(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
<< SL2Invariant.m
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant\\k=2"];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

DirectoryName: String expected at position 1 in DirectoryName[File].

ParentDirectory: Argument DirectoryName[File] should be a positive machine–size integer, a nonempty string, or a File specification.

DeclarePackage: Symbol CreateWikiConnection in DeclarePackage[WikiLink`, {CreateWikiConnection, WikiGetPageText, WikiGetPageTexts, WikiSetPageText, WikiSetPageTexts, WikiUploadFile, WikiUserName, WikiPageMatchQ, WikiPageFreeQ, WikiStringReplace, ≪2≫}] has already been declared.

DeclarePackage: Symbol WikiGetPageText in DeclarePackage[WikiLink`, {CreateWikiConnection, WikiGetPageText, WikiGetPageTexts, WikiSetPageText, WikiSetPageTexts, WikiUploadFile, WikiUserName, WikiPageMatchQ, WikiPageFreeQ, WikiStringReplace, ≪2≫}] has already been declared.

DeclarePackage: Symbol WikiGetPageTexts in DeclarePackage[WikiLink`, {CreateWikiConnection, WikiGetPageText, WikiGetPageTexts, WikiSetPageText, WikiSetPageTexts, WikiUploadFile, WikiUserName, WikiPageMatchQ, WikiPageFreeQ, WikiStringReplace, ≪2≫}] has already been declared.

General: Further output of DeclarePackage::aldec will be suppressed during this calculation.

```
Loading QuantumGroups` version 2.0
Read more at http://katlas.math.toronto.edu/wiki/QuantumGroups
```

F: Symbol F appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

G: Symbol G appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

n: Symbol n appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

x: Symbol x appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

c: Symbol c appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

StringDrop: String or list of strings expected at position 1 in StringDrop[File, –14].

StringTake: String or list of strings expected at position 1 in StringTake[StringDrop[File, –14], –7].

Get: StringDrop[File, –14] in $Path is not a string.

Get: StringDrop[File, –14] in $Path is not a string.

Get: StringDrop[File, –14] in $Path is not a string.

General: Further output of Get::path will be suppressed during this calculation.

n$: Symbol n$ appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

x$: Symbol x$ appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

c$: Symbol c$ appears in multiple contexts {QuantumGroups`, Global`}; definitions in context QuantumGroups` may shadow or be shadowed by other definitions.

WikiGetPageText: You must call CreateWikiConnection before using WikiGetPageText or WikiSetPageText

StringSplit: String or list of strings expected at position 1 in StringSplit[Null, <tr>].

StringSplit: StringSplit called with 0 arguments; between 1 and 3 arguments are expected.

Join: Heads StringSplit and List at positions 1 and 2 are expected to be the same.

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

(Alt) In[ ]:=
```
OverbayP2Data = Get["C:\\drorbn\\AcademicPensieve\\People\\Overbay\\OverbayP2Data.m"];
```

(Alt) In[ ]:=
```
$k = 2;
ħ = γ = 1;
$QZipFail = True;
tab = Table[
  fname = "Data/" <> (K /. {
        Knot[n_, k_] :> ToString[n] <> "_" <> ToString[k],
        Knot[n_, Alternating, k_] :> ToString[n] <> "a" <> ToString[k],
        Knot[n_, NonAlternating, k_] :> ToString[n] <> "n" <> ToString[k]
       }) <> ".m";
  Get[fname],
  {K, AllKnots[{3, 12}]}
 ]
```

(Alt) Out[ ]=
$$\left\{\text{Knot}[3,1] \to \left\{23.6875, \mathbb{E}_{\{\}\to\{0\}}\left[\frac{1-T+T^2}{T}, \cdots 2 \cdots, \left\{1, \cdots 1 \cdots, \frac{4-27T+90T^2-198T^3+310T^4-357T^5+292T^6-141T^7-18\cdots 1 \cdots+115T^9-132T^{10}+99T^{11}-54T^{12}+22T^{13}-6T^{14}+T^{15}}{2T^8} + \frac{a\left(\cdots 21 \cdots+4\cdots 1 \cdots\right)}{T^8} + \cdots 1 \cdots + \cdots 1 \cdots + \frac{\left(\cdots 1 \cdots\right)\cdots 1 \cdots\cdots 1 \cdots y}{T^8} + \frac{\left(3-9T+12T^2+\cdots 16 \cdots+3T^{14}\right)x^2y^2}{T^8}\right\}\right], \cdots 2975 \cdots, \cdots 1 \cdots\right\}\right\}$$

large output | show less | show more | show all | set size limit...

