

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
<< SL2Invariant.m
```

Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: June 2018. Original version: July 1994.

```
In[ ]:= dSi // Short
```

$$\text{Out[]//Short} = \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[-a_i \alpha_i - b_i \beta_i, \langle\langle 1 \rangle\rangle, 1 + \left(\langle\langle 17 \rangle\rangle + \frac{x_i (\langle\langle 1 \rangle\rangle - \langle\langle 1 \rangle\rangle) \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle \xi_i^2}{2 B_i} + \frac{\langle\langle 1 \rangle\rangle}{4 \hbar B_i^2} \right) \epsilon + \left(-\frac{\gamma^2 \hbar^2 \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle \eta_i}{2 B_i} + \langle\langle 102 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} \right) \langle\langle 1 \rangle\rangle + O[\epsilon]^3 \right]$$

```
In[ ]:= dSi // E4 // Short
```

$$\text{Out[]//Short} = \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[1, -a_i \alpha_i - \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle, \langle\langle 1 \rangle\rangle, \left\{ 1, \langle\langle 17 \rangle\rangle + \langle\langle 1 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, -\frac{\gamma^2 \hbar^2 y_i \mathcal{A}_i \eta_i}{2 B_i} + \langle\langle 101 \rangle\rangle + \frac{x_i \langle\langle 2 \rangle\rangle \xi_i^4}{8 \hbar B_i^3} + \frac{(\langle\langle 6 \rangle\rangle + \gamma^2 B_i^4 \mathcal{A}_i^4) \eta_i^4 \xi_i^4}{32 \hbar^2 B_i^4} \right\} \right]$$

```
In[ ]:= Timing[z3 = Z[Knot[3, 1]]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

QZip4 fail at {L,Q,P}={ħ a₃ b₁,

$$\begin{aligned} & \hbar x_3 y_n \xi_{n\$22981[1]} + y_1 \eta_n \xi_{n\$22981[1]} + y_1 \eta_n \xi_{n\$22981[2]} + x_1 \xi_n \xi_{n\$22981[1]} + \frac{(1 - B_1) \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]}}{\hbar} + x_1 \xi_n \xi_{n\$22981[2]}, \\ & \frac{1}{\sqrt{B_1}} + \left(\frac{\hbar a_1}{2 \sqrt{B_1}} - \frac{\gamma \hbar^3 x_3^2 y_n^2 \xi_{n\$22981[1]}}{4 \sqrt{B_1}} - \frac{\hbar a_3 y_1 \eta_n \xi_{n\$22981[2]}}{\sqrt{B_1}} - \frac{\gamma \hbar x_1 \xi_n \xi_{n\$22981[1]}}{\sqrt{B_1}} + \right. \\ & a_1 \sqrt{B_1} \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]} + \frac{\gamma \hbar x_1 y_1 \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]}}{\sqrt{B_1}} + \frac{(\gamma - 3 \gamma B_1) y_1 \eta_n^2 \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]}}{2 \sqrt{B_1}} + \\ & \left. \frac{(\gamma - 3 \gamma B_1) x_1 \eta_n \xi_{n\$22981[2]} \xi_n^2 \xi_{n\$22981[1]}}{2 \sqrt{B_1}} + \frac{(\gamma - 4 \gamma B_1 + 3 \gamma B_1^2) \eta_n^2 \xi_{n\$22981[2]} \xi_n^2 \xi_{n\$22981[1]}}{4 \hbar \sqrt{B_1}} \right) \epsilon + \\ & \left(\frac{\hbar^2 a_1^2}{8 \sqrt{B_1}} - \frac{\gamma \hbar^4 a_1 x_3^2 y_n^2 \xi_{n\$22981[1]}}{8 \sqrt{B_1}} + \frac{\gamma^2 \hbar^5 x_3^3 y_n^3 \xi_{n\$22981[1]}}{9 \sqrt{B_1}} + \frac{\gamma^2 \hbar^6 x_3^4 y_n^4 \xi_{n\$22981[1]}}{32 \sqrt{B_1}} - \frac{\hbar^2 a_1 a_3 y_1 \eta_n \xi_{n\$22981[2]}}{2 \sqrt{B_1}} + \right. \\ & \frac{\hbar^2 a_3^2 y_1 \eta_n \xi_{n\$22981[2]}}{2 \sqrt{B_1}} + \frac{\gamma \hbar^4 a_3 x_3^2 y_1 \eta_n^2 \xi_{n\$22981[1]} \eta_n \xi_{n\$22981[2]}}{4 \sqrt{B_1}} + \frac{\hbar^2 a_3^2 y_1^2 \eta_n^2 \xi_{n\$22981[2]}}{2 \sqrt{B_1}} + \frac{\gamma^2 \hbar^2 x_1 \xi_n \xi_{n\$22981[1]}}{2 \sqrt{B_1}} - \\ & \frac{\gamma \hbar^2 a_1 x_1 \xi_n \xi_{n\$22981[1]}}{2 \sqrt{B_1}} + \frac{\gamma^2 \hbar^4 x_1 x_3^2 y_n^2 \xi_{n\$22981[1]} \xi_n \xi_{n\$22981[1]}}{4 \sqrt{B_1}} - \frac{\gamma^2 \hbar^2 x_1 y_1 \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]}}{2 \sqrt{B_1}} + \\ & \left. \frac{\gamma \hbar^2 a_1 x_1 y_1 \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]}}{2 \sqrt{B_1}} - \frac{1}{4} \gamma \hbar^3 a_1 \sqrt{B_1} x_3^2 y_n^2 \xi_{n\$22981[1]} \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]} - \right. \\ & \left. \frac{\gamma^2 \hbar^4 x_1 x_3^2 y_1 \eta_n^2 \xi_{n\$22981[1]} \eta_n \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]}}{4 \sqrt{B_1}} - \hbar a_1 a_3 \sqrt{B_1} y_1 \eta_n^2 \xi_{n\$22981[2]} \xi_n \xi_{n\$22981[1]} + \right. \end{aligned}$$

