

Pensieve header: Testing Profile.m using 1-smidgen computations.

## Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Profile"];
Once[<< KnotTheory`];
Once[<< "Profile-161127-1.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

### ? BeginProfile

Info43689257550-9616937

BeginProfile[] begins a profiling session, with root name ProfileRoot. BeginProfile[root] does same with root name root.

### ? ProfileData

Info53689257550-9616937

The result of EndProfile. Has format ProfileData[root,calls,total,self], where root is the profile rootlabel, calls is a linear combination of tags with head Called, andtotal and self are linear combinations of tags with head TimeUnder.

## Rotational Virtual Knots

**RVK::usage =**

"RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through 2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting operator converting to the RVK presentation from other knot presentations.";

```
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List @@ pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n}];
  front = {0};
  For[k = 0, k < 2 n, ++k,
    If[k == 0 || FreeQ[front, -k],
      front = Flatten[front /. k -> Catch[xs /. {
        Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L}],
        Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L}])
      }]],
      If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
    ]
  ];
  RVK[xs, rots]
];
RVK[K_] := RVK[PD[K]];
```

## NOE-1t

Logos

$$\Lambda[k_] := (1 - t_k) (\alpha^2 \beta^2 + 4 \alpha \beta \delta \mu + 2 \delta^2 \mu^2) / 2 + 2 \mu^2 (\alpha \beta + \delta \mu) c_k - \beta (2 \mu - 1) (\alpha \beta + 2 \delta \mu) u_k + 2 \beta \delta \mu^2 c_k u_k - \beta^2 \delta (3 \mu - 1) * u_k^2 / 2 + \alpha (\alpha \beta + 2 \delta \mu) w_k + 2 \alpha \delta \mu^2 c_k w_k - 2 (t_k - 1) \delta^2 (\alpha \beta + \delta \mu) u_k w_k + 2 \delta^2 \mu^2 c_k u_k w_k - \beta \delta^2 (2 \mu - 1) * u_k^2 w_k + \alpha^2 \delta (1 + \mu) w_k^2 / 2 + \alpha \delta^2 u_k * w_k^2 - (t_k - 1) \delta^4 * u_k^2 * w_k^2 / 2;$$

```

DPx→α, y→β[P-][f-] := Profile[DP, (* means P[∂α, ∂β][f] *)
  Total[CoefficientRules[P, {x, y}] /. ({m-, n-} → c-) ⇒ c D[f, {α, m}, {β, n}]]
]

```

```

CF[ $\mathbb{E}[\omega, L, Q, P]$ ] := Profile[CF,
  Expand /@ Together /@
     $\mathbb{E}[\omega /. b_L \Rightarrow \text{Log}[t_L], L, Q /. b_L \Rightarrow \text{Log}[t_L], P /. b_L \Rightarrow \text{Log}[t_L]]$ 
];
 $\mathbb{E} /:$   $\mathbb{E}[\omega 1, L 1, Q 1, P 1] \mathbb{E}[\omega 2, L 2, Q 2, P 2] := \text{CF} @ \mathbb{E}[\omega 1 \omega 2, L 1 + L 2, \omega 2 Q 1 + \omega 1 Q 2, \omega 2^4 P 1 + \omega 1^4 P 2];$ 

```

```

Nui, cj → k[ $\mathbb{E}[\omega, L, Q, P]$ ] := Profile[Nuc,
  With[{q = e-γ β uk + γ ck}, CF[
     $\mathbb{E}[\omega, \gamma c_k + (L /. c_j \rightarrow \theta), \omega e^{-\gamma} \beta u_k + (Q /. u_i \rightarrow \theta), e^{-q} \text{DP}_{c_j \rightarrow D_\gamma, u_i \rightarrow D_\beta}[P][e^q]] /. \{\gamma \rightarrow \partial_{c_j} L, \beta \rightarrow \omega^{-1} \partial_{u_i} Q\}$ 
]];
Nwi, cj → k[ $\mathbb{E}[\omega, L, Q, P]$ ] := Profile[Nwc,
  With[{q = eγ α wk + γ ck}, CF[
     $\mathbb{E}[\omega, \gamma c_k + (L /. c_j \rightarrow \theta), \omega e^\gamma \alpha w_k + (Q /. w_i \rightarrow \theta), e^{-q} \text{DP}_{c_j \rightarrow D_\gamma, w_i \rightarrow D_\alpha}[P][e^q]] /. \{\gamma \rightarrow \partial_{c_j} L, \alpha \rightarrow \omega^{-1} \partial_{w_i} Q\}$ 
]];