In[ ]:=
```
Length[tab]
```

Out[ ]= 2977

In[•]:=
```
Ks = First /@ tab
```

Out[•]=
```
{Knot[3, 1], Knot[4, 1], Knot[5, 1], Knot[5, 2],
 Knot[6, 1], Knot[6, 2], Knot[6, 3], Knot[7, 1], Knot[7, 2],
 Knot[7, 3], Knot[7, 4], ··· 2955 ···, Knot[12, NonAlternating, 878],
 Knot[12, NonAlternating, 879], Knot[12, NonAlternating, 880],
 Knot[12, NonAlternating, 881], Knot[12, NonAlternating, 882],
 Knot[12, NonAlternating, 883], Knot[12, NonAlternating, 884],
 Knot[12, NonAlternating, 885], Knot[12, NonAlternating, 886],
 Knot[12, NonAlternating, 887], Knot[12, NonAlternating, 888]}
```

large output | show less | show more | show all | set size limit...

In[•]:=
```
Knot[3, 1] /. tab
```

Out[•]=
$$\left\{23.6875, \mathbb{E}_{\{\}\to\{0\}}\left[\frac{1-T+T^2}{T}, 0, 0, \left\{1, \frac{-2+7T-14T^2+18T^3-16T^4+10T^5-4T^6+T^7}{T^4}+\right.\right.\right.$$
$$\frac{a\left(-2+6T-10T^2+8T^3-8T^5+10T^6-6T^7+2T^8\right)}{T^4}+$$
$$\left.\left.\frac{\left(-2+4T-6T^2+2T^3+2T^4-6T^5+4T^6-2T^7\right)xy}{T^4},\right.\right.$$
$$\frac{1}{2T^8}\left(4-27T+90T^2-198T^3+310T^4-357T^5+292T^6-141T^7-18T^8+115T^9-132T^{10}+\right.$$
$$99T^{11}-54T^{12}+22T^{13}-6T^{14}+T^{15}\Big)+\frac{1}{T^8}a\left(4-24T+66T^2-94T^3+330T^5-896T^6+\right.$$
$$\left.1494T^7-1836T^8+1750T^9-1320T^{10}+786T^{11}-364T^{12}+126T^{13}-30T^{14}+4T^{15}\right)+$$
$$\frac{1}{T^8}a^2\left(2-10T+18T^2+16T^3-182T^4+558T^5-1108T^6+1622T^7-1836T^8+\right.$$
$$\left.1622T^9-1108T^{10}+558T^{11}-182T^{12}+16T^{13}+18T^{14}-10T^{15}+2T^{16}\right)+$$
$$\frac{1}{T^8}a\left(4-12T+116T^3-444T^4+1008T^5-1628T^6+1980T^7-1836T^8+1264T^9-\right.$$
$$\left.588T^{10}+108T^{11}+80T^{12}-84T^{13}+36T^{14}-8T^{15}\right)xy+\frac{1}{T^8}$$
$$\left(2-12T+36T^2-74T^3+108T^4-120T^5+92T^6-36T^7-36T^8+92T^9-120T^{10}+108T^{11}-\right.$$
$$\left.74T^{12}+36T^{13}-12T^{14}+2T^{15}\right)xy+\frac{1}{T^8}\left(3-9T+12T^2+21T^3-117T^4+282T^5-\right.$$
$$\left.\left.\left.441T^6+513T^7-441T^8+282T^9-117T^{10}+21T^{11}+12T^{12}-9T^{13}+3T^{14}\right)x^2y^2\right\}\right]\right\}$$

(Alt) In[•]:=
```
ρ1[K_] := ϵSeries @@ (K /. tab)[[2, 4, 1 ;; 2]];
ρ2[K_] := ϵSeries @@ (K /. tab)[[2, 4, 1 ;; 3]]
```

