Oregon Handout as of August 1, 2011

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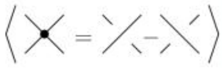
The Pure Virtual Braid Group is Quadratic, II

Examples and Interpretations

Dror Bar-Natan and Peter Lee in Oregon, August 2011
http://www.math.toronto.edu/~drorbn/Talks/Oregon-1108/

Example. (goes back to [Koh])

T. Kohno

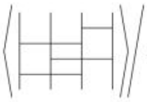
$K =$  $I =$ 

$(K/I^{p+1})^\star =$ (invariants of type p) $=: \mathcal{V}_p$

$(I^p/I^{p+1})^\star = \mathcal{V}_p/\mathcal{V}_{p-1}$ $\cancel{\cong}$ $\langle t^{ij} | t^{ij} = t^{ji}\rangle = \langle | \ \vdash\!\dashv \rangle$

$\ker \bar\mu_2 = \langle [t^{ij}, t^{kl}] = 0 = [t^{ij}, t^{ik} + t^{jk}]\rangle = \langle 4T \text{ relations}\rangle$

$A = q K =$

$A = \left(\begin{array}{l}\text{horizontal chord dia-}\\ \text{grams mod 4T}\end{array}\right) =$  $/4T$

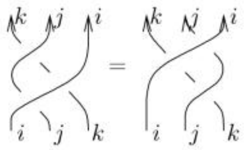
Z: universal finite type invariant, the Kontsevich integral.

PvB_n is the group

$$\langle \sigma_{ij} : 1 \le i \ne j \le n \rangle \Big/ \begin{array}{c}\sigma_{ij}\sigma_{ik}\sigma_{jk} = \sigma_{jk}\sigma_{ik}\sigma_{ij}\\ \sigma_{ij}\sigma_{kl} = \sigma_{kl}\sigma_{ij}\end{array}$$

L. Kauffman [Kau, KL]

of "pure virtual braids" ("braids when you look", "blunder braids"):

$\sigma_{24} =$  R3: 

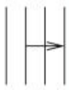
The Main Theorem [Lee]. PvB_n is quadratic.

$A_n = q(PvB_n)$. [GPV]

Goussarov-Polyak-Viro

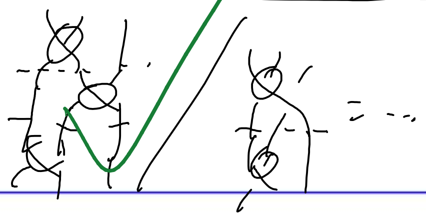
$I =$  with $\bar\sigma_{ij} = \sigma_{ij} - 1$, the "semi-virtual crossing".

$V = I/I^2 = \left\langle \begin{array}{c}\text{v-braids}\\ \text{with one}\end{array} \right\rangle \Big/ (\ = \ )$

$= \langle a_{ij} \rangle_{1 \le i \ne j \le n}$ $a_{24} =$ 

So $A_n = TV/[a_{ij}, a_{ik}] + \dots$ etc.

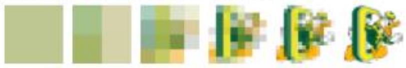
draw 6T

$I^{:3} =$  $= \dots$

Just for fun. $\mathcal{K} =$  $=$ (The set of all 2D projections of reality)

$\mathcal{K}/\mathcal{K}_1 \leftarrow \mathcal{K}/\mathcal{K}_2 \leftarrow \mathcal{K}/\mathcal{K}_3 \leftarrow \mathcal{K}/\mathcal{K}_4 \leftarrow \dots$

Crop Rotate Adjoin



An expansion Z is a choice of a "progressive scan" algorithm.

crop rotate adjoin

$\mathcal{K}/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \mathcal{K}_4/\mathcal{K}_5 \oplus \mathcal{K}_5/\mathcal{K}_6 \oplus \cdots$

$\mathcal{K}/\mathcal{K}_1 = \mathbb{R}^3$, $\mathcal{K}_3/\mathcal{K}_4 = \ker(\mathcal{K}/\mathcal{K}_4 \to \mathcal{K}/\mathcal{K}_3)$

Figuring out R_2:

$$\begin{array}{ccc} J^{\otimes 2} & \xrightarrow[(-1)]{\mu^F} & J \supset M \\ \downarrow \pi_2 & & \downarrow \pi_1 \\ I^{\otimes 2} & \xrightarrow{\mu} & I = J/M \end{array}$$

$\ker \mu = \pi_2(\mu_F^{-1}(M))$

in principle computable.

Add one sample syzygy.

Change \frakR_2 to virtuals!!!!!