$$\begin{aligned}
 & \frac{a_3 (-\gamma \hbar + 3 \gamma \hbar B_1) y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}}{2 \sqrt{B_1}} + \frac{a_1 (\gamma \hbar + 3 \gamma \hbar B_1) y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}}{4 \sqrt{B_1}} + \\
 & \frac{(\gamma^2 \hbar - 5 \gamma^2 \hbar B_1) y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}}{4 \sqrt{B_1}} + \frac{\gamma^2 \hbar^2 x_1 y_1^2 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}}{2 \sqrt{B_1}} - \\
 & \frac{\gamma \hbar^2 a_3 x_1 y_1^2 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}}{\sqrt{B_1}} + \frac{1}{8 \sqrt{B_1}} (-\gamma^2 \hbar^3 + 3 \gamma^2 \hbar^3 B_1) x_3^2 y_1 y_{n\$22981[1]}^2 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]} + \\
 & \frac{a_3 (-\gamma \hbar + 3 \gamma \hbar B_1) y_1^2 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}}{2 \sqrt{B_1}} + \frac{(\gamma^2 \hbar - 7 \gamma^2 \hbar B_1) y_1^2 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}}{6 \sqrt{B_1}} + \\
 & \frac{\gamma^2 \hbar^2 x_1^2 \xi_{n\$22981[1]}^2}{2 \sqrt{B_1}} + \frac{a_1 (\gamma \hbar - \gamma \hbar B_1) x_1 \eta_{n\$22981[2]} \xi_{n\$22981[1]}^2}{4 \sqrt{B_1}} + \\
 & \frac{(-\gamma^2 \hbar + \gamma^2 \hbar B_1) x_1 \eta_{n\$22981[2]} \xi_{n\$22981[1]}^2}{4 \sqrt{B_1}} - \frac{\gamma^2 \hbar^2 x_1^2 y_1 \eta_{n\$22981[2]} \xi_{n\$22981[1]}^2}{2 \sqrt{B_1}} + \frac{1}{8 \sqrt{B_1}} \\
 & (-\gamma^2 \hbar^3 + 3 \gamma^2 \hbar^3 B_1) x_1 x_3^2 y_{n\$22981[1]}^2 \eta_{n\$22981[2]} \xi_{n\$22981[1]}^2 + \frac{1}{2} a_1^2 B_1^{3/2} \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2 + \\
 & \frac{a_1 (\gamma + 4 \gamma B_1 - 9 \gamma B_1^2) \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2}{8 \sqrt{B_1}} + \frac{(\gamma^2 - 6 \gamma^2 B_1 + 5 \gamma^2 B_1^2) \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2}{8 \sqrt{B_1}} + \\
 & \gamma \hbar a_1 \sqrt{B_1} x_1 y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2 + \frac{a_3 (-\gamma \hbar + 3 \gamma \hbar B_1) x_1 y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2}{2 \sqrt{B_1}} + \\