```

```

Nwi, uj → k[ $\mathbb{E}[\omega, L, Q, P]$ ] := Profile[Nwu,
  With[{q = (1 - tk) μ-1 α β + μ-1 β uk + μ-1 δ uk wk + μ-1 α wk}, CF[
     $\mathbb{E}[\mu \omega, L, \mu \omega q + \mu (Q /. w_i | u_j \rightarrow \theta), \mu^4 (\text{DP}_{w_i \rightarrow D_\alpha, u_j \rightarrow D_\beta}[P][e^q] /. e \rightarrow 1) + \omega^4 \Delta[k]] /. \mu \rightarrow 1 + (t_k - 1) \delta /. \{\alpha \rightarrow \omega^{-1} (\partial_{w_i} Q /. u_j \rightarrow \theta), \beta \rightarrow \omega^{-1} (\partial_{u_j} Q /. w_i \rightarrow \theta), \delta \rightarrow \omega^{-1} \partial_{w_i, u_j} Q\}$ 
]];

```

```

mi, j → k[Z-] := Profile[m,
  Module[{x, y, z},
    Z // Nwi, cj → x // Nwx, uj → y // ReplaceAll[{cx|y → cx, wj → wy}] // Nui, cx → x //
    ReplaceAll[Z-i|j|x|y → zk] // CF
];

```

```

Ri, j-+ :=  $\mathbb{E}[1, b_i c_j, u_i w_j, -c_i (t_i - 1)^2 / 2 - c_i^2 (t_i - 1)^2 / 2 + c_i c_j (t_j^2 - t_i - 2) / 2 - c_j u_i w_i / 2 + c_i (1 - t_i) u_i w_i - u_i^2 w_i^2 / 2 + u_i w_j + c_j t_i u_i w_j / 2 + c_i (t_i - 2) t_i u_i w_j + c_i (1 + t_j) u_j w_j / 2 + (t_i - 1) u_i^2 w_i w_j - (t_i - 2) t_i u_i^2 w_j^2 / 2];$ 
Ri, j-- :=  $\mathbb{E}[1, -b_i c_j, -t_i^{-1} u_i w_j, c_i (t_i - 1)^2 / 2 + c_i^2 (t_i - 1)^2 / 2 + c_i c_j (2 + t_i - t_j^2) / 2 + c_j u_i w_i / 2 + c_i (t_i - 1) u_i w_i + u_i^2 w_i^2 / 2 + (1 - t_i^{-1}) u_i w_j / 2 + c_i (2 t_i - 5 + 3 t_i^{-1}) u_i w_j / 2 + c_j (t_i^{-1} + 1 - t_i^{-1} t_j^2) u_i w_j / 2 - c_i (t_j + 1) u_j w_j / 2 + (2 - 3 t_i^{-1}) u_i^2 w_i w_j / 2 + (1 + 2 t_i^{-2} - 3 t_i^{-1}) u_i^2 w_j^2 / 2 - t_i^{-1} (1 + t_j) u_i u_j w_j^2 / 2];$ 
uri :=  $\mathbb{E}[t_i^{-1/4}, \theta, \theta, c_i t_i / 4 + u_i w_i / 8];$ 
nri :=  $\mathbb{E}[t_i^{1/4}, \theta, \theta, -c_i t_i^3 / 4 - t_i^2 u_i w_i / 8];$ 
uli :=  $\mathbb{E}[t_i^{1/4}, \theta, \theta, c_i t_i (4 + t_i) / 4 - t_i^2 u_i w_i / 8];$ 
nli :=  $\mathbb{E}[t_i^{-1/4}, \theta, \theta, -c_i (1 + 4 t_i^{-1}) / 4 + u_i w_i / 8];$ 

