

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
<< SL2Invariant.m
```

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In[*]:= Kink[0, $k_] = E[0, 0, 1] $k;
Kink[n_, $k_] /; n ≠ 0 := Kink[n, $k] = If[n > 0,
  (Kink[n - 1, $k] Kink[2, $k]) ~B1,2 ~dm1,2→1,
  (Kink[n + 1, $k] Kink[2, $k]) ~B1,2 ~dm1,2→1]
Kink[n_] := Kink[n, $k]
```

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In[*]:= Kink[1]
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Booted @ \$k=0 in 1.735 sec.

Booted @ \$k=1 in 2.64 sec.

Booted @ \$k=2 in 9.188 sec.

$$\text{Out[*]} = \mathbb{E} \left[\hbar a_1 b_1, \hbar x_1 y_1, \frac{1}{\sqrt{B_1}} + \frac{(2 \hbar a_1 - \gamma \hbar^3 x_1^2 y_1^2) \epsilon}{4 \sqrt{B_1}} + \frac{1}{288 \sqrt{B_1}} \right. \\ \left. (36 \hbar^2 a_1^2 - 36 \gamma \hbar^4 a_1 x_1^2 y_1^2 + 32 \gamma^2 \hbar^5 x_1^3 y_1^3 + 9 \gamma^2 \hbar^6 x_1^4 y_1^4) \epsilon^2 + O[\epsilon]^3 \right]$$

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In[*]:= Kink[3]
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$$\text{Out[*]} = \mathbb{E} \left[3 \hbar a_1 b_1, \hbar x_1 y_1 + \hbar B_1 x_1 y_1 + \hbar B_1^2 x_1 y_1, \frac{1}{B_1^{3/2}} + \frac{1}{4 B_1^{3/2}} \right. \\ \left. (6 \hbar a_1 - 4 \hbar^2 a_1 B_1 x_1 y_1 - 8 \hbar^2 a_1 B_1^2 x_1 y_1 - \gamma \hbar^3 x_1^2 y_1^2 + 3 \gamma \hbar^3 B_1 x_1^2 y_1^2 + 8 \gamma \hbar^3 B_1^2 x_1^2 y_1^2 + 7 \gamma \hbar^3 B_1^4 x_1^2 y_1^2) \right. \\ \left. \epsilon + \frac{1}{288 B_1^{3/2}} (324 \hbar^2 a_1^2 - 288 \hbar^3 a_1^2 B_1 x_1 y_1 - 288 \hbar^3 a_1^2 B_1^2 x_1 y_1 - 108 \gamma \hbar^4 a_1 x_1^2 y_1^2 + 144 \gamma^2 \hbar^4 B_1 x_1^2 y_1^2 - \right. \\ \left. 108 \gamma \hbar^4 a_1 B_1^2 x_1^2 y_1^2 + 144 \hbar^4 a_1^2 B_1 x_1^2 y_1^2 + 576 \gamma^2 \hbar^4 B_1^3 x_1^2 y_1^2 - 864 \gamma \hbar^4 a_1 B_1^3 x_1^2 y_1^2 + 576 \hbar^4 a_1^2 B_1^3 x_1^2 y_1^2 + \right. \\ \left. 864 \gamma^2 \hbar^4 B_1^4 x_1^2 y_1^2 - 1260 \gamma \hbar^4 a_1 B_1^4 x_1^2 y_1^2 + 576 \hbar^4 a_1^2 B_1^4 x_1^2 y_1^2 + 32 \gamma^2 \hbar^5 x_1^3 y_1^3 + 72 \gamma \hbar^5 a_1 B_1 x_1^3 y_1^3 + \right. \\ \left. 144 \gamma \hbar^5 a_1 B_1^2 x_1^3 y_1^3 + 320 \gamma^2 \hbar^5 B_1^3 x_1^3 y_1^3 - 216 \gamma \hbar^5 a_1 B_1^3 x_1^3 y_1^3 + 1440 \gamma^2 \hbar^5 B_1^4 x_1^3 y_1^3 - \right. \\ \left. 1008 \gamma \hbar^5 a_1 B_1^4 x_1^3 y_1^3 + 2592 \gamma^2 \hbar^5 B_1^5 x_1^3 y_1^3 - 1656 \gamma \hbar^5 a_1 B_1^5 x_1^3 y_1^3 + 1760 \gamma^2 \hbar^5 B_1^6 x_1^3 y_1^3 - \right. \\ \left. 1008 \gamma \hbar^5 a_1 B_1^6 x_1^3 y_1^3 + 9 \gamma^2 \hbar^6 x_1^4 y_1^4 - 54 \gamma^2 \hbar^6 B_1^2 x_1^4 y_1^4 - 144 \gamma^2 \hbar^6 B_1^3 x_1^4 y_1^4 - 45 \gamma^2 \hbar^6 B_1^4 x_1^4 y_1^4 + \right. \\ \left. 432 \gamma^2 \hbar^6 B_1^5 x_1^4 y_1^4 + 954 \gamma^2 \hbar^6 B_1^6 x_1^4 y_1^4 + 1008 \gamma^2 \hbar^6 B_1^7 x_1^4 y_1^4 + 441 \gamma^2 \hbar^6 B_1^8 x_1^4 y_1^4) \epsilon^2 + O[\epsilon]^3 \right]$$

