The GPV/BHLR Intuition

May-08-11
11:08 AM

All ~~should~~ may be viewed within a free algebra F, and I_F is the ideal of degree ≥ 1 elements.

All is concrete. This must be easy.

$$\overbrace{a_{ij}a_{ik}a_{jk} - a_{jk}a_{ik}a_{ij} + \underbrace{[a_{ij}, a_{ik}] + [a_{ij}, a_{jk}] + [a_{ik}, a_{jk}]}_{y_{ijk}}}^{Y_{ijk}} = 0$$

$$c^{ij}_{kl} \leftrightarrow [a_{ij}, a_{kl}]$$

$$\mathcal{D}: FA(a_{ij}, \eta_{ijk}, \gamma^{ij}_{kl})$$

	a_{ij}	$\eta_{ijk}, \gamma^{ij}_{kl}$
height	0	1
deg	1	2/indefinite

$\mathcal{D}_0$: height 0

$\mathcal{D}_1$: height 1

$\partial^{loc}: \mathcal{D}_1 \longrightarrow \mathcal{D}_0$ by $(\eta, \gamma) \longmapsto (y, c)$

$\partial^{glob}: \mathcal{D}_1 \longrightarrow \mathcal{D}_0$ by $(\eta, \gamma) \longmapsto (Y, c)$

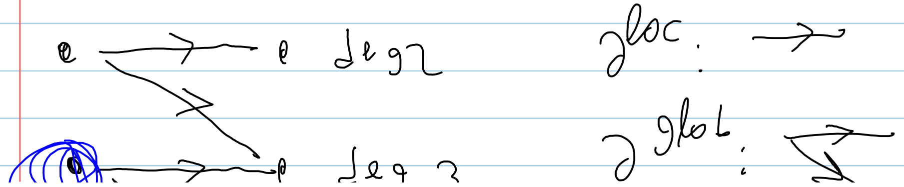
Do these have the same rank?

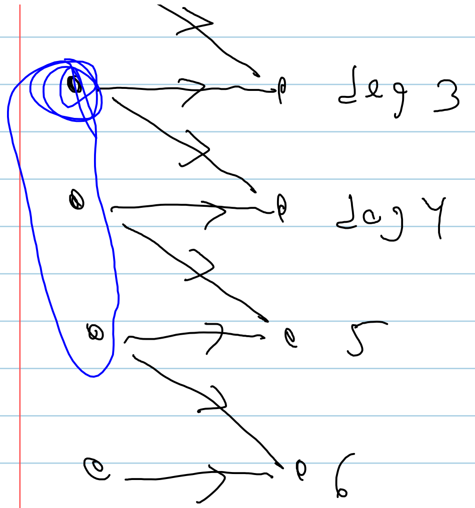
Really, there's a^+_{ij} & a^-_{ij} with the additional relation $a^+ + a^- + a^+a^- = 0$; namely

$a^- = -a^+ + (a^+)^2 - (a^+)^3 + \ldots$,

but this makes no difference.

(key: PvB_n has only "positive relations")





∂^{glob}:

claim rank $\partial^{loc} \le$ rank ∂^{glob}

$$\begin{tikzcd} \mathcal{D}_1 \arrow[r, "\partial^{loc}"] \arrow[dr, "\partial^{glob}"'] & \mathcal{D}_0 \\ & \mathcal{D}_0 \arrow[u, "\exists"'] \end{tikzcd} \quad \text{ok}$$

$$\text{nullity}(\partial^{loc}) \ge \text{nullity}(\partial^{glob})$$

$$\begin{tikzcd} \mathcal{D}_1 \arrow[dr, "\partial^{loc}"] \arrow[d, "\exists"'] & \\ \mathcal{D}_1 \arrow[r, "\partial^{glob}"'] & \mathcal{D}_0 \end{tikzcd} \quad ?$$

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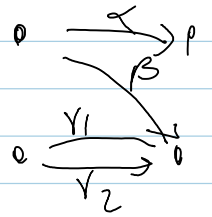
claim 1 

if rank $\gamma_1 \le$ rank γ_2 then

$$\text{rank}\begin{pmatrix}\alpha & 0\\ 0 & \gamma_1\end{pmatrix} \le \text{rank}\begin{pmatrix}\alpha & 0\\ \beta & \gamma_2\end{pmatrix}$$

always.

claim 2 If in addition rank $\gamma_1 =$ rank γ_2 and $\ker\alpha \subset \ker\beta$, then rank$\begin{pmatrix}\alpha & 0\\ 0 & \gamma_1\end{pmatrix} =$ rank$\begin{pmatrix}\alpha & 0\\ \beta & \gamma_2\end{pmatrix}$

Added Aug 16, 2011. It seems that instead of

$$\text{im}\,\beta|_{\ker\alpha} = 0$$

we only get

$$\operatorname{im} \beta|_{\ker \alpha} \subset \operatorname{im} \begin{pmatrix} \gamma_1 & 0 \\ \gamma_2 & 0 \end{pmatrix}$$

is this enough? Aug 19: Yes, by a mini spectral sequence argument for the filtered complex

$$0 \longrightarrow \mathcal{D}_1 \xrightarrow{\partial^{glob}} \mathcal{D}_0 \longrightarrow 0 .$$