Feb 2 meeting with Peter

February-16-11
10:08 AM

$I/I = I \otimes_Q I$

$I:I = I \otimes_k I$

A an aug alg over Q gen'd by S

F a free — — — — —

I_A, I_F the aug. ideals. $M \subseteq I_F^2$ 2 sided ideal

st. $0 \to M \to F \to A \to 0$

$0 \to M \to I_F \to I_A \to 0$

(and I_F is free and a free F module (2-sided))

Let $\{y_q . q \in Q\}$ be a min. set of gens for M as 2 sided F module

and suppose $\{y_q + I_F^3 ; q \in Q\}$ are lin. indep. in $\frac{N + I_F^3}{I_F^3}$.

Put $R^A := \ker(\mu_A : I_A \otimes_A I_A \to I_A^2)$, $\mathcal{R} := \ker\left(I_A/I_A^2 \otimes I_A/I_A^2 \to I_A^2/I_A^3\right)$

Then $R_A \simeq \mathcal{R}$ as Q v.sp.

$$M_m := \sum_j I_F^{\otimes_F j} \otimes_F M \otimes_F I_F^{\otimes_F m-j-1} = \sum_j I_F^{:j} : M : I_F^{:m-j-1}$$

$\mu_{FF} : I_F : I_F \twoheadrightarrow I_F^2$ 2 sided F-mod homom.

$R^F := \mu^{-1}(M)$ (so $\mu_{FF}(R^F) = M$).

CLAIM $R^A = \frac{R^F}{M_2}$ (and $M_2 \subseteq R^F$) Recall $M \subseteq I_F^2$

b/c $I_A^2 = \frac{I_F^2}{M} \xleftarrow[\sim]{\mu} \frac{I_F : I_F}{R^F} \simeq \frac{(I_F : I_F)/M_2}{R^F/M_2} \simeq \frac{I_A : I_A}{R^F/M_2}$

CLAIM $\mathcal{R} \simeq \frac{M}{I_F^3 \cap M} \simeq \frac{M + I_F^3}{I_F^3}$.

b/c $\frac{I_A^2}{I_A^3} \simeq \frac{I_F^2/M}{I_F^3/(M \cap I_F^3)} \simeq \frac{I_F^2/M}{(I_F^3+M)/M} = \frac{I_F^2}{I_F^3+M} = \frac{I_F^2/I_F^3}{(I_F^3+M)/I_F^3}$

But also $I_F^2/I_F^3 \simeq I_A/I_A^2 \otimes I_A/I_A^2$

b/c $\frac{I_F : I_F}{I_F . I_F . I_F} \sim \frac{(I_F : I_F)/M_2}{I_F . I_F^2/M_2}$

$\sim \frac{I_A : I_A}{I_A . I_A^2} \simeq I_A/I_A^2 \mid I_A/I_A^2$