 & \frac{(3 \gamma^2 \hbar - 15 \gamma^2 \hbar B_1) x_1 y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2}{4 \sqrt{B_1}} + \frac{\gamma^2 \hbar^2 x_1^2 y_1^2 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2}{2 \sqrt{B_1}} + \\
 & \frac{1}{16 \sqrt{B_1}} (-\gamma^2 \hbar^2 + 4 \gamma^2 \hbar^2 B_1 - 3 \gamma^2 \hbar^2 B_1^2) x_3^2 y_{n\$22981[1]}^2 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^2 + \\
 & \frac{1}{2} a_1 (\gamma \sqrt{B_1} - 3 \gamma B_1^{3/2}) y_1 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^2 + \frac{a_3 (-\gamma + 4 \gamma B_1 - 3 \gamma B_1^2) y_1 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^2}{4 \sqrt{B_1}} + \\
 & \frac{(5 \gamma^2 - 34 \gamma^2 B_1 + 41 \gamma^2 B_1^2) y_1 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^2}{12 \sqrt{B_1}} + \frac{(\gamma^2 \hbar - 3 \gamma^2 \hbar B_1) x_1 y_1^2 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^2}{2 \sqrt{B_1}} + \\
 & \frac{(\gamma^2 - 6 \gamma^2 B_1 + 9 \gamma^2 B_1^2) y_1^2 \eta_{n\$22981[2]}^4 \xi_{n\$22981[1]}^2}{8 \sqrt{B_1}} + \frac{(-\gamma^2 \hbar + \gamma^2 \hbar B_1) x_1^2 \eta_{n\$22981[2]} \xi_{n\$22981[1]}^3}{3 \sqrt{B_1}} + \\
 & \frac{1}{2} a_1 (\gamma \sqrt{B_1} - 3 \gamma B_1^{3/2}) x_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^3 + \frac{(\gamma^2 - 11 \gamma^2 B_1 + 16 \gamma^2 B_1^2) x_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^3}{6 \sqrt{B_1}} + \\
 & \frac{(\gamma^2 \hbar - 3 \gamma^2 \hbar B_1) x_1^2 y_1 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^3}{2 \sqrt{B_1}} + \frac{1}{4 \hbar} a_1 (\gamma \sqrt{B_1} - 4 \gamma B_1^{3/2} + 3 \gamma B_1^{5/2}) \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^3 + \\
 & \frac{1}{36 \hbar \sqrt{B_1}} (5 \gamma^2 - 39 \gamma^2 B_1 + 75 \gamma^2 B_1^2 - 41 \gamma^2 B_1^3) \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^3 + \\
 & \frac{(\gamma^2 - 5 \gamma^2 B_1 + 6 \gamma^2 B_1^2) x_1 y_1 \eta_{n\$22981[2]}^3 \xi_{n\$22981[1]}^3}{2 \sqrt{B_1}} + \frac{1}{8 \hbar \sqrt{B_1}} \\
 & (\gamma^2 - 7 \gamma^2 B_1 + 15 \gamma^2 B_1^2 - 9 \gamma^2 B_1^3) y_1 \eta_{n\$22981[2]}^4 \xi_{n\$22981[1]}^3 + \frac{(\gamma^2 - 6 \gamma^2 B_1 + 9 \gamma^2 B_1^2) x_1^2 \eta_{n\$22981[2]}^2 \xi_{n\$22981[1]}^4}{8 \sqrt{B_1}} +
 \end{aligned}$$

