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POLY-TIME KNOT POLYNOMIALS VIA SOLVABLE APPROXIMATIONS

DROR BAR-NATAN AND ROLAND VAN DER VEEN

ABSTRACT. Following Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], we construct the first poly-time-computable knot polynomials since Alexander's [Al, 1928]. We use some new commutator-calculus techniques and a family of Lie algebra $\mathfrak{g}_k$ which are solvable yet at the same time progressively better approximations of the simple Lie algebra sl_2. The resulting invariants are the strongest genuinely-computable knot invariants presently available, and they seem to contain information about some classical topologically-defined knot invariants.

CONTENTS

1. INTRODUCTION

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(Computations below)

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Plan.

Introduction. The "algebras to invariants" story (minus cuaps) and its deficiencies. "Spaces of formulas" and meta-algebras. Solvable approximations and realization within thus. Section by section summary. Acknowledgement.

The First Example: Algorithm Only. Elves/yaks following Talks/GWU-1612.

The Zeroth Example in Detail. g_0 and the Alexander polynomial, with full derivations.

The Lie Algebra g_\beta and its Quantization. Following Doubling.nb. (Also comment on 2-parameter quantum groups).

The General g_k Invariant. Following Double.nb and more.

Tables and Experimental Results.

Odds and Ends.