Oregon Handout as of July 7, 2011

July-06-11
9:49 PM

Consider including A-~~D~~C of 2011-05/Handout as of May 20

Foot 1

The Pure Virtual Braid Group is Quadratic[1]

Dror Bar–Natan and Peter Lee in Oregon, August 2011

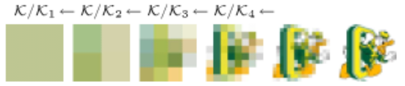
http://www.math.toronto.edu/~drorbn/Talks/Oregon-1108/
Foots & refs on PDF version, page 2.

Just for fun.

$\mathcal{K} = \{$  $\} = \left(\begin{array}{c}\text{The set of all}\\ \text{2D projec-}\\ \text{tions of reality}\end{array}\right)$

$\mathcal{K}/\mathcal{K}_1 \leftarrow \mathcal{K}/\mathcal{K}_2 \leftarrow \mathcal{K}/\mathcal{K}_3 \leftarrow \mathcal{K}/\mathcal{K}_4 \leftarrow \cdots$

Crop
Rotate
Adjoin



An expansion Z is a choice of a "progressive scan" algorithm.

crop
rotate
adjoin

$\mathcal{K}/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \mathcal{K}_4/\mathcal{K}_5 \oplus \mathcal{K}_5/\mathcal{K}_6 \oplus \cdots$

$\| \quad \mathbb{R}^3 \qquad \| \quad \ker(\mathcal{K}/\mathcal{K}_4 \to \mathcal{K}/\mathcal{K}_3)$

move to bottom.

Let K be an algebra over a field $\mathbb{F}$ with char $\mathbb{F}=0$, and let $I \subset \mathbb{F}$ be an "augmentation ideal"; $K/I = \mathbb{F}$.

Definition We say that K is quadratic if its associated graded $\operatorname{gr} K := \bigoplus_{m=0} I^m/I^{m+1}$ is a quadratic algebra. Alternatively, let $A = Q(K) = \langle I/I^2 \rangle / \ker(\mu_{11} : I/I^2 \otimes I/I^2 \to I^2/I^3)$ be the "quadratic approximation" to K (Q is a lovely functor). Then K is quadratic iff the obvious $\mu : A \to \operatorname{gr} K$ is an isomorphism.

Why care for quadraticity?

* In abstract generality, $\operatorname{gr} K$ is a simplified version of K, and if it is quadratic, it is as simple as it may be w/o being silly.
* In some concrete somewhat generalized knot-theoretic cases, A is a space of "universal Lie algebraic formulas"; and the "primary approach" Z to proving quadraticity becomes wonderful mathematics:

include many refs.

	u knots/braids	v knots (in progress)	w- knots.
A describes	metrized Lie algs.	Lie bialgebras	arbit. f.d. Lie algs
		Etingof-Kazh.	Alek-tor-

	u knots/braids	v knots (in progress)	w-knots.
A describes	metrized Lie algs.	Lie bialgebras	arbit. f.d. Lie algs
Z describes	Associators	Etingof-Kazh. quantization	Alek-tor-kash-Vergne.

DoF of homomorphic exp.

include many refs.

α: Work presented to the great algebra masters of the Oregon school, in humble pursuit of their wisdom and advice, in humble acceptance that they know all and have seen all, and in humble dread that we will inflict boredom upon them.

Footnotes

1. ~~foot1.~~ following an $\underbrace{\text{equi-named}}_{}$ word? paper and thesis by Peter Lee; see ___; → all serious work here is his, page design by DBN.

References

[BEER] L. Bartholdi, B. Enriquez, P. Etingof, and E. Rains, *Groups and Lie algebras corresponding to the YangBaxter equations,* Jornal of Algebra **305-2** (2006) 742-764, arXiv:math.RA/0509661.