$$\frac{1}{8 \hbar \sqrt{B_1}} (\gamma^2 - 7 \gamma^2 B_1 + 15 \gamma^2 B_1^2 - 9 \gamma^2 B_1^3) x_1 \eta_n^3 \zeta_n^4 + \frac{1}{32 \hbar^2 \sqrt{B_1}} (\gamma^2 - 8 \gamma^2 B_1 + 22 \gamma^2 B_1^2 - 24 \gamma^2 B_1^3 + 9 \gamma^2 B_1^4) \eta_n^4 \zeta_n^4 \left\{ \epsilon^2 + O[\epsilon]^3 \right\}$$

$$\text{Out[*]} = \left\{ 44.5313, \mathbb{E}_{\{\}} \rightarrow \{\emptyset\} \left[\mathbf{0}, \mathbf{0}, \frac{T}{1 - T + T^2} + \left(T \hbar (2 a (-1 + T - T^3 + T^4) + \gamma (-2 + 3 T - 2 T^2 + T^3 - 2 x y \hbar - 2 T^3 x y \hbar)) \right) \epsilon \right] / (1 - T + T^2)^3 + \frac{1}{2 (1 - T + T^2)^5} T \hbar^2 \left(4 a^2 (1 - T + T^2)^2 (1 + T - 6 T^2 + T^3 + T^4) - 4 a (1 - T + T^2) \gamma (T (2 - 4 x y \hbar) - 2 (1 + x y \hbar) - 8 T^3 (1 + x y \hbar) + T^4 (5 + 2 x y \hbar) + T^5 (-2 + 4 x y \hbar) + T^2 (7 + 10 x y \hbar)) + \gamma^2 (4 + 4 x y \hbar + 6 x^2 y^2 \hbar^2 + T^7 (1 + 4 x y \hbar) + 2 T^3 x y \hbar (-2 + 15 x y \hbar) + T^2 (6 - 12 x^2 y^2 \hbar^2) + T^5 (4 + 6 x^2 y^2 \hbar^2) + T (-11 - 8 x y \hbar + 6 x^2 y^2 \hbar^2) + T^6 (-2 - 8 x y \hbar + 6 x^2 y^2 \hbar^2) - 2 T^4 (1 + 2 x y \hbar + 6 x^2 y^2 \hbar^2)) \right) \epsilon^2 + O[\epsilon]^3 \right\}$$