```

```

rot[_, 0] =  $\mathbb{E}[1, \theta, \theta, \theta];$ 
rot[i-, 1] := Module[{y}, nli ury // mi, y → i];
rot[i-, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi, y → i];
rot[i-, -1] := Module[{y}, nri uly // mi, y → i];
rot[i-, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi, y → i];

```

Z

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Profile[Z,
Module[{todo, n, rots,  $\xi$ , done, st, x,  $\xi$ 1, i, j, k, k1, k2, k3},
{todo, rots} = List@@rvk;
AppendTo[rots, 0];
n = Length[todo];
 $\xi$  =  $\mathbb{E}$ [1, 0, 0, 0];
done = {0};
st = Range[0, 2 n + 1];
While[todo != {},
{x} = MaximalBy[todo, Length[done  $\cap$  {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
Z$todo = todo; Z$x = x;
{i, j} = List@@x;
 $\xi$ 1 = Switch[Head[x],
Xp, mj,k→j [R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ],
Xm, mj,k→j [R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ]
];
 $\xi$ 1 = rot[k, rots[[i]]  $\xi$ 1 // mk,i→i; rots[[i]] = 0;
 $\xi$ 1 =  $\xi$ 1 rot[k, rots[[i + 1]] // mi,k→i; rots[[i + 1]] = 0;
 $\xi$ 1 = rot[k, rots[[j]]  $\xi$ 1 // mk,j→j; rots[[j]] = 0;
 $\xi$ 1 =  $\xi$ 1 rot[k, rots[[j + 1]] // mj,k→j; rots[[j + 1]] = 0;
 $\xi$  *=  $\xi$ 1;
If[MemberQ[done, i],  $\xi$  =  $\xi$  // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]];
If[MemberQ[done, i - 1],  $\xi$  =  $\xi$  // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]];
If[MemberQ[done, j],  $\xi$  =  $\xi$  // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]];
If[MemberQ[done, j - 1],  $\xi$  =  $\xi$  // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]];
done = done  $\cup$  {i - 1, i, j - 1, j};
todo = DeleteCases[todo, x]
];
 $\xi$  /. {u0 → u, c0 → c, w0 → w}
]
]

```

## Testing ...

