}_3 \mathcal{A}_4} \right) \eta_4 \xi_2 \xi_3 + \\
& \quad \left( \frac{\gamma}{2 \hbar \mathcal{A}_3} - \frac{2 T \gamma}{\hbar \mathcal{A}_3} + \frac{3 T^2 \gamma}{2 \hbar \mathcal{A}_3} \right) \eta_4^2 \xi_2 \xi_3 + \mathbf{x}_1 \left( \frac{\gamma}{2 \mathcal{A}_4} - \frac{3 T \gamma}{2 \mathcal{A}_4} \right) \eta_4 \xi_3^2 + \left( \frac{\gamma}{4 \hbar} - \frac{T \gamma}{\hbar} + \frac{3 T^2 \gamma}{4 \hbar} \right) \eta_4^2 \xi_3^2 \Big] \in + \mathbf{0} [\epsilon]^2
\end{aligned}$$

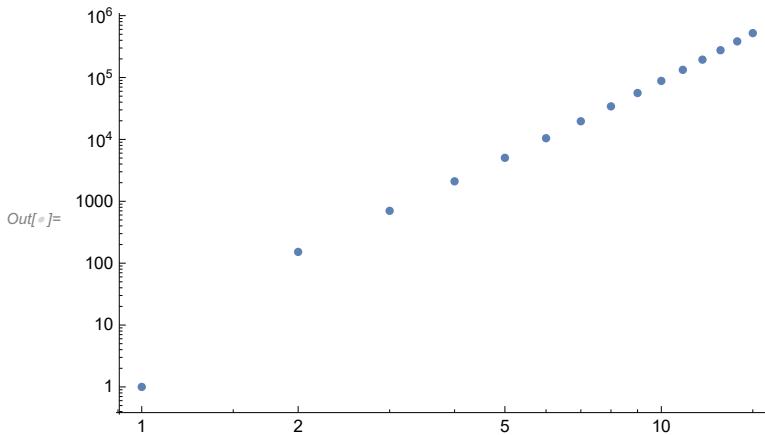
```
In[=]:= Table[Echo[Prepend[LeafCount /@ List @@ km[n], n]], {n, 15}] // MatrixForm
" {1, 13, 15, 1}
" {2, 25, 56, 152}
" {3, 37, 135, 701}
" {4, 49, 262, 2104}
" {5, 61, 447, 5039}
" {6, 73, 700, 10459}
" {7, 85, 1031, 19637}
" {8, 97, 1450, 34211}
" {9, 109, 1967, 56229}
" {10, 121, 2592, 88194}
" {11, 133, 3335, 133109}
" {12, 145, 4206, 194522}
" {13, 157, 5215, 276571}
" {14, 169, 6372, 384029}
" {15, 181, 7687, 522349}

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 13 & 15 & 1 \\ 2 & 25 & 56 & 152 \\ 3 & 37 & 135 & 701 \\ 4 & 49 & 262 & 2104 \\ 5 & 61 & 447 & 5039 \\ 6 & 73 & 700 & 10459 \\ 7 & 85 & 1031 & 19637 \\ 8 & 97 & 1450 & 34211 \\ 9 & 109 & 1967 & 56229 \\ 10 & 121 & 2592 & 88194 \\ 11 & 133 & 3335 & 133109 \\ 12 & 145 & 4206 & 194522 \\ 13 & 157 & 5215 & 276571 \\ 14 & 169 & 6372 & 384029 \\ 15 & 181 & 7687 & 522349 \end{pmatrix}$$

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```
In[6]:= ListLogLogPlot[Table[LeafCount[km[n][3]], {n, 15}]]
```



```
In[7]:= N@Table[ $\frac{\text{Log}@{\text{LeafCount}[\text{km}[n][3]]}}{\text{Log}[n]}$ , {n, 2, 15}]
```

```
Out[7]= {7.24793, 5.96435, 5.51946, 5.29686, 5.16544, 5.07997,  
5.02072, 4.97773, 4.94544, 4.92053, 4.90091, 4.88517, 4.87237, 4.86183}
```

```
In[6]:= PrintProfile[]

Out[6]= ProfileRoot is root. Profiled time: 234.094
( 15) 0.688/ 231.950 above B
( 1) 0.016/ 2.141 above Boot[1]
CF: called 1272 times, time in 86.437/95.955
( 240) 55.573/ 62.954 under LZip
( 1032) 30.864/ 33.001 under QZip
( 31484) 2.313/ 9.518 above CCF
LZip: called 31 times, time in 59.362/125.051
( 31) 59.362/ 125.050 under B
( 240) 55.573/ 62.954 above CF
( 30) 0.656/ 2.735 above Zip