In[]:= **Kink**[7]

$$\begin{aligned}
 \text{Out[]:= } & \mathbb{E} \left[7 \hbar a_1 b_1, \hbar x_1 y_1 + \hbar B_1 x_1 y_1 + \hbar B_1^2 x_1 y_1 + \hbar B_1^3 x_1 y_1 + \hbar B_1^4 x_1 y_1 + \hbar B_1^5 x_1 y_1 + \hbar B_1^6 x_1 y_1, \right. \\
 & \frac{1}{B_1^{7/2}} + \frac{1}{4 B_1^{7/2}} \left(14 \hbar a_1 - 4 \hbar^2 a_1 B_1 x_1 y_1 - 8 \hbar^2 a_1 B_1^2 x_1 y_1 - 12 \hbar^2 a_1 B_1^3 x_1 y_1 - 16 \hbar^2 a_1 B_1^4 x_1 y_1 - \right. \\
 & 20 \hbar^2 a_1 B_1^5 x_1 y_1 - 24 \hbar^2 a_1 B_1^6 x_1 y_1 - \gamma \hbar^3 x_1^2 y_1^2 + 3 \gamma \hbar^3 B_1^2 x_1^2 y_1^2 + 8 \gamma \hbar^3 B_1^3 x_1^2 y_1^2 + \\
 & 15 \gamma \hbar^3 B_1^4 x_1^2 y_1^2 + 24 \gamma \hbar^3 B_1^5 x_1^2 y_1^2 + 35 \gamma \hbar^3 B_1^6 x_1^2 y_1^2 + 48 \gamma \hbar^3 B_1^7 x_1^2 y_1^2 + 55 \gamma \hbar^3 B_1^8 x_1^2 y_1^2 + \\
 & \left. 56 \gamma \hbar^3 B_1^9 x_1^2 y_1^2 + 51 \gamma \hbar^3 B_1^{10} x_1^2 y_1^2 + 40 \gamma \hbar^3 B_1^{11} x_1^2 y_1^2 + 23 \gamma \hbar^3 B_1^{12} x_1^2 y_1^2 \right) \epsilon + \\
 & \frac{1}{288 B_1^{7/2}} \left(1764 \hbar^2 a_1^2 - 864 \hbar^3 a_1^2 B_1 x_1 y_1 - 1440 \hbar^3 a_1^2 B_1^2 x_1 y_1 - 1728 \hbar^3 a_1^2 B_1^3 x_1 y_1 - \right. \\
 & 1728 \hbar^3 a_1^2 B_1^4 x_1 y_1 - 1440 \hbar^3 a_1^2 B_1^5 x_1 y_1 - 864 \hbar^3 a_1^2 B_1^6 x_1 y_1 - 252 \gamma \hbar^4 a_1 x_1^2 y_1^2 + 144 \gamma^2 \hbar^4 B_1^2 x_1^2 y_1^2 + \\
 & 324 \gamma \hbar^4 a_1 B_1^2 x_1^2 y_1^2 + 144 \hbar^4 a_1^2 B_1^2 x_1^2 y_1^2 + 576 \gamma^2 \hbar^4 B_1^3 x_1^2 y_1^2 + 288 \gamma \hbar^4 a_1 B_1^3 x_1^2 y_1^2 + \\
 & 576 \hbar^4 a_1^2 B_1^3 x_1^2 y_1^2 + 1440 \gamma^2 \hbar^4 B_1^4 x_1^2 y_1^2 - 540 \gamma \hbar^4 a_1 B_1^4 x_1^2 y_1^2 + 1440 \hbar^4 a_1^2 B_1^4 x_1^2 y_1^2 + \\
 & 2880 \gamma^2 \hbar^4 B_1^5 x_1^2 y_1^2 - 2592 \gamma \hbar^4 a_1 B_1^5 x_1^2 y_1^2 + 2880 \hbar^4 a_1^2 B_1^5 x_1^2 y_1^2 + 5040 \gamma^2 \hbar^4 B_1^6 x_1^2 y_1^2 - \\