In[•]:=
```
{ρ1[Knot[3, 1]], ρ2[Knot[3, 1]]}
```

Out[•]=
$$\Big\{\in\text{Series}\Big[1,\ \frac{-2+7\,T-14\,T^2+18\,T^3-16\,T^4+10\,T^5-4\,T^6+T^7}{T^4}+\frac{a\left(-2+6\,T-10\,T^2+8\,T^3-8\,T^5+10\,T^6-6\,T^7+2\,T^8\right)}{T^4}+\frac{\left(-2+4\,T-6\,T^2+2\,T^3+2\,T^4-6\,T^5+4\,T^6-2\,T^7\right)x\,y}{T^4}\Big],\ \in\text{Series}\Big[1,\ \frac{-2+7\,T-14\,T^2+18\,T^3-16\,T^4+10\,T^5-4\,T^6+T^7}{T^4}+\frac{a\left(-2+6\,T-10\,T^2+8\,T^3-8\,T^5+10\,T^6-6\,T^7+2\,T^8\right)}{T^4}+\frac{\left(-2+4\,T-6\,T^2+2\,T^3+2\,T^4-6\,T^5+4\,T^6-2\,T^7\right)x\,y}{T^4},$$
$$\frac{1}{2\,T^8}\left(4-27\,T+90\,T^2-198\,T^3+310\,T^4-357\,T^5+292\,T^6-141\,T^7-18\,T^8+115\,T^9-132\,T^{10}+99\,T^{11}-54\,T^{12}+22\,T^{13}-6\,T^{14}+T^{15}\right)+\frac{1}{T^8}a\left(4-24\,T+66\,T^2-94\,T^3+330\,T^5-896\,T^6+1494\,T^7-1836\,T^8+1750\,T^9-1320\,T^{10}+786\,T^{11}-364\,T^{12}+126\,T^{13}-30\,T^{14}+4\,T^{15}\right)+$$
$$\frac{1}{T^8}a^2\left(2-10\,T+18\,T^2+16\,T^3-182\,T^4+558\,T^5-1108\,T^6+1622\,T^7-1836\,T^8+1622\,T^9-1108\,T^{10}+558\,T^{11}-182\,T^{12}+16\,T^{13}+18\,T^{14}-10\,T^{15}+2\,T^{16}\right)+$$
$$\frac{1}{T^8}a\left(4-12\,T+116\,T^3-444\,T^4+1008\,T^5-1628\,T^6+1980\,T^7-1836\,T^8+1264\,T^9-588\,T^{10}+108\,T^{11}+80\,T^{12}-84\,T^{13}+36\,T^{14}-8\,T^{15}\right)x\,y+\frac{1}{T^8}$$
$$\left(2-12\,T+36\,T^2-74\,T^3+108\,T^4-120\,T^5+92\,T^6-36\,T^7-36\,T^8+92\,T^9-120\,T^{10}+108\,T^{11}-74\,T^{12}+36\,T^{13}-12\,T^{14}+2\,T^{15}\right)x\,y+\frac{1}{T^8}\left(3-9\,T+12\,T^2+21\,T^3-117\,T^4+282\,T^5-441\,T^6+513\,T^7-441\,T^8+282\,T^9-117\,T^{10}+21\,T^{11}+12\,T^{12}-9\,T^{13}+3\,T^{14}\right)x^2\,y^2\Big]\Big\}$$