In[*] = **E4@z3**

$$\text{Out[*]} = \mathbb{E}_{\{\}} \rightarrow \{\emptyset\} \left[\frac{1 - T + T^2}{T}, \mathbf{0}, \mathbf{0}, \left\{ 1, \frac{1}{T^4} a (-2 \hbar + 6 T \hbar - 10 T^2 \hbar + 8 T^3 \hbar - 8 T^5 \hbar + 10 T^6 \hbar - 6 T^7 \hbar + 2 T^8 \hbar) + \frac{1}{T^4} (-2 \gamma \hbar + 7 T \gamma \hbar - 14 T^2 \gamma \hbar + 18 T^3 \gamma \hbar - 16 T^4 \gamma \hbar + 10 T^5 \gamma \hbar - 4 T^6 \gamma \hbar + T^7 \gamma \hbar) + \frac{1}{T^4} x y (-2 \gamma \hbar^2 + 4 T \gamma \hbar^2 - 6 T^2 \gamma \hbar^2 + 2 T^3 \gamma \hbar^2 + 2 T^4 \gamma \hbar^2 - 6 T^5 \gamma \hbar^2 + 4 T^6 \gamma \hbar^2 - 2 T^7 \gamma \hbar^2), \frac{1}{T^8} a^2 (2 \hbar^2 - 10 T \hbar^2 + 18 T^2 \hbar^2 + 16 T^3 \hbar^2 - 182 T^4 \hbar^2 + 558 T^5 \hbar^2 - 1108 T^6 \hbar^2 + 1622 T^7 \hbar^2 - 1836 T^8 \hbar^2 + 1622 T^9 \hbar^2 - 1108 T^{10} \hbar^2 + 558 T^{11} \hbar^2 - 182 T^{12} \hbar^2 + 16 T^{13} \hbar^2 + 18 T^{14} \hbar^2 - 10 T^{15} \hbar^2 + 2 T^{16} \hbar^2) + \frac{1}{T^8} a (4 \gamma \hbar^2 - 24 T \gamma \hbar^2 + 66 T^2 \gamma \hbar^2 - 94 T^3 \gamma \hbar^2 + 330 T^5 \gamma \hbar^2 - 896 T^6 \gamma \hbar^2 + 1494 T^7 \gamma \hbar^2 - 1836 T^8 \gamma \hbar^2 + 1750 T^9 \gamma \hbar^2 - 1320 T^{10} \gamma \hbar^2 + 786 T^{11} \gamma \hbar^2 - 364 T^{12} \gamma \hbar^2 + 126 T^{13} \gamma \hbar^2 - 30 T^{14} \gamma \hbar^2 + 4 T^{15} \gamma \hbar^2) + \frac{1}{2 T^8} (4 \gamma^2 \hbar^2 - 27 T \gamma^2 \hbar^2 + 90 T^2 \gamma^2 \hbar^2 - 198 T^3 \gamma^2 \hbar^2 + 310 T^4 \gamma^2 \hbar^2 - 357 T^5 \gamma^2 \hbar^2 + 292 T^6 \gamma^2 \hbar^2 - 141 T^7 \gamma^2 \hbar^2 - 18 T^8 \gamma^2 \hbar^2 + 115 T^9 \gamma^2 \hbar^2 - 132 T^{10} \gamma^2 \hbar^2 + 99 T^{11} \gamma^2 \hbar^2 - 54 T^{12} \gamma^2 \hbar^2 + 22 T^{13} \gamma^2 \hbar^2 - 6 T^{14} \gamma^2 \hbar^2 + T^{15} \gamma^2 \hbar^2) + \frac{1}{T^8} a x y (4 \gamma \hbar^3 - 12 T \gamma \hbar^3 + 116 T^3 \gamma \hbar^3 - 444 T^4 \gamma \hbar^3 + 1008 T^5 \gamma \hbar^3 - 1628 T^6 \gamma \hbar^3 + 1980 T^7 \gamma \hbar^3 - 1836 T^8 \gamma \hbar^3 + 1264 T^9 \gamma \hbar^3 - 588 T^{10} \gamma \hbar^3 + 108 T^{11} \gamma \hbar^3 + 80 T^{12} \gamma \hbar^3 - 84 T^{13} \gamma \hbar^3 + 36 T^{14} \gamma \hbar^3 - 8 T^{15} \gamma \hbar^3) + \frac{1}{T^8} x y (2 \gamma^2 \hbar^3 - 12 T \gamma^2 \hbar^3 + 36 T^2 \gamma^2 \hbar^3 - 74 T^3 \gamma^2 \hbar^3 + 108 T^4 \gamma^2 \hbar^3 - 120 T^5 \gamma^2 \hbar^3 + 92 T^6 \gamma^2 \hbar^3 - 36 T^7 \gamma^2 \hbar^3 - 36 T^8 \gamma^2 \hbar^3 + 92 T^9 \gamma^2 \hbar^3 - 120 T^{10} \gamma^2 \hbar^3 + 108 T^{11} \gamma^2 \hbar^3 - 74 T^{12} \gamma^2 \hbar^3 + 36 T^{13} \gamma^2 \hbar^3 - 12 T^{14} \gamma^2 \hbar^3 + 2 T^{15} \gamma^2 \hbar^3) + \frac{1}{T^8} x^2 y^2 (3 \gamma^2 \hbar^4 - 9 T \gamma^2 \hbar^4 + 12 T^2 \gamma^2 \hbar^4 + 21 T^3 \gamma^2 \hbar^4 - 117 T^4 \gamma^2 \hbar^4 + 282 T^5 \gamma^2 \hbar^4 - 441 T^6 \gamma^2 \hbar^4 + 513 T^7 \gamma^2 \hbar^4 - 441 T^8 \gamma^2 \hbar^4 + 282 T^9 \gamma^2 \hbar^4 - 117 T^{10} \gamma^2 \hbar^4 + 21 T^{11} \gamma^2 \hbar^4 + 12 T^{12} \gamma^2 \hbar^4 - 9 T^{13} \gamma^2 \hbar^4 + 3 T^{14} \gamma^2 \hbar^4) \right\}$$