QZip: called 30 times, time in 42.089/108.292
( 30) 42.089/ 108.290 under B
( 1032) 30.864/ 33.001 above CF
( 30) 26.219/ 33.202 above Zip
Zip: called 235 times, time in 29./40.933
( 30) 0.656/ 2.735 under LZip
( 30) 26.219/ 33.202 under QZip
( 175) 2.125/ 4.996 under Zip
( 235) 6.937/ 6.937 above Collect
( 175) 2.125/ 4.996 above Zip
Collect: called 235 times, time in 6.937/6.937
( 235) 6.937/ 6.937 under Zip
Exp: called 59820 times, time in 6.209/8.596
( 59820) 6.209/ 8.596 under CCF
( 28336) 0.996/ 2.387 above CCF
CCF: called 59820 times, time in 3.309/11.905
( 31484) 2.313/ 9.518 under CF
( 28336) 0.996/ 2.387 under Exp
( 59820) 6.209/ 8.596 above Exp
B: called 31 times, time in 0.688/234.031
( 15) 0.688/ 231.950 under ProfileRoot
( 16) 0/ 2.078 under Boot[1]
( 31) 59.362/ 125.050 above LZip
( 30) 42.089/ 108.290 above QZip
Boot[1]: called 13 times, time in 0.063/5.249
( 1) 0.016/ 2.141 under ProfileRoot
( 12) 0.047/ 3.108 under Boot[1]
( 16) 0/ 2.078 above B
( 3) 0/ 0 above Boot[0]
( 12) 0.047/ 3.108 above Boot[1]
Boot[0]: called 3 times, time in 0./0.
( 3) 0/ 0 under Boot[1]
```

The Trefoil

```
In[1]:= Timing@Block[{$k = 1},  

Z = R1,5 R6,2 R3,7 C4 Kink8 Kink9 Kink10;  

Do[Z = Z ~ B1,r ~ dm1,r → 1, {r, 2, 10}];  

Simplify /@ Z, Simplify /@ (Z ~ B1 ~ b2t1 /. T1 → T)]]  

Out[1]= {18.1406, {E{ } → {1} [0, 0,  $\frac{B_1}{1 - B_1 + B_1^2}$  -  

 $\frac{1}{(1 - B_1 + B_1^2)^3} \hbar B_1 (-a_1 (-1 + B_1 - B_1^3 + B_1^4) + \gamma (B_1 - 2 B_1^2 - 2 B_1^4 + 2 \hbar x_1 y_1 + B_1^3 (3 + 2 \hbar x_1 y_1))) \in +$   

0[ε]2], E{ } → {1} [0, 0,  $\frac{T}{1 - T + T^2} + \frac{1}{(1 - T + T^2)^3}$   

T  $\hbar (T (-1 + 2 T - 3 T^2 + 2 T^3) \gamma + 2 (-1 + T - T^3 + T^4) a_1 - 2 (1 + T^3) \gamma \hbar x_1 y_1) \in + 0[\epsilon]^2$ ]}}
```

```
In[2]:= Timing@Block[{$k = 1},  

Z = kR1,5 kR6,2 kC4 KKink8 KKink9 KKink10;  

Do[Z = Z ~ B1,r ~ km1,r → 1, {r, 2, 10}];  

Simplify /@ Z]  

Out[2]= {9.67188, E{ } → {1} [0, 0,  $\frac{T}{1 - T + T^2} + \frac{1}{(1 - T + T^2)^3}$   

T  $\hbar (T (-1 + 2 T - 3 T^2 + 2 T^3) \gamma + 2 (-1 + T - T^3 + T^4) a_1 - 2 (1 + T^3) \gamma \hbar x_1 y_1) \in + 0[\epsilon]^2$ ]}
```

In[3]:= **Z@Knot** [10, 100]

Part: The expression Knot[10, 100] cannot be used as a part specification.

