Hutchings with Unrelated Relations

January-18-11
12:51 PM

should be onto.

$$\begin{tikzcd}
 & \ker b \arrow[r, "\pi'"] \arrow[d] & \ker \bar b \arrow[d] & \\
(\ker \pi) \arrow[r] \arrow[d, "o"] & K'_n \arrow[r, "\pi"] \arrow[d, "b"] & \mathcal{D}'_n \arrow[r] \arrow[d, "\bar b"] \arrow[dl, red, "\exists!"'] & 0 \\
0 \arrow[r] & \frac{K^0_{n+1}}{bK'_{n+1}} \arrow[r, "f"] \arrow[d, "I"] & K^0_n \arrow[r, "\pi"] \arrow[d] & \mathcal{D}^0_n \arrow[r] \arrow[d] & 0 \\
 & K^0_{n+1}/bK'_{n+1} \arrow[r] \arrow[d] & K^0_n/bK'_n \arrow[r] \arrow[d] & A_n \arrow[d] & \\
 & 0 & 0 & 0 &
\end{tikzcd}$$

Want 1-1

The Snake Lemma is the wrong tool to use here. Which is the right?

Need: 1. bK'_n, bK'_{n+1} to be "all the relations that we want".

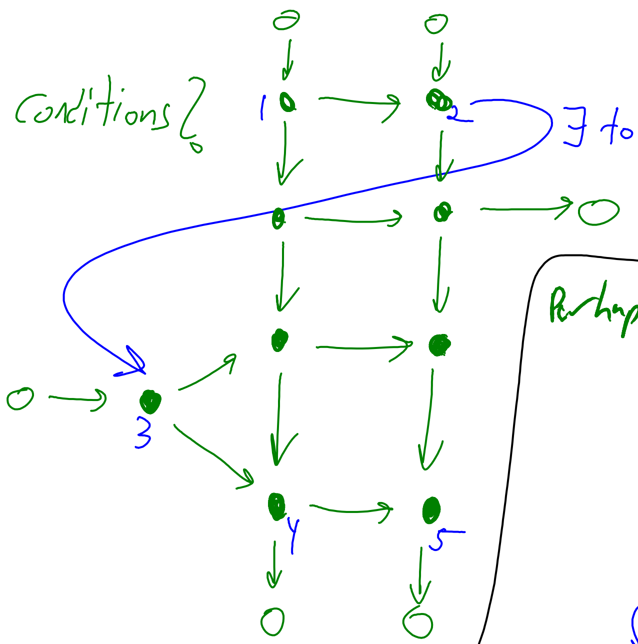
Inductive meaning would be

$$K^0_n / bK'_n \xrightarrow{1-1} K$$

or, "any once-integrable invariant is integrable".

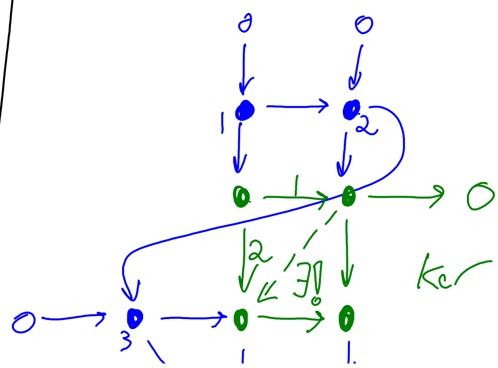
The Green Stuff.

Conditions?



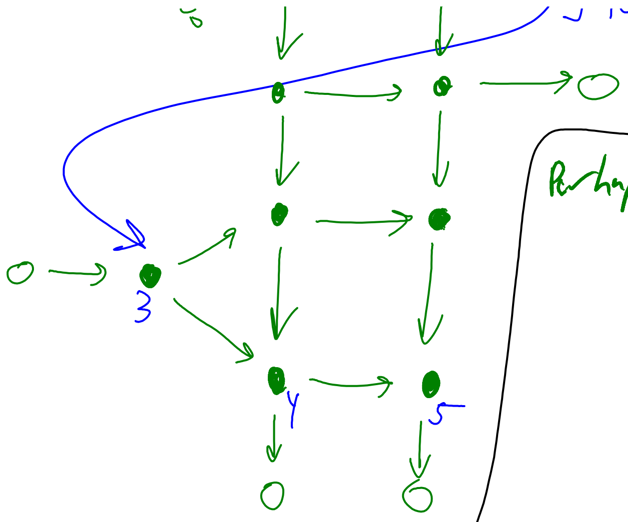
∃ to make 1→5 exact.

Perhaps even,

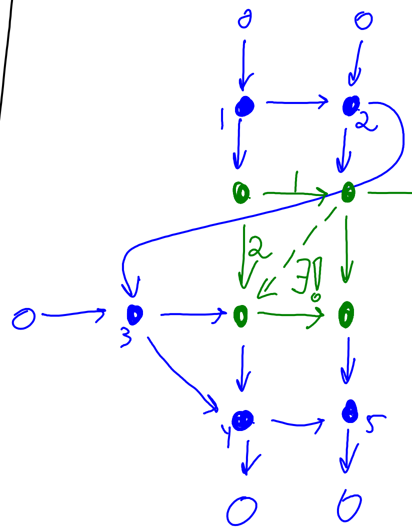


ker 1 ⊂ ker 2

exact.