In[•]:=
```
Union[ρ1 /@ Ks] // Length
```

Out[•]=
```
2882
```

In[•]:=
```
Union[ρ2 /@ Ks] // Length
```

Out[•]=
```
2893
```

In[•]:=
```
classes1 = DeleteCases[GatherBy[Ks, ρ1], {_}]
```

Out[•]=
```
{{Knot[10, 106], Knot[12, NonAlternating, 369]},
 {Knot[11, Alternating, 19], Knot[11, Alternating, 25]},
 {Knot[11, Alternating, 24], Knot[11, Alternating, 26]},
 {Knot[11, Alternating, 44], Knot[11, Alternating, 47]},
 {Knot[11, Alternating, 57], Knot[11, Alternating, 231]},
 {Knot[11, Alternating, 251], Knot[11, Alternating, 253]},
 {Knot[11, Alternating, 252], Knot[11, Alternating, 254]},
 {Knot[11, NonAlternating, 34], Knot[11, NonAlternating, 42]},
```

```
{Knot[11, NonAlternating, 35], Knot[11, NonAlternating, 43]},
{Knot[11, NonAlternating, 36], Knot[11, NonAlternating, 44]},
{Knot[11, NonAlternating, 39], Knot[11, NonAlternating, 45]},
{Knot[11, NonAlternating, 40], Knot[11, NonAlternating, 46]},
{Knot[11, NonAlternating, 41], Knot[11, NonAlternating, 47]},
{Knot[11, NonAlternating, 73], Knot[11, NonAlternating, 74]},
{Knot[11, NonAlternating, 151], Knot[11, NonAlternating, 152]},
{Knot[12, Alternating, 7], Knot[12, Alternating, 14]},
{Knot[12, Alternating, 13], Knot[12, Alternating, 15]},
{Knot[12, Alternating, 30], Knot[12, Alternating, 33]},
{Knot[12, Alternating, 36], Knot[12, Alternating, 694]},
{Knot[12, Alternating, 44], Knot[12, Alternating, 64]},
{Knot[12, Alternating, 45], Knot[12, Alternating, 65]},
{Knot[12, Alternating, 48], Knot[12, Alternating, 60]},
{Knot[12, Alternating, 59], Knot[12, Alternating, 63]},
{Knot[12, Alternating, 67], Knot[12, Alternating, 136]},
{Knot[12, Alternating, 91], Knot[12, Alternating, 111]},
{Knot[12, Alternating, 101], Knot[12, Alternating, 115]},
{Knot[12, Alternating, 102], Knot[12, Alternating, 107]},
{Knot[12, Alternating, 108], Knot[12, Alternating, 120]},
{Knot[12, Alternating, 114], Knot[12, Alternating, 117]},
{Knot[12, Alternating, 116], Knot[12, Alternating, 122], Knot[12, Alternating, 182]},
{Knot[12, Alternating, 126], Knot[12, Alternating, 132]},
{Knot[12, Alternating, 131], Knot[12, Alternating, 133]},
{Knot[12, Alternating, 134], Knot[12, Alternating, 188]},
{Knot[12, Alternating, 154], Knot[12, Alternating, 162]},
{Knot[12, Alternating, 164], Knot[12, Alternating, 166]},
{Knot[12, Alternating, 167], Knot[12, Alternating, 692]},
{Knot[12, Alternating, 195], Knot[12, Alternating, 693]},
{Knot[12, Alternating, 273], Knot[12, Alternating, 890]},
{Knot[12, Alternating, 341], Knot[12, Alternating, 627]},
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]},
{Knot[12, Alternating, 458], Knot[12, Alternating, 887]},
{Knot[12, Alternating, 510], Knot[12, Alternating, 821]},
{Knot[12, Alternating, 639], Knot[12, Alternating, 680]},
{Knot[12, Alternating, 675], Knot[12, Alternating, 688]},
{Knot[12, Alternating, 707], Knot[12, Alternating, 935]},
{Knot[12, Alternating, 811], Knot[12, Alternating, 817]},