```
Out[3]= {0,  $\left( -\hbar - T^2 \hbar + \frac{\hbar}{1 - T + T^2} - \frac{T \hbar}{1 - T + T^2} + \frac{2 T^2 \hbar}{1 - T + T^2} - \frac{T^3 \hbar}{1 - T + T^2} + \frac{T^4 \hbar}{1 - T + T^2} \right) x_1 y_1,$   

 $\left( \frac{T}{(1 - T + T^2)^3} - \frac{2 T^2}{(1 - T + T^2)^3} + \frac{3 T^3}{(1 - T + T^2)^3} - \frac{2 T^4}{(1 - T + T^2)^3} + \frac{T^5}{(1 - T + T^2)^3} \right) +$   

 $\left( -\frac{T^4 \gamma \hbar}{(1 - T + T^2)^5} + \frac{3 T^5 \gamma \hbar}{(1 - T + T^2)^5} - \frac{5 T^6 \gamma \hbar}{(1 - T + T^2)^5} + \frac{5 T^7 \gamma \hbar}{(1 - T + T^2)^5} - \frac{3 T^8 \gamma \hbar}{(1 - T + T^2)^5} + \frac{T^9 \gamma \hbar}{(1 - T + T^2)^5} -$   

 $\frac{T^2 \gamma \hbar}{(1 - T + T^2)^4} + \frac{3 T^3 \gamma \hbar}{(1 - T + T^2)^4} - \frac{5 T^4 \gamma \hbar}{(1 - T + T^2)^4} + \frac{5 T^5 \gamma \hbar}{(1 - T + T^2)^4} - \frac{3 T^6 \gamma \hbar}{(1 - T + T^2)^4} + \frac{T^7 \gamma \hbar}{(1 - T + T^2)^4} +$   

 $\left( -\frac{2 T^2 \hbar}{(1 - T + T^2)^4} + \frac{8 T^3 \hbar}{(1 - T + T^2)^4} - \frac{14 T^4 \hbar}{(1 - T + T^2)^4} + \frac{16 T^5 \hbar}{(1 - T + T^2)^4} - \frac{10 T^6 \hbar}{(1 - T + T^2)^4} + \frac{4 T^7 \hbar}{(1 - T + T^2)^4} -$   

 $\frac{2 T \hbar}{(1 - T + T^2)^3} + \frac{4 T^2 \hbar}{(1 - T + T^2)^3} - \frac{6 T^3 \hbar}{(1 - T + T^2)^3} + \frac{4 T^4 \hbar}{(1 - T + T^2)^3} - \frac{2 T^5 \hbar}{(1 - T + T^2)^3} \right) a_1 +$   

 $\left( \frac{T^2 \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{8 T^3 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{26 T^4 \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{53 T^5 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{71 T^6 \gamma \hbar^2}{(1 - T + T^2)^6} -$   

 $\frac{62 T^7 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{26 T^8 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{13 T^9 \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{31 T^{10} \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{26 T^{11} \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{12 T^{12} \gamma \hbar^2}{(1 - T + T^2)^6} +$   

 $\frac{3 T^{13} \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{T^2 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{5 T^3 \gamma \hbar^2}{(1 - T + T^2)^5} - \frac{10 T^4 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{5 T^5 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{16 T^6 \gamma \hbar^2}{(1 - T + T^2)^5} -$ 
```

$$\begin{aligned}