 & 6300 \gamma \hbar^4 a_1 B_1^6 x_1^2 y_1^2 + 5040 \hbar^4 a_1^2 B_1^6 x_1^2 y_1^2 + 8064 \gamma^2 \hbar^4 B_1^7 x_1^2 y_1^2 - 12096 \gamma \hbar^4 a_1 B_1^7 x_1^2 y_1^2 + \\
 & 8064 \hbar^4 a_1^2 B_1^7 x_1^2 y_1^2 + 11520 \gamma^2 \hbar^4 B_1^8 x_1^2 y_1^2 - 17820 \gamma \hbar^4 a_1 B_1^8 x_1^2 y_1^2 + 10080 \hbar^4 a_1^2 B_1^8 x_1^2 y_1^2 + \\
 & 14400 \gamma^2 \hbar^4 B_1^9 x_1^2 y_1^2 - 22176 \gamma \hbar^4 a_1 B_1^9 x_1^2 y_1^2 + 10944 \hbar^4 a_1^2 B_1^9 x_1^2 y_1^2 + 15696 \gamma^2 \hbar^4 B_1^{10} x_1^2 y_1^2 - \\
 & 23868 \gamma \hbar^4 a_1 B_1^{10} x_1^2 y_1^2 + 10512 \hbar^4 a_1^2 B_1^{10} x_1^2 y_1^2 + 14400 \gamma^2 \hbar^4 B_1^{11} x_1^2 y_1^2 - 21600 \gamma \hbar^4 a_1 B_1^{11} x_1^2 y_1^2 + \\
 & 8640 \hbar^4 a_1^2 B_1^{11} x_1^2 y_1^2 + 9504 \gamma^2 \hbar^4 B_1^{12} x_1^2 y_1^2 - 14076 \gamma \hbar^4 a_1 B_1^{12} x_1^2 y_1^2 + 5184 \hbar^4 a_1^2 B_1^{12} x_1^2 y_1^2 + \\
 & 32 \gamma^2 \hbar^5 x_1^3 y_1^3 + 72 \gamma \hbar^5 a_1 B_1 x_1^3 y_1^3 + 144 \gamma \hbar^5 a_1 B_1^2 x_1^3 y_1^3 + 320 \gamma^2 \hbar^5 B_1^3 x_1^3 y_1^3 + 1440 \gamma^2 \hbar^5 B_1^4 x_1^3 y_1^3 - \\
 & 720 \gamma \hbar^5 a_1 B_1^4 x_1^3 y_1^3 + 4032 \gamma^2 \hbar^5 B_1^5 x_1^3 y_1^3 - 2520 \gamma \hbar^5 a_1 B_1^5 x_1^3 y_1^3 + 8960 \gamma^2 \hbar^5 B_1^6 x_1^3 y_1^3 - \\
 & 6048 \gamma \hbar^5 a_1 B_1^6 x_1^3 y_1^3 + 17280 \gamma^2 \hbar^5 B_1^7 x_1^3 y_1^3 - 12600 \gamma \hbar^5 a_1 B_1^7 x_1^3 y_1^3 + 30240 \gamma^2 \hbar^5 B_1^8 x_1^3 y_1^3 - \\
 & 22176 \gamma \hbar^5 a_1 B_1^8 x_1^3 y_1^3 + 47840 \gamma^2 \hbar^5 B_1^9 x_1^3 y_1^3 - 34200 \gamma \hbar^5 a_1 B_1^9 x_1^3 y_1^3 + 68832 \gamma^2 \hbar^5 B_1^{10} x_1^3 y_1^3 - \\
 & 47520 \gamma \hbar^5 a_1 B_1^{10} x_1^3 y_1^3 + 90720 \gamma^2 \hbar^5 B_1^{11} x_1^3 y_1^3 - 60408 \gamma \hbar^5 a_1 B_1^{11} x_1^3 y_1^3 + 109760 \gamma^2 \hbar^5 B_1^{12} x_1^3 y_1^3 - \\