In[]:= **E3@E4@z3**

$$\text{Out[]:= } \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[\emptyset, \emptyset, \frac{\mathbb{T}}{1 - \mathbb{T} + \mathbb{T}^2} + \left(\frac{\mathbf{a} (-2 \mathbb{T} \hbar + 2 \mathbb{T}^3 \hbar)}{1 - 2 \mathbb{T} + 3 \mathbb{T}^2 - 2 \mathbb{T}^3 + \mathbb{T}^4} + \frac{-2 \mathbb{T} \gamma \hbar + 3 \mathbb{T}^2 \gamma \hbar - 2 \mathbb{T}^3 \gamma \hbar + \mathbb{T}^4 \gamma \hbar}{1 - 3 \mathbb{T} + 6 \mathbb{T}^2 - 7 \mathbb{T}^3 + 6 \mathbb{T}^4 - 3 \mathbb{T}^5 + \mathbb{T}^6} + \frac{\mathbf{x} \mathbf{y} (-2 \mathbb{T} \gamma \hbar^2 - 2 \mathbb{T}^2 \gamma \hbar^2)}{1 - 2 \mathbb{T} + 3 \mathbb{T}^2 - 2 \mathbb{T}^3 + \mathbb{T}^4} \right) \epsilon + \left(\frac{\mathbf{a}^2 (2 \mathbb{T} \hbar^2 + 2 \mathbb{T}^2 \hbar^2 - 12 \mathbb{T}^3 \hbar^2 + 2 \mathbb{T}^4 \hbar^2 + 2 \mathbb{T}^5 \hbar^2)}{1 - 3 \mathbb{T} + 6 \mathbb{T}^2 - 7 \mathbb{T}^3 + 6 \mathbb{T}^4 - 3 \mathbb{T}^5 + \mathbb{T}^6} + \left(\mathbf{a} (4 \mathbb{T} \gamma \hbar^2 - 4 \mathbb{T}^2 \gamma \hbar^2 - 14 \mathbb{T}^3 \gamma \hbar^2 + 16 \mathbb{T}^4 \gamma \hbar^2 - 10 \mathbb{T}^5 \gamma \hbar^2 + 4 \mathbb{T}^6 \gamma \hbar^2) \right) / \left(1 - 4 \mathbb{T} + 10 \mathbb{T}^2 - 16 \mathbb{T}^3 + 19 \mathbb{T}^4 - 16 \mathbb{T}^5 + 10 \mathbb{T}^6 - 4 \mathbb{T}^7 + \mathbb{T}^8 \right) + \left(4 \mathbb{T} \gamma^2 \hbar^2 - 11 \mathbb{T}^2 \gamma^2 \hbar^2 + 6 \mathbb{T}^3 \gamma^2 \hbar^2 - 2 \mathbb{T}^5 \gamma^2 \hbar^2 + 4 \mathbb{T}^6 \gamma^2 \hbar^2 - 2 \mathbb{T}^7 \gamma^2 \hbar^2 + \mathbb{T}^8 \gamma^2 \hbar^2 \right) / \left(2 - 10 \mathbb{T} + 30 \mathbb{T}^2 - 60 \mathbb{T}^3 + 90 \mathbb{T}^4 - 102 \mathbb{T}^5 + 90 \mathbb{T}^6 - 60 \mathbb{T}^7 + 30 \mathbb{T}^8 - 10 \mathbb{T}^9 + 2 \mathbb{T}^{10} \right) + \left(\mathbf{a} \mathbf{x} \mathbf{y} (4 \mathbb{T} \gamma \hbar^3 + 12 \mathbb{T}^2 \gamma \hbar^3 - 12 \mathbb{T}^3 \gamma \hbar^3 - 8 \mathbb{T}^4 \gamma \hbar^3) \right) / \left(1 - 3 \mathbb{T} + 6 \mathbb{T}^2 - 7 \mathbb{T}^3 + 6 \mathbb{T}^4 - 3 \mathbb{T}^5 + \mathbb{T}^6 \right) + \left(\mathbf{x} \mathbf{y} (2 \mathbb{T} \gamma^2 \hbar^3 - 2 \mathbb{T}^2 \gamma^2 \hbar^3 - 4 \mathbb{T}^3 \gamma^2 \hbar^3 - 4 \mathbb{T}^4 \gamma^2 \hbar^3 - 2 \mathbb{T}^5 \gamma^2 \hbar^3 + 2 \mathbb{T}^6 \gamma^2 \hbar^3) \right) / \left(1 - 4 \mathbb{T} + 10 \mathbb{T}^2 - 16 \mathbb{T}^3 + 19 \mathbb{T}^4 - 16 \mathbb{T}^5 + 10 \mathbb{T}^6 - 4 \mathbb{T}^7 + \mathbb{T}^8 \right) + \frac{\mathbf{x}^2 \mathbf{y}^2 (3 \mathbb{T} \gamma^2 \hbar^4 + 9 \mathbb{T}^2 \gamma^2 \hbar^4 + 3 \mathbb{T}^3 \gamma^2 \hbar^4)}{1 - 3 \mathbb{T} + 6 \mathbb{T}^2 - 7 \mathbb{T}^3 + 6 \mathbb{T}^4 - 3 \mathbb{T}^5 + \mathbb{T}^6} \right) \epsilon^2 + \mathcal{O}[\epsilon]^3]$$

In[]:= **HL [(E3@E4@z3) == z3]**

Out[]:= **True**