{Knot[12, Alternating, 829], Knot[12, Alternating, 832]},
{Knot[12, Alternating, 830], Knot[12, Alternating, 831]},
{Knot[12, Alternating, 844], Knot[12, Alternating, 846]},
{Knot[12, Alternating, 908], Knot[12, Alternating, 1185]},
{Knot[12, NonAlternating, 21], Knot[12, NonAlternating, 29]},
{Knot[12, NonAlternating, 22], Knot[12, NonAlternating, 30]},
{Knot[12, NonAlternating, 23], Knot[12, NonAlternating, 31]},
{Knot[12, NonAlternating, 26], Knot[12, NonAlternating, 32]},
{Knot[12, NonAlternating, 27], Knot[12, NonAlternating, 33]},
{Knot[12, NonAlternating, 28], Knot[12, NonAlternating, 34]},
{Knot[12, NonAlternating, 55], Knot[12, NonAlternating, 223]},
{Knot[12, NonAlternating, 56], Knot[12, NonAlternating, 57]},
{Knot[12, NonAlternating, 59], Knot[12, NonAlternating, 220]},
```

```
{Knot[12, NonAlternating, 60],
 Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]},
{Knot[12, NonAlternating, 62], Knot[12, NonAlternating, 66]},
{Knot[12, NonAlternating, 64], Knot[12, NonAlternating, 261]},
{Knot[12, NonAlternating, 85], Knot[12, NonAlternating, 130]},
{Knot[12, NonAlternating, 86], Knot[12, NonAlternating, 131]},
{Knot[12, NonAlternating, 87], Knot[12, NonAlternating, 132]},
{Knot[12, NonAlternating, 88], Knot[12, NonAlternating, 133]},
{Knot[12, NonAlternating, 89], Knot[12, NonAlternating, 134]},
{Knot[12, NonAlternating, 90], Knot[12, NonAlternating, 135]},
{Knot[12, NonAlternating, 91], Knot[12, NonAlternating, 136]},
{Knot[12, NonAlternating, 92], Knot[12, NonAlternating, 137]},
{Knot[12, NonAlternating, 93], Knot[12, NonAlternating, 138]},
{Knot[12, NonAlternating, 98], Knot[12, NonAlternating, 125]},
{Knot[12, NonAlternating, 99], Knot[12, NonAlternating, 126]},
{Knot[12, NonAlternating, 122], Knot[12, NonAlternating, 127]},
{Knot[12, NonAlternating, 123], Knot[12, NonAlternating, 128]},
{Knot[12, NonAlternating, 124], Knot[12, NonAlternating, 129]},
{Knot[12, NonAlternating, 144], Knot[12, NonAlternating, 507]},
{Knot[12, NonAlternating, 205], Knot[12, NonAlternating, 226]},
{Knot[12, NonAlternating, 206], Knot[12, NonAlternating, 227]},
{Knot[12, NonAlternating, 207], Knot[12, NonAlternating, 228]},
{Knot[12, NonAlternating, 208], Knot[12, NonAlternating, 212]},
{Knot[12, NonAlternating, 209], Knot[12, NonAlternating, 213]},
{Knot[12, NonAlternating, 210], Knot[12, NonAlternating, 214]},
{Knot[12, NonAlternating, 231], Knot[12, NonAlternating, 232]},
{Knot[12, NonAlternating, 252], Knot[12, NonAlternating, 262]},
{Knot[12, NonAlternating, 255], Knot[12, NonAlternating, 263]},
{Knot[12, NonAlternating, 256], Knot[12, NonAlternating, 264]},
{Knot[12, NonAlternating, 313], Knot[12, NonAlternating, 430]},
{Knot[12, NonAlternating, 670], Knot[12, NonAlternating, 681]},
{Knot[12, NonAlternating, 671], Knot[12, NonAlternating, 682]},
{Knot[12, NonAlternating, 691], Knot[12, NonAlternating, 692]},
{Knot[12, NonAlternating, 693], Knot[12, NonAlternating, 696]}}
```