 & 70560 \gamma \hbar^5 a_1 B_1^{12} x_1^3 y_1^3 + 120960 \gamma^2 \hbar^5 B_1^{13} x_1^3 y_1^3 - 75096 \gamma \hbar^5 a_1 B_1^{13} x_1^3 y_1^3 + 118080 \gamma^2 \hbar^5 B_1^{14} x_1^3 y_1^3 - \\
 & 70560 \gamma \hbar^5 a_1 B_1^{14} x_1^3 y_1^3 + 102560 \gamma^2 \hbar^5 B_1^{15} x_1^3 y_1^3 - 59040 \gamma \hbar^5 a_1 B_1^{15} x_1^3 y_1^3 + 77472 \gamma^2 \hbar^5 B_1^{16} x_1^3 y_1^3 - \\
 & 43056 \gamma \hbar^5 a_1 B_1^{16} x_1^3 y_1^3 + 47520 \gamma^2 \hbar^5 B_1^{17} x_1^3 y_1^3 - 25560 \gamma \hbar^5 a_1 B_1^{17} x_1^3 y_1^3 + 19040 \gamma^2 \hbar^5 B_1^{18} x_1^3 y_1^3 - \\
 & 9936 \gamma \hbar^5 a_1 B_1^{18} x_1^3 y_1^3 + 9 \gamma^2 \hbar^6 x_1^4 y_1^4 - 54 \gamma^2 \hbar^6 B_1^2 x_1^4 y_1^4 - 144 \gamma^2 \hbar^6 B_1^3 x_1^4 y_1^4 - 189 \gamma^2 \hbar^6 B_1^4 x_1^4 y_1^4 + \\
 & 756 \gamma^2 \hbar^6 B_1^6 x_1^4 y_1^4 + 2592 \gamma^2 \hbar^6 B_1^7 x_1^4 y_1^4 + 6381 \gamma^2 \hbar^6 B_1^8 x_1^4 y_1^4 + 13104 \gamma^2 \hbar^6 B_1^9 x_1^4 y_1^4 + \\
 & 23598 \gamma^2 \hbar^6 B_1^{10} x_1^4 y_1^4 + 38304 \gamma^2 \hbar^6 B_1^{11} x_1^4 y_1^4 + 57015 \gamma^2 \hbar^6 B_1^{12} x_1^4 y_1^4 + 78624 \gamma^2 \hbar^6 B_1^{13} x_1^4 y_1^4 + \\
 & 100350 \gamma^2 \hbar^6 B_1^{14} x_1^4 y_1^4 + 118944 \gamma^2 \hbar^6 B_1^{15} x_1^4 y_1^4 + 131229 \gamma^2 \hbar^6 B_1^{16} x_1^4 y_1^4 + 134640 \gamma^2 \hbar^6 B_1^{17} x_1^4 y_1^4 + \\
 & 127764 \gamma^2 \hbar^6 B_1^{18} x_1^4 y_1^4 + 110880 \gamma^2 \hbar^6 B_1^{19} x_1^4 y_1^4 + 86499 \gamma^2 \hbar^6 B_1^{20} x_1^4 y_1^4 + 59904 \gamma^2 \hbar^6 B_1^{21} x_1^4 y_1^4 + \\
 & \left. 35514 \gamma^2 \hbar^6 B_1^{22} x_1^4 y_1^4 + 16560 \gamma^2 \hbar^6 B_1^{23} x_1^4 y_1^4 + 4761 \gamma^2 \hbar^6 B_1^{24} x_1^4 y_1^4 \right) \epsilon^2 + O[\epsilon]^3]
 \end{aligned}$$