& \left( \frac{47 T^7 \gamma h^2}{(1 - T + T^2)^5} + \frac{61 T^8 \gamma h^2}{(1 - T + T^2)^5} - \frac{52 T^9 \gamma h^2}{(1 - T + T^2)^5} + \frac{26 T^{10} \gamma h^2}{(1 - T + T^2)^5} - \frac{8 T^{11} \gamma h^2}{(1 - T + T^2)^5} - \right. \\
& \left. \frac{2 T^2 \gamma h^2}{(1 - T + T^2)^4} + \frac{8 T^3 \gamma h^2}{(1 - T + T^2)^4} - \frac{16 T^4 \gamma h^2}{(1 - T + T^2)^4} + \frac{27 T^5 \gamma h^2}{(1 - T + T^2)^4} - \frac{30 T^6 \gamma h^2}{(1 - T + T^2)^4} + \right. \\
& \left. \frac{29 T^7 \gamma h^2}{(1 - T + T^2)^4} - \frac{16 T^8 \gamma h^2}{(1 - T + T^2)^4} + \frac{7 T^9 \gamma h^2}{(1 - T + T^2)^4} - \frac{2 T \gamma h^2}{(1 - T + T^2)^3} + \frac{2 T^2 \gamma h^2}{(1 - T + T^2)^3} - \right. \\
& \left. \frac{4 T^3 \gamma h^2}{(1 - T + T^2)^3} + \frac{2 T^4 \gamma h^2}{(1 - T + T^2)^3} - \frac{4 T^5 \gamma h^2}{(1 - T + T^2)^3} + \frac{2 T^6 \gamma h^2}{(1 - T + T^2)^3} - \frac{2 T^7 \gamma h^2}{(1 - T + T^2)^3} \right) x_1 y_1 + \\
& \left( \frac{2 T^3 h^2}{(1 - T + T^2)^5} - \frac{8 T^4 h^2}{(1 - T + T^2)^5} + \frac{20 T^5 h^2}{(1 - T + T^2)^5} - \frac{32 T^6 h^2}{(1 - T + T^2)^5} + \frac{38 T^7 h^2}{(1 - T + T^2)^5} - \right. \\
& \left. \frac{32 T^8 h^2}{(1 - T + T^2)^5} + \frac{20 T^9 h^2}{(1 - T + T^2)^5} - \frac{8 T^{10} h^2}{(1 - T + T^2)^5} + \frac{2 T^{11} h^2}{(1 - T + T^2)^5} - \frac{6 T^3 h^2}{(1 - T + T^2)^4} + \frac{18 T^4 h^2}{(1 - T + T^2)^4} - \right. \\
& \left. \frac{36 T^5 h^2}{(1 - T + T^2)^4} + \frac{42 T^6 h^2}{(1 - T + T^2)^4} - \frac{36 T^7 h^2}{(1 - T + T^2)^4} + \frac{18 T^8 h^2}{(1 - T + T^2)^4} - \frac{6 T^9 h^2}{(1 - T + T^2)^4} + \right. \\
& \left. \frac{4 T^3 h^2}{(1 - T + T^2)^3} - \frac{8 T^4 h^2}{(1 - T + T^2)^3} + \frac{12 T^5 h^2}{(1 - T + T^2)^3} - \frac{8 T^6 h^2}{(1 - T + T^2)^3} + \frac{4 T^7 h^2}{(1 - T + T^2)^3} \right) a_1 x_1 y_1 + \\
& \left( \frac{T \gamma h^3}{4 (1 - T + T^2)^7} - \frac{2 T^2 \gamma h^3}{(1 - T + T^2)^7} + \frac{7 T^3 \gamma h^3}{(1 - T + T^2)^7} - \frac{15 T^4 \gamma h^3}{(1 - T + T^2)^7} + \frac{19 T^5 \gamma h^3}{(1 - T + T^2)^7} - \right. \\
& \left. \frac{8 T^6 \gamma h^3}{(1 - T + T^2)^7} - \frac{47 T^7 \gamma h^3}{2 (1 - T + T^2)^7} + \frac{63 T^8 \gamma h^3}{(1 - T + T^2)^7} - \frac{345 T^9 \gamma h^3}{4 (1 - T + T^2)^7} + \frac{76 T^{10} \gamma h^3}{(1 - T + T^2)^7} - \right. \\
& \left. \frac{79 T^{11} \gamma h^3}{2 (1 - T + T^2)^7} + \frac{T^{12} \gamma h^3}{(1 - T + T^2)^7} + \frac{39 T^{13} \gamma h^3}{2 (1 - T + T^2)^7} - \frac{20 T^{14} \gamma h^3}{(1 - T + T^2)^7} + \frac{23 T^{15} \gamma h^3}{2 (1 - T + T^2)^7} - \right. \\