Perhaps even,



ker 1 ⊂ ker 2

$$\begin{tikzcd}
 & \ker b \arrow[r] \arrow[d] & \ker \bar{b} \arrow[d] & \\
K^n_{n+1} \arrow[r] \arrow[d, "0"] & K^1_n \arrow[r] \arrow[d, "b"] & \mathcal{D}^1_n \arrow[r] \arrow[d, "\bar{b}"] & 0 \\
0 \arrow[r] & K^0_{n+1}/bK^1_{n+1} \arrow[r] \arrow[d] & K^0_n \arrow[r] \arrow[d] & \mathcal{D}^0_n \arrow[r] \arrow[d] & 0 \\
 & K^0_{n+1}/bK^1_{n+1} \arrow[r] \arrow[d] & K^0_n/\delta K^1_n \arrow[r] \arrow[d] & \mathcal{A}_n \arrow[d] & \\
 & 0 & 0 & 0 &
\end{tikzcd}$$

Diffability in (R,I)?

$I \otimes_R I^2 \otimes_R I \to$

$\alpha \otimes \beta\gamma \otimes \delta$

$I \,|\, I/R \,|\, I \,|\, I/R \,|\, R$

$R \otimes_Q I \otimes_Q R$, I } non-isomorphic R-bimodules.

$\exists!\ \mathcal{D}'_n \xrightarrow{\lambda} K^0_n$? $\qquad K^0_n = \underbrace{I \otimes_R I \otimes_R \cdots \otimes_R I}_{n \text{ times}}$

$$\mathcal{D}'_n = \bigoplus (I/I^2) \otimes_Q \cdots \otimes_Q (I/I^2) \otimes_Q \underline{\left[\ker\left(\tfrac{I}{I^2} \otimes \tfrac{I}{I^2} \xrightarrow{\mu} \tfrac{I^2}{I^3}\right)\right]} \otimes_Q \cdots \otimes_Q I/I^2$$

$$\lambda([\alpha_1] \otimes \cdots \underline{\alpha_i \otimes \alpha_{i+1}} \otimes \cdots \otimes [\alpha_n]) = \alpha_1 \otimes \cdots \otimes \alpha_n$$ well defined?

$$K'_n = \bigoplus I^{\otimes_R (i-1)} \otimes_R \left[\ker\left(I \otimes_R I \xrightarrow{\mu} I^2\right)\right] \otimes_R I^{\otimes_R (n-i-1)}$$

The people want a well-defined $\delta : K'_n \to K'_{n-1}$!

$\alpha_1 | \alpha_2 \alpha_3 | \alpha_4^j \alpha_5^j | \alpha_6 | \alpha_7 | \alpha_8$

The Prerequisites:

$$\begin{tikzcd}
\cdots \arrow[r, "\delta"] & \mathcal{K}^1_{n+1} \arrow[r, "\delta"] \arrow[d, "b"] & \mathcal{K}^1_n \arrow[r, "\delta"] \arrow[d, "b"] & \mathcal{K}^1_{n-1} \arrow[r, "\delta"] \arrow[d, "b"] & \cdots \\
\cdots \arrow[r, "\delta"] & \mathcal{K}_{n+1} \arrow[r, "\delta"] & \mathcal{K}_n \arrow[r, "\delta"] & \mathcal{K}_{n-1} \arrow[r, "\delta"] & \cdots,
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathcal{K}^1_n \arrow[r, "\delta"] \arrow[d, "b"] & \mathcal{K}^1_{n-1} \arrow[r, "\pi"] \arrow[d, "b"] & \bar{\mathcal{K}}^1_{n-1} \arrow[r] \arrow[d, "\bar{b}"] & 0 \\
\mathcal{K}_n \arrow[r, "\delta"] & \mathcal{K}_{n-1} \arrow[r, "\pi"] & \bar{\mathcal{K}}_{n-1} \arrow[r] & 0
\end{tikzcd}
\quad \text{(exact rows).}$$

δ'

Pasted from <http://www.math.toronto.edu/~drorbn/papers/Species/1_4Hutchings_theory_integra.html>

Why not take $K_n' = \ker\delta$?

$$\begin{tikzcd}
\ker\delta \arrow[r, dashed, "0?"'] \arrow[d, hook] & \ker\delta \arrow[r, dashed, "0?"'] \arrow[d, hook] & \ker\delta \arrow[d, hook] \\
K_{n+1} \arrow[r] & K_n \arrow[r, "\delta"] & K_{n-1}
\end{tikzcd}$$

If $\delta' = 0$, this makes π an isomorphism, so $\bar{K}_n' = \ker\delta : K_n \to K_{n-1}$

That may work!

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
 & & \ker\delta_n \arrow[r] \arrow[d] & \ker\rho \arrow[d] & \\
0 \arrow[r] & \frac{K_{n+1}}{\ker\delta_{n+1}} \arrow[r, "\delta"] \arrow[d, "I"] & K_n \arrow[r, "\pi"] \arrow[d] & D_n \arrow[r] \arrow[d, "\rho"] & 0 \\
 & \frac{K_{n+1}}{\ker\delta_{n+1}} \arrow[r, "\delta", "1-1?"'] & K_n/\ker\delta_n \arrow[r] \arrow[d] & A_n \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 &
\end{tikzcd}$$

Dah.

Dah. Finding $\ker\delta$ should generally be hard. The most we can hope for is to find a generating set for $\ker\delta$.