In[]:= **Kink**[-3]

$$\text{Out[]:= } \mathbb{E} \left[-3 \hbar a_1 b_1, \frac{-\hbar x_1 y_1 - \hbar B_1 x_1 y_1 - \hbar B_1^2 x_1 y_1}{B_1^3}, \right.$$

$$\begin{aligned} & B_1^{3/2} + \frac{1}{4 B_1^{9/2}} \left(-6 \hbar a_1 B_1^6 - 12 \hbar^2 a_1 B_1^3 x_1 y_1 - 8 \hbar^2 a_1 B_1^4 x_1 y_1 - 4 \hbar^2 a_1 B_1^5 x_1 y_1 - \right. \\ & \quad \left. 11 \gamma \hbar^3 x_1^2 y_1^2 - 16 \gamma \hbar^3 B_1 x_1^2 y_1^2 - 15 \gamma \hbar^3 B_1^2 x_1^2 y_1^2 - 8 \gamma \hbar^3 B_1^3 x_1^2 y_1^2 - 3 \gamma \hbar^3 B_1^4 x_1^2 y_1^2 \right) \epsilon + \\ & \frac{1}{288 B_1^{21/2}} \left(324 \hbar^2 a_1^2 B_1^{12} + 288 \hbar^3 a_1^2 B_1^{10} x_1 y_1 + 288 \hbar^3 a_1^2 B_1^{11} x_1 y_1 + 2160 \gamma^2 \hbar^4 B_1^6 x_1^2 y_1^2 - \right. \\ & \quad 3564 \gamma \hbar^4 a_1 B_1^6 x_1^2 y_1^2 + 1296 \hbar^4 a_1^2 B_1^6 x_1^2 y_1^2 + 2304 \gamma^2 \hbar^4 B_1^7 x_1^2 y_1^2 - 4032 \gamma \hbar^4 a_1 B_1^7 x_1^2 y_1^2 + \\ & \quad 1728 \hbar^4 a_1^2 B_1^7 x_1^2 y_1^2 + 1440 \gamma^2 \hbar^4 B_1^8 x_1^2 y_1^2 - 2700 \gamma \hbar^4 a_1 B_1^8 x_1^2 y_1^2 + 1440 \hbar^4 a_1^2 B_1^8 x_1^2 y_1^2 + \\ & \quad 576 \gamma^2 \hbar^4 B_1^9 x_1^2 y_1^2 - 864 \gamma \hbar^4 a_1 B_1^9 x_1^2 y_1^2 + 576 \hbar^4 a_1^2 B_1^9 x_1^2 y_1^2 + 144 \gamma^2 \hbar^4 B_1^{10} x_1^2 y_1^2 - \\ & \quad 108 \gamma \hbar^4 a_1 B_1^{10} x_1^2 y_1^2 + 144 \hbar^4 a_1^2 B_1^{10} x_1^2 y_1^2 - 4352 \gamma^2 \hbar^5 B_1^3 x_1^3 y_1^3 + 2376 \gamma \hbar^5 a_1 B_1^3 x_1^3 y_1^3 - \\ & \quad 8640 \gamma^2 \hbar^5 B_1^4 x_1^3 y_1^3 + 5040 \gamma \hbar^5 a_1 B_1^4 x_1^3 y_1^3 - 10080 \gamma^2 \hbar^5 B_1^5 x_1^3 y_1^3 + 6336 \gamma \hbar^5 a_1 B_1^5 x_1^3 y_1^3 - \\ & \quad 7520 \gamma^2 \hbar^5 B_1^6 x_1^3 y_1^3 + 5040 \gamma \hbar^5 a_1 B_1^6 x_1^3 y_1^3 - 4032 \gamma^2 \hbar^5 B_1^7 x_1^3 y_1^3 + 2880 \gamma \hbar^5 a_1 B_1^7 x_1^3 y_1^3 - \\ & \quad 1440 \gamma^2 \hbar^5 B_1^8 x_1^3 y_1^3 + 1008 \gamma \hbar^5 a_1 B_1^8 x_1^3 y_1^3 - 320 \gamma^2 \hbar^5 B_1^9 x_1^3 y_1^3 + 216 \gamma \hbar^5 a_1 B_1^9 x_1^3 y_1^3 + \\ & \quad 1089 \gamma^2 \hbar^6 x_1^4 y_1^4 + 3168 \gamma^2 \hbar^6 B_1 x_1^4 y_1^4 + 5274 \gamma^2 \hbar^6 B_1^2 x_1^4 y_1^4 + 5904 \gamma^2 \hbar^6 B_1^3 x_1^4 y_1^4 + 4923 \gamma^2 \hbar^6 B_1^4 x_1^4 y_1^4 + \\ & \quad \left. 3024 \gamma^2 \hbar^6 B_1^5 x_1^4 y_1^4 + 1386 \gamma^2 \hbar^6 B_1^6 x_1^4 y_1^4 + 432 \gamma^2 \hbar^6 B_1^7 x_1^4 y_1^4 + 81 \gamma^2 \hbar^6 B_1^8 x_1^4 y_1^4 \right) \epsilon^2 + \mathcal{O}[\epsilon]^3 \end{aligned}$$