& \left. \frac{4 T^{16} \gamma h^3}{(1 - T + T^2)^7} + \frac{3 T^{17} \gamma h^3}{4 (1 - T + T^2)^7} + \frac{T^2 \gamma h^3}{2 (1 - T + T^2)^6} - \frac{5 T^3 \gamma h^3}{2 (1 - T + T^2)^6} + \frac{6 T^4 \gamma h^3}{(1 - T + T^2)^6} - \right. \\
& \left. \frac{21 T^5 \gamma h^3}{2 (1 - T + T^2)^6} + \frac{15 T^6 \gamma h^3}{(1 - T + T^2)^6} - \frac{51 T^7 \gamma h^3}{2 (1 - T + T^2)^6} + \frac{45 T^8 \gamma h^3}{(1 - T + T^2)^6} - \frac{147 T^9 \gamma h^3}{2 (1 - T + T^2)^6} + \right. \\
& \left. \frac{93 T^{10} \gamma h^3}{(1 - T + T^2)^6} - \frac{185 T^{11} \gamma h^3}{2 (1 - T + T^2)^6} + \frac{68 T^{12} \gamma h^3}{(1 - T + T^2)^6} - \frac{75 T^{13} \gamma h^3}{2 (1 - T + T^2)^6} + \frac{27 T^{14} \gamma h^3}{2 (1 - T + T^2)^6} - \right. \\
& \left. \frac{3 T^{15} \gamma h^3}{(1 - T + T^2)^6} - \frac{T \gamma h^3}{2 (1 - T + T^2)^5} + \frac{2 T^2 \gamma h^3}{(1 - T + T^2)^5} - \frac{3 T^3 \gamma h^3}{4 (1 - T + T^2)^5} - \frac{6 T^4 \gamma h^3}{(1 - T + T^2)^5} + \right. \\
& \left. \frac{27 T^5 \gamma h^3}{(1 - T + T^2)^5} - \frac{54 T^6 \gamma h^3}{(1 - T + T^2)^5} + \frac{351 T^7 \gamma h^3}{4 (1 - T + T^2)^5} - \frac{105 T^8 \gamma h^3}{(1 - T + T^2)^5} + \frac{108 T^9 \gamma h^3}{(1 - T + T^2)^5} - \right. \\
& \left. \frac{83 T^{10} \gamma h^3}{(1 - T + T^2)^5} + \frac{209 T^{11} \gamma h^3}{4 (1 - T + T^2)^5} - \frac{21 T^{12} \gamma h^3}{(1 - T + T^2)^5} + \frac{6 T^{13} \gamma h^3}{(1 - T + T^2)^5} - \frac{6 T^3 \gamma h^3}{(1 - T + T^2)^4} + \right. \\
& \left. \frac{14 T^4 \gamma h^3}{(1 - T + T^2)^4} - \frac{30 T^5 \gamma h^3}{(1 - T + T^2)^4} + \frac{36 T^6 \gamma h^3}{(1 - T + T^2)^4} - \frac{44 T^7 \gamma h^3}{(1 - T + T^2)^4} + \frac{36 T^8 \gamma h^3}{(1 - T + T^2)^4} - \right.
\end{aligned}$$

$$\left. \left( \frac{30 T^9 \gamma h^3}{(1 - T + T^2)^4} + \frac{14 T^{10} \gamma h^3}{(1 - T + T^2)^4} - \frac{6 T^{11} \gamma h^3}{(1 - T + T^2)^4} + \frac{T \gamma h^3}{4 (1 - T + T^2)^3} - \frac{T^2 \gamma h^3}{2 (1 - T + T^2)^3} + \right. \right. \\ \left. \left. \frac{15 T^3 \gamma h^3}{4 (1 - T + T^2)^3} - \frac{9 T^4 \gamma h^3}{2 (1 - T + T^2)^3} + \frac{15 T^5 \gamma h^3}{2 (1 - T + T^2)^3} - \frac{9 T^6 \gamma h^3}{2 (1 - T + T^2)^3} + \frac{23 T^7 \gamma h^3}{4 (1 - T + T^2)^3} - \right. \right. \\ \left. \left. \frac{5 T^8 \gamma h^3}{2 (1 - T + T^2)^3} + \frac{9 T^9 \gamma h^3}{4 (1 - T + T^2)^3} \right) x_1^2 y_1^2 \right) \in + 0 [\epsilon]^2 \} \text{Knot}[10, 100] \]$$

*In[1]:= \$k = 1; Timing@Knot[10, 100]*

