Pensieve header: A unified verification notebook for the PPSA project; continued pensieve://Projects/SL2Portfolio/.

Continues pensieve://2017-06/ and pensieve://2017-08/.

Prolog

```
Go;
```

```
wdir = SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\PPSA"];
NotebookOpen[wdir <> "\\MakeVSnips.nb"];
```

```
HL[ℰ_] := Style[ℰ, Background → Yellow];
```

Initialization / Utilities

The “degree carrier / filtration parameter” is ℏ, and all “coupling constants” are proportional to it.

TD

```
$TℏD = 3; $TϵD = 2; ϵ /: ϵ^d_· /; d > $TϵD := 0;
(* $TϵD can't be ∞ at least because of Quesne. Can't be ≤
 1 at least because of the explicit ϵ^2 in S𝔻$g. *)
SetAttributes[{SS, SST}, HoldAll];
SS[ℰ_] := Block[{ℏ, ϵ}, (* Shielded Series *)
   Collect[Normal@Series[ℰ, {ℏ, 0, $TℏD}], ℏ, Together] ];
SST[ℰ_] := Block[{ℏ, ϵ},
   Collect[Normal@Series[ℰ /. {T_i_ → e^(ℏ t_i/2), T → e^(ℏ t/2)}, {ℏ, 0, $TℏD}], ℏ, Together] ];
Simp[ℰ_, op_] := Collect[ℰ, _CU | _QU, op];
Simp[ℰ_] := Simp[ℰ, Collect[Normal@Series[#, {ℏ, 0, $TℏD}], ℏ, Expand] &];
SimpT[ℰ_] := Collect[ℰ, _CU | _QU,
   Collect[Normal@Series[# /. {T_i_ → e^(ℏ t_i/2), T → e^(ℏ t/2)}, {ℏ, 0, $TℏD}], ℏ, Expand] &];
```

Differential polynomials (DP):

Utils

```
DP_{α_→D_x_, β_→D_y_}[P_][λ_] :=
 Total[CoefficientRules[P, {α, β