In[•]:=
```
Tally[Length /@ classes1]
```

Out[•]=
```
{{2, 89}, {3, 3}}
```

In[•]:=
```
classes2 = DeleteCases[GatherBy[Ks, ρ2], {_}]
```

Out[•]=
```
{{Knot[11, Alternating, 19], Knot[11, Alternating, 25]},
 {Knot[11, Alternating, 24], Knot[11, Alternating, 26]},
 {Knot[11, Alternating, 44], Knot[11, Alternating, 47]},
 {Knot[11, Alternating, 57], Knot[11, Alternating, 231]},
 {Knot[11, Alternating, 251], Knot[11, Alternating, 253]},
 {Knot[11, Alternating, 252], Knot[11, Alternating, 254]},
 {Knot[11, NonAlternating, 34], Knot[11, NonAlternating, 42]},
 {Knot[11, NonAlternating, 35], Knot[11, NonAlternating, 43]},
 {Knot[11, NonAlternating, 36], Knot[11, NonAlternating, 44]},
 {Knot[11, NonAlternating, 39], Knot[11, NonAlternating, 45]},
 {Knot[11, NonAlternating, 40], Knot[11, NonAlternating, 46]},
 {Knot[11, NonAlternating, 41], Knot[11, NonAlternating, 47]},
```

```
{Knot[11, NonAlternating, 73], Knot[11, NonAlternating, 74]},
{Knot[11, NonAlternating, 151], Knot[11, NonAlternating, 152]},
{Knot[12, Alternating, 7], Knot[12, Alternating, 14]},
{Knot[12, Alternating, 13], Knot[12, Alternating, 15]},
{Knot[12, Alternating, 30], Knot[12, Alternating, 33]},
{Knot[12, Alternating, 36], Knot[12, Alternating, 694]},
{Knot[12, Alternating, 44], Knot[12, Alternating, 64]},
{Knot[12, Alternating, 45], Knot[12, Alternating, 65]},
{Knot[12, Alternating, 48], Knot[12, Alternating, 60]},
{Knot[12, Alternating, 59], Knot[12, Alternating, 63]},
{Knot[12, Alternating, 67], Knot[12, Alternating, 136]},
{Knot[12, Alternating, 91], Knot[12, Alternating, 111]},
{Knot[12, Alternating, 101], Knot[12, Alternating, 115]},
{Knot[12, Alternating, 102], Knot[12, Alternating, 107]},
{Knot[12, Alternating, 108], Knot[12, Alternating, 120]},
{Knot[12, Alternating, 114], Knot[12, Alternating, 117]},
{Knot[12, Alternating, 116], Knot[12, Alternating, 122], Knot[12, Alternating, 182]},
{Knot[12, Alternating, 126], Knot[12, Alternating, 132]},
{Knot[12, Alternating, 131], Knot[12, Alternating, 133]},
{Knot[12, Alternating, 134], Knot[12, Alternating, 188]},
{Knot[12, Alternating, 154], Knot[12, Alternating, 162]},
{Knot[12, Alternating, 164], Knot[12, Alternating, 166]},
{Knot[12, Alternating, 167], Knot[12, Alternating, 692]},
{Knot[12, Alternating, 195], Knot[12, Alternating, 693]},
{Knot[12, Alternating, 639], Knot[12, Alternating, 680]},
{Knot[12, Alternating, 675], Knot[12, Alternating, 688]},
{Knot[12, Alternating, 811], Knot[12, Alternating, 817]},
{Knot[12, Alternating, 829], Knot[12, Alternating, 832]},
{Knot[12, Alternating, 830], Knot[12, Alternating, 831]},
{Knot[12, Alternating, 844], Knot[12, Alternating, 846]},
{Knot[12, NonAlternating, 21], Knot[12, NonAlternating, 29]},
{Knot[12, NonAlternating, 22], Knot[12, NonAlternating, 30]},
{Knot[12, NonAlternating, 23], Knot[12, NonAlternating, 31]},
{Knot[12, NonAlternating, 26], Knot[12, NonAlternating, 32]},
{Knot[12, NonAlternating, 27], Knot[12, NonAlternating, 33]},
{Knot[12, NonAlternating, 28], Knot[12, NonAlternating, 34]},
{Knot[12, NonAlternating, 55], Knot[12, NonAlternating, 223]},
{Knot[12, NonAlternating, 56], Knot[12, NonAlternating, 57]},
{Knot[12, NonAlternating, 59], Knot[12, NonAlternating, 220]},
{Knot[12, NonAlternating, 60],
 Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]},
{Knot[12, NonAlternating, 62], Knot[12, NonAlternating, 66]},
{Knot[12, NonAlternating, 64], Knot[12, NonAlternating, 261]},
{Knot[12, NonAlternating, 85], Knot[12, NonAlternating, 130]},
{Knot[12, NonAlternating, 86], Knot[12, NonAlternating, 131]},
{Knot[12, NonAlternating, 87], Knot[12, NonAlternating, 132]},
{Knot[12, NonAlternating, 88], Knot[12, NonAlternating, 133]},
{Knot[12, NonAlternating, 89], Knot[12, NonAlternating, 134]},
{Knot[12, NonAlternating, 90], Knot[12, NonAlternating, 135]},
{Knot[12, NonAlternating, 91], Knot[12, NonAlternating, 136]},
{Knot[12, NonAlternating, 92], Knot[12, NonAlternating, 137]},
```

```
 {Knot[12, NonAlternating, 93], Knot[12, NonAlternating, 138]},
 {Knot[12, NonAlternating, 98], Knot[12, NonAlternating, 125]},
 {Knot[12, NonAlternating, 99], Knot[12, NonAlternating, 126]},
 {Knot[12, NonAlternating, 122], Knot[12, NonAlternating, 127]},
 {Knot[12, NonAlternating, 123], Knot[12, NonAlternating, 128]},
 {Knot[12, NonAlternating, 124], Knot[12, NonAlternating, 129]},
 {Knot[12, NonAlternating, 205], Knot[12, NonAlternating, 226]},
 {Knot[12, NonAlternating, 206], Knot[12, NonAlternating, 227]},
 {Knot[12, NonAlternating, 207], Knot[12, NonAlternating, 228]},
 {Knot[12, NonAlternating, 208], Knot[12, NonAlternating, 212]},
 {Knot[12, NonAlternating, 209], Knot[12, NonAlternating, 213]},
 {Knot[12, NonAlternating, 210], Knot[12, NonAlternating, 214]},
 {Knot[12, NonAlternating, 231], Knot[12, NonAlternating, 232]},
 {Knot[12, NonAlternating, 252], Knot[12, NonAlternating, 262]},
 {Knot[12, NonAlternating, 255], Knot[12, NonAlternating, 263]},
 {Knot[12, NonAlternating, 256], Knot[12, NonAlternating, 264]},
 {Knot[12, NonAlternating, 670], Knot[12, NonAlternating, 681]},
 {Knot[12, NonAlternating, 671], Knot[12, NonAlternating, 682]},
 {Knot[12, NonAlternating, 691], Knot[12, NonAlternating, 692]},
 {Knot[12, NonAlternating, 693], Knot[12, NonAlternating, 696]}}
```