**Part:** The expression Knot[10, 100] cannot be used as a part specification.

$$\begin{aligned} \text{Out[1]= } & \{0, \left( -\frac{\hbar}{1 - T + T^2} - \frac{T \hbar}{1 - T + T^2} + \frac{2 T^2 \hbar}{1 - T + T^2} - \frac{T^3 \hbar}{1 - T + T^2} + \frac{T^4 \hbar}{1 - T + T^2} \right) x_1 y_1, \\ & \left( \frac{T}{(1 - T + T^2)^3} - \frac{2 T^2}{(1 - T + T^2)^3} + \frac{3 T^3}{(1 - T + T^2)^3} - \frac{2 T^4}{(1 - T + T^2)^3} + \frac{T^5}{(1 - T + T^2)^3} \right) + \\ & \left( -\frac{T^4 \gamma \hbar}{(1 - T + T^2)^5} + \frac{3 T^5 \gamma \hbar}{(1 - T + T^2)^5} - \frac{5 T^6 \gamma \hbar}{(1 - T + T^2)^5} + \frac{5 T^7 \gamma \hbar}{(1 - T + T^2)^5} - \frac{3 T^8 \gamma \hbar}{(1 - T + T^2)^5} + \frac{T^9 \gamma \hbar}{(1 - T + T^2)^5} - \right. \\ & \left. \frac{T^2 \gamma \hbar}{(1 - T + T^2)^4} + \frac{3 T^3 \gamma \hbar}{(1 - T + T^2)^4} - \frac{5 T^4 \gamma \hbar}{(1 - T + T^2)^4} + \frac{5 T^5 \gamma \hbar}{(1 - T + T^2)^4} - \frac{3 T^6 \gamma \hbar}{(1 - T + T^2)^4} + \frac{T^7 \gamma \hbar}{(1 - T + T^2)^4} + \right. \\ & \left. \left( -\frac{2 T^2 \hbar}{(1 - T + T^2)^4} + \frac{8 T^3 \hbar}{(1 - T + T^2)^4} - \frac{14 T^4 \hbar}{(1 - T + T^2)^4} + \frac{16 T^5 \hbar}{(1 - T + T^2)^4} - \frac{10 T^6 \hbar}{(1 - T + T^2)^4} + \frac{4 T^7 \hbar}{(1 - T + T^2)^4} - \right. \right. \\ & \left. \left. \frac{2 T \hbar}{(1 - T + T^2)^3} + \frac{4 T^2 \hbar}{(1 - T + T^2)^3} - \frac{6 T^3 \hbar}{(1 - T + T^2)^3} + \frac{4 T^4 \hbar}{(1 - T + T^2)^3} - \frac{2 T^5 \hbar}{(1 - T + T^2)^3} \right) a_1 + \right. \\ & \left( \frac{T^2 \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{8 T^3 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{26 T^4 \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{53 T^5 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{71 T^6 \gamma \hbar^2}{(1 - T + T^2)^6} - \right. \\ & \left. \frac{62 T^7 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{26 T^8 \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{13 T^9 \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{31 T^{10} \gamma \hbar^2}{(1 - T + T^2)^6} + \frac{26 T^{11} \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{12 T^{12} \gamma \hbar^2}{(1 - T + T^2)^6} + \right. \\ & \left. \frac{3 T^{13} \gamma \hbar^2}{(1 - T + T^2)^6} - \frac{T^2 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{5 T^3 \gamma \hbar^2}{(1 - T + T^2)^5} - \frac{10 T^4 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{5 T^5 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{16 T^6 \gamma \hbar^2}{(1 - T + T^2)^5} - \right. \\ & \left. \frac{47 T^7 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{61 T^8 \gamma \hbar^2}{(1 - T + T^2)^5} - \frac{52 