In[]:= **Table**[**Kink**[**n**, **1**], {**n**, **0**, **5**}] // **Column**

$\mathbb{E} [0, 0, 1]$

$$\mathbb{E} \left[\hbar a_1 b_1, \hbar x_1 y_1, \frac{1}{\sqrt{B_1}} + \frac{(2 \hbar a_1 - \gamma \hbar^3 x_1^2 y_1^2) \epsilon}{4 \sqrt{B_1}} + \mathcal{O}[\epsilon]^2 \right]$$

$$\mathbb{E} \left[2 \hbar a_1 b_1, \hbar x_1 y_1 + \hbar B_1 x_1 y_1, \frac{1}{B_1} + \frac{(4 \hbar a_1 - 4 \hbar^2 a_1 B_1 x_1 y_1 - \gamma \hbar^3 x_1^2 y_1^2 + 3 \gamma \hbar^3 B_1^2 x_1^2 y_1^2) \epsilon}{4 B_1} + \mathcal{O}[\epsilon]^2 \right]$$

$\mathbb{E} [3 \hbar a_1 b_1, \hbar x_1 y_1 + \hbar B_1 x_1 y_1 + \hbar B_1^2 x_1 y_1,$

$$\frac{1}{B_1^{3/2}} + \frac{1}{4 B_1^{3/2}} \left(6 \hbar a_1 - 4 \hbar^2 a_1 B_1 x_1 y_1 - 8 \hbar^2 a_1 B_1^2 x_1 y_1 - \gamma \hbar^3 x_1^2 y_1^2 + \right. \\ \left. 3 \gamma \hbar^3 B_1^2 x_1^2 y_1^2 + 8 \gamma \hbar^3 B_1^3 x_1^2 y_1^2 + 7 \gamma \hbar^3 B_1^4 x_1^2 y_1^2 \right) \epsilon + \mathcal{O}[\epsilon]^2]$$

Out[]:=

$\mathbb{E} [4 \hbar a_1 b_1, \hbar x_1 y_1 + \hbar B_1 x_1 y_1 + \hbar B_1^2 x_1 y_1 + \hbar B_1^3 x_1 y_1,$

$$\frac{1}{B_1^2} + \frac{1}{4 B_1^2} \left(8 \hbar a_1 - 4 \hbar^2 a_1 B_1 x_1 y_1 - 8 \hbar^2 a_1 B_1^2 x_1 y_1 - 12 \hbar^2 a_1 B_1^3 x_1 y_1 - \gamma \hbar^3 x_1^2 y_1^2 + 3 \gamma \hbar^3 B_1^2 x_1^2 y_1^2 + \right. \\ \left. 8 \gamma \hbar^3 B_1^3 x_1^2 y_1^2 + 15 \gamma \hbar^3 B_1^4 x_1^2 y_1^2 + 16 \gamma \hbar^3 B_1^5 x_1^2 y_1^2 + 11 \gamma \hbar^3 B_1^6 x_1^2 y_1^2 \right) \epsilon + \mathcal{O}[\epsilon]^2]$$

$\mathbb{E} [5 \hbar a_1 b_1, \hbar x_1 y_1 + \hbar B_1 x_1 y_1 + \hbar B_1^2 x_1 y_1 + \hbar B_1^3 x_1 y_1 + \hbar B_1^4 x_1 y_1,$

$$\frac{1}{B_1^{5/2}} + \frac{1}{4 B_1^{5/2}} \left(10 \hbar a_1 - 4 \hbar^2 a_1 B_1 x_1 y_1 - 8 \hbar^2 a_1 B_1^2 x_1 y_1 - 12 \hbar^2 a_1 B_1^3 x_1 y_1 - \right. \\ \left. 16 \hbar^2 a_1 B_1^4 x_1 y_1 - \gamma \hbar^3 x_1^2 y_1^2 + 3 \gamma \hbar^3 B_1^2 x_1^2 y_1^2 + 8 \gamma \hbar^3 B_1^3 x_1^2 y_1^2 + 15 \gamma \hbar^3 B_1^4 x_1^2 y_1^2 + \right. \\ \left. 24 \gamma \hbar^3 B_1^5 x_1^2 y_1^2 + 27 \gamma \hbar^3 B_1^6 x_1^2 y_1^2 + 24 \gamma \hbar^3 B_1^7 x_1^2 y_1^2 + 15 \gamma \hbar^3 B_1^8 x_1^2 y_1^2 \right) \epsilon + \mathcal{O}[\epsilon]^2]$$