Formulas

December-16-10
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$(z_1, z_2) \sim (e^{i\alpha} z_1, \underbrace{e^{i\alpha} z_2}_{\text{can be made real.}})$

Problem Find "the right" map $S^2 \xrightarrow{\sim} D^2/\partial D^2$.

Best so far —

$$(x,y,z) \mapsto (\operatorname{atg} y/x, z) \mapsto (\operatorname{atg} y/z, \tfrac{1+z}{2}) \text{ in polar.}$$

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longitude $= \phi \in [-180°, 180°]$ latitude $= \theta \in [-90°, 90°]$

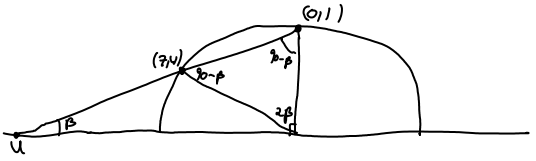
$S^2 \longmapsto D^2/\partial D^2$ by

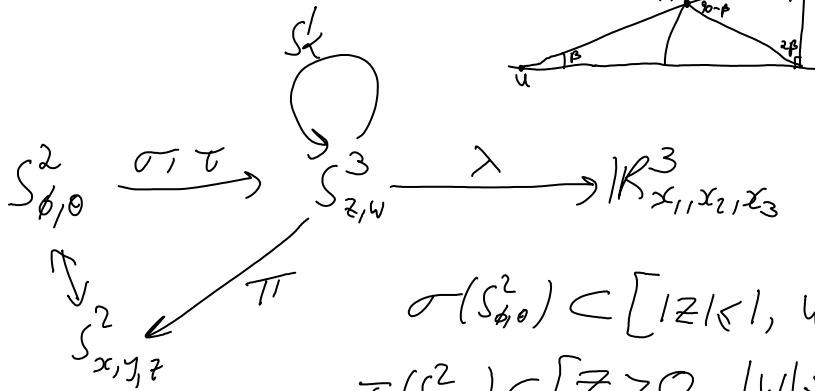
$(\phi, \theta) \longmapsto r(\cos\phi, \sin\phi)$, where $r = \sqrt{\frac{1-\sin\theta}{2}}$

$S^2_{\phi,\theta} \times S^1_\alpha \longmapsto S^3$ by

$(\phi, \theta, \alpha) \longmapsto e^{i\alpha}\left(r e^{i\phi}, \sqrt{\frac{1+\sin\theta}{2}}\right)$ $\quad [N \to (0,1)]$

$S^3 \longmapsto \mathbb{R}^3$ by $(z,w) \mapsto \frac{1}{1-\operatorname{Im} w}(z, \operatorname{Re}(w))$





$\sigma(S^2_{\phi,\theta}) \subset [|z| \le 1, w > 0]$

$\tau(S^2_{\phi,\theta}) \subset [z > 0, |w| \le 1]$