T^9 \gamma \hbar^2}{(1 - T + T^2)^5} + \frac{26 T^{10} \gamma \hbar^2}{(1 - T + T^2)^5} - \frac{8 T^{11} \gamma \hbar^2}{(1 - T + T^2)^5} - \right. \\ & \left. \frac{2 T^2 \gamma \hbar^2}{(1 - T + T^2)^4} + \frac{8 T^3 \gamma \hbar^2}{(1 - T + T^2)^4} - \frac{16 T^4 \gamma \hbar^2}{(1 - T + T^2)^4} + \frac{27 T^5 \gamma \hbar^2}{(1 - T + T^2)^4} - \frac{30 T^6 \gamma \hbar^2}{(1 - T + T^2)^4} + \right. \\ & \left. \frac{29 T^7 \gamma \hbar^2}{(1 - T + T^2)^4} - \frac{16 T^8 \gamma \hbar^2}{(1 - T + T^2)^4} + \frac{7 T^9 \gamma \hbar^2}{(1 - T + T^2)^4} - \frac{2 T \gamma \hbar^2}{(1 - T + T^2)^3} + \frac{2 T^2 \gamma \hbar^2}{(1 - T + T^2)^3} - \right. \\ & \left. \frac{4 T^3 \gamma \hbar^2}{(1 - T + T^2)^3} + \frac{2 T^4 \gamma \hbar^2}{(1 - T + T^2)^3} - \frac{4 T^5 \gamma \hbar^2}{(1 - T + T^2)^3} + \frac{2 T^6 \gamma \hbar^2}{(1 - T + T^2)^3} - \frac{2 T^7 \gamma \hbar^2}{(1 - T + T^2)^3} \right) x_1 y_1 + \\ & \left( \frac{2 T^3 \hbar^2}{(1 - T + T^2)^5} - \frac{8 T^4 \hbar^2}{(1 - T + T^2)^5} + \frac{20 T^5 \hbar^2}{(1 - T + T^2)^5} - \frac{32 T^6 \hbar^2}{(1 - T + T^2)^5} + \frac{38 T^7 \hbar^2}{(1 - T + T^2)^5} - \right. \end{aligned}$$

$$\begin{aligned}
& \left( \frac{32 T^8 \hbar^2}{(1 - T + T^2)^5} + \frac{20 T^9 \hbar^2}{(1 - T + T^2)^5} - \frac{8 T^{10} \hbar^2}{(1 - T + T^2)^5} + \frac{2 T^{11} \hbar^2}{(1 - T + T^2)^5} - \frac{6 T^3 \hbar^2}{(1 - T + T^2)^4} + \frac{18 T^4 \hbar^2}{(1 - T + T^2)^4} - \right. \\
& \left. \frac{36 T^5 \hbar^2}{(1 - T + T^2)^4} + \frac{42 T^6 \hbar^2}{(1 - T + T^2)^4} - \frac{36 T^7 \hbar^2}{(1 - T + T^2)^4} + \frac{18 T^8 \hbar^2}{(1 - T + T^2)^4} - \frac{6 T^9 \hbar^2}{(1 - T + T^2)^4} + \right. \\
& \left. \frac{4 T^3 \hbar^2}{(1 - T + T^2)^3} - \frac{8 T^4 \hbar^2}{(1 - T + T^2)^3} + \frac{12 T^5 \hbar^2}{(1 - T + T^2)^3} - \frac{8 T^6 \hbar^2}{(1 - T + T^2)^3} + \frac{4 T^7 \hbar^2}{(1 - T + T^2)^3} \right) a_1 x_1 y_1 + \\
& \left( \frac{T \gamma \hbar^3}{4 (1 - T + T^2)^7} - \frac{2 T^2 \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{7 T^3 \gamma \hbar^3}{(1 - T + T^2)^7} - \frac{15 T^4 \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{19 T^5 \gamma \hbar^3}{(1 - T + T^2)^7} - \right. \\
& \left. \frac{8 T^6 \gamma \hbar^3}{(1 - T + T^2)^7} - \frac{47 T^7 \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{63 T^8 \gamma \hbar^3}{(1 - T + T^2)^7} - \frac{345 T^9 \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{76 T^{10} \gamma \hbar^3}{(1 - T + T^2)^7} - \right. \\
& \left. \frac{79 T^{11} \gamma \hbar^3}{2 (1 - T + T^2)^7} + \frac{T^{12} \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{39 T^{13} \gamma \hbar^3}{2 (1 - T + T^2)^7} - \frac{20 T^{14} \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{23 T^{15} \gamma \hbar^3}{2 (1 - T + T^2)^7} - \right. \\