```
BeginProfile[];
Timing[Z[Knot[10, 100]]]
EndProfile[];
```

 KnotTheory: Loading precomputed data in PD4Knots` 

$$\left\{ 16.4688, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$\left. \frac{2563146 + 667500c + \frac{6}{t^{16}} - \frac{8c}{t^{16}} - \frac{92}{t^{15}} + \frac{118c}{t^{15}} + \frac{723}{t^{14}} - \frac{892c}{t^{14}} - \frac{3818}{t^{13}} + \frac{4523c}{t^{13}} + \frac{15133}{t^{12}} - \frac{17161c}{t^{12}} - \frac{47848}{t^{11}} + \frac{51709c}{t^{11}} + \frac{125539}{t^{10}} - \frac{128505c}{t^{10}} - \frac{281054}{t^9} + \frac{270279c}{t^9} + \frac{548129}{t^8} - \frac{489715c}{t^8} - \frac{945756}{t^7} + \frac{772841c}{t^7} + \frac{1460263}{t^6} - \frac{1066829c}{t^6} - \frac{2034106}{t^5} + \frac{1282861c}{t^5} + \frac{2570432}{t^4} - \frac{1320331c}{t^4} - \frac{2956518}{t^3} + \frac{1107336c}{t^3} + \frac{3099338}{t^2} - \frac{640054c}{t^2} - \frac{2958726}{t} - \frac{540c}{t} - 2000454t - 1197840ct + 1387610t^2 + 1472160ct^2 - 832998t^3 - 1456020ct^3 + 407256t^4 + 1204364ct^4 - 132546t^5 - 829886ct^5 - 9557t^6 + 453636ct^6 + 59220t^7 - 162131ct^7 - 58859t^8 - 11711ct^8 + 40498t^9 + 81439ct^9 - 22001t^{10} - 84595ct^{10} + 9704t^{11} + 59721ct^{11} - 3455t^{12} - 32685ct^{12} + 966t^{13} + 14251ct^{13} - 201t^{14} - 4919ct^{14} + 28t^{15} + 1307ct^{15} - 2t^{16} - 253ct^{16} + 32ct^{17} - 2ct^{18} - 493132uw + \frac{8uw}{t^{16}} - \frac{110uw}{t^{15}} + \frac{782uw}{t^{14}} - \frac{3741uw}{t^{13}} + \frac{13420uw}{t^{12}} - \frac{38289uw}{t^{11}} + \frac{90216uw}{t^{10}} - \frac{180063uw}{t^9} + \frac{309652uw}{t^8} - \frac{463189uw}{t^7} + \frac{603640uw}{t^6} - \frac{679221uw}{t^5} + \frac{641110uw}{t^4} - \frac{466226uw}{t^3} + \frac{173828uw}{t^2} + \frac{174368uw}{t} + 704708t uw - 767452t^2 uw + 688568t^3 uw - 515796t^4 uw + 314090t^5 uw - 139546t^6 uw + 22585t^7 uw + 34296t^8 uw - 47143t^9 uw + 37452t^{10} uw - 22269t^{11} uw + 10416t^{12} uw - 3835t^{13} uw + 1084t^{14} uw - 223t^{15} uw + 30t^{16} uw - 2t^{17} uw \right] \}$$

```
PrintProfile[];
```

CF: called 490 times, time in 14.654/14.654

Parents:

- ( 102) 1.168/ 1.168 under m
- ( 102) 2.799/ 2.799 under Nuc
- ( 102) 2.436/ 2.436 under Nwc
- ( 102) 7.640/ 7.640 under Nwu
- ( 82) 0.611/ 0.611 under Z

DP: called 306 times, time in 1.124/1.124

Parents:

- ( 102) 0.359/ 0.359 under Nuc
- ( 102) 0.342/ 0.342 under Nwc
- ( 102) 0.423/ 0.423 under Nwu

Nwu: called 102 times, time in 0.268/8.331

Parents:

- ( 102) 0.268/ 8.331 under m

Children:

- ( 102) 7.640/ 7.640 above CF

( 102) 0.423/ 0.423 above DP

m: called 102 times, time in 0.219/15.748

Parents:

( 102) 0.219/ 15.750 under Z

Children:

( 102) 1.168/ 1.168 above CF

( 102) 0.079/ 3.237 above Nuc

( 102) 0.015/ 2.793 above Nwc

( 102) 0.268/ 8.331 above Nwu

Nuc: called 102 times, time in 0.079/3.237

Parents:

( 102) 0.079/ 3.237 under m

Children:

( 102) 2.799/ 2.799 above CF

( 102) 0.359/ 0.359 above DP

Z: called 1 times, time in 0.062/16.421

Parents:

( 1) 0.062/ 16.420 under ProfileRoot

Children:

( 82) 0.611/ 0.611 above CF

( 102) 0.219/ 15.750 above m

Nwc: called 102 times, time in 0.015/2.793

Parents:

( 102) 0.015/ 2.793 under m

Children:

( 102) 2.436/ 2.436 above CF

( 102) 0.342/ 0.342 above DP

ProfileRoot: called 0 times, time in 0./0.

Children:

( 1) 0.062/ 16.420 above Z