In[ ]:=
```
Tally[Length /@ classes2]
```

Out[ ]=
```
{{2, 80}, {3, 2}}
```

In[ ]:=
```
Complement[classes1, classes2]
```

Out[ ]=
```
{{Knot[10, 106], Knot[12, NonAlternating, 369]},
 {Knot[12, Alternating, 273], Knot[12, Alternating, 890]},
 {Knot[12, Alternating, 341], Knot[12, Alternating, 627]},
 {Knot[12, Alternating, 458], Knot[12, Alternating, 887]},
 {Knot[12, Alternating, 510], Knot[12, Alternating, 821]},
 {Knot[12, Alternating, 707], Knot[12, Alternating, 935]},
 {Knot[12, Alternating, 908], Knot[12, Alternating, 1185]},
 {Knot[12, NonAlternating, 144], Knot[12, NonAlternating, 507]},
 {Knot[12, NonAlternating, 313], Knot[12, NonAlternating, 430]},
 {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]}}
```

In[ ]:=
```
Complement[classes2, classes1]
```

Out[ ]=
```
{}
```

In[ ]:=
```
equiv =
  {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[ρ1 /@ equiv]
Length@Union[ρ2 /@ equiv]
```

Out[ ]=
```
1
```

Out[ ]=
```
3
```

```
In[•]:= equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
        Length@Union[ρ1 /@ equiv]
        Length@Union[ρ2 /@ equiv]
Out[•]= 1

Out[•]= 2

(Alt) In[•]:= Total[First /@ ({Knot[10, 106], Knot[12, NonAlternating, 369]} /. tab)]
(Alt) Out[•]= 6277.86

(Alt) In[•]:= First /@ ({Knot[12, Alternating, 427], Knot[12, Alternating, 435],
                Knot[12, Alternating, 990], Knot[10, 106], Knot[12, NonAlternating, 369]} /. tab)
(Alt) Out[•]= {6234.28, 17 602.3, 20 933.4, 2165.66, 4112.2}

(Alt) In[•]:= Total[First /@ ({Knot[12, Alternating, 427], Knot[12, Alternating, 435],
                 Knot[12, Alternating, 990], Knot[10, 106], Knot[12, NonAlternating, 369]} /. tab)]
(Alt) Out[•]= 51 047.9

(Alt) In[•]:= Total[First /@ ({Knot[12, NonAlternating, 60],
                 Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]} /. tab)]
(Alt) Out[•]= 10 544.
```