& \left. \frac{4 T^{16} \gamma \hbar^3}{(1 - T + T^2)^7} + \frac{3 T^{17} \gamma \hbar^3}{4 (1 - T + T^2)^7} + \frac{T^2 \gamma \hbar^3}{2 (1 - T + T^2)^6} - \frac{5 T^3 \gamma \hbar^3}{2 (1 - T + T^2)^6} + \frac{6 T^4 \gamma \hbar^3}{(1 - T + T^2)^6} - \right. \\
& \left. \frac{21 T^5 \gamma \hbar^3}{2 (1 - T + T^2)^6} + \frac{15 T^6 \gamma \hbar^3}{(1 - T + T^2)^6} - \frac{51 T^7 \gamma \hbar^3}{2 (1 - T + T^2)^6} + \frac{45 T^8 \gamma \hbar^3}{(1 - T + T^2)^6} - \frac{147 T^9 \gamma \hbar^3}{2 (1 - T + T^2)^6} + \right. \\
& \left. \frac{93 T^{10} \gamma \hbar^3}{(1 - T + T^2)^6} - \frac{185 T^{11} \gamma \hbar^3}{2 (1 - T + T^2)^6} + \frac{68 T^{12} \gamma \hbar^3}{(1 - T + T^2)^6} - \frac{75 T^{13} \gamma \hbar^3}{2 (1 - T + T^2)^6} + \frac{27 T^{14} \gamma \hbar^3}{2 (1 - T + T^2)^6} - \right. \\
& \left. \frac{3 T^{15} \gamma \hbar^3}{(1 - T + T^2)^6} - \frac{T \gamma \hbar^3}{2 (1 - T + T^2)^5} + \frac{2 T^2 \gamma \hbar^3}{(1 - T + T^2)^5} - \frac{3 T^3 \gamma \hbar^3}{4 (1 - T + T^2)^5} - \frac{6 T^4 \gamma \hbar^3}{(1 - T + T^2)^5} + \right. \\
& \left. \frac{27 T^5 \gamma \hbar^3}{(1 - T + T^2)^5} - \frac{54 T^6 \gamma \hbar^3}{(1 - T + T^2)^5} + \frac{351 T^7 \gamma \hbar^3}{4 (1 - T + T^2)^5} - \frac{105 T^8 \gamma \hbar^3}{(1 - T + T^2)^5} + \frac{108 T^9 \gamma \hbar^3}{(1 - T + T^2)^5} - \right. \\
& \left. \frac{83 T^{10} \gamma \hbar^3}{(1 - T + T^2)^5} + \frac{209 T^{11} \gamma \hbar^3}{4 (1 - T + T^2)^5} - \frac{21 T^{12} \gamma \hbar^3}{(1 - T + T^2)^5} + \frac{6 T^{13} \gamma \hbar^3}{(1 - T + T^2)^5} - \frac{6 T^3 \gamma \hbar^3}{(1 - T + T^2)^4} + \right. \\
& \left. \frac{14 T^4 \gamma \hbar^3}{(1 - T + T^2)^4} - \frac{30 T^5 \gamma \hbar^3}{(1 - T + T^2)^4} + \frac{36 T^6 \gamma \hbar^3}{(1 - T + T^2)^4} - \frac{44 T^7 \gamma \hbar^3}{(1 - T + T^2)^4} + \frac{36 T^8 \gamma \hbar^3}{(1 - T + T^2)^4} - \right. \\
& \left. \frac{30 T^9 \gamma \hbar^3}{(1 - T + T^2)^4} + \frac{14 T^{10} \gamma \hbar^3}{(1 - T + T^2)^4} - \frac{6 T^{11} \gamma \hbar^3}{(1 - T + T^2)^4} + \frac{T \gamma \hbar^3}{4 (1 - T + T^2)^3} - \frac{T^2 \gamma \hbar^3}{2 (1 - T + T^2)^3} + \right. \\
& \left. \frac{15 T^3 \gamma \hbar^3}{4 (1 - T + T^2)^3} - \frac{9 T^4 \gamma \hbar^3}{2 (1 - T + T^2)^3} + \frac{15 T^5 \gamma \hbar^3}{2 (1 - T + T^2)^3} - \frac{9 T^6 \gamma \hbar^3}{2 (1 - T + T^2)^3} + \frac{23 T^7 \gamma \hbar^3}{4 (1 - T + T^2)^3} - \right. \\
& \left. \frac{5 T^8 \gamma \hbar^3}{2 (1 - T + T^2)^3} + \frac{9 T^9 \gamma \hbar^3}{4 (1 - T + T^2)^3} \right) x_1^2 y_1^2 \Bigg) \in + O[\epsilon]^2 \} [\text{Knot}[10, 100]] \}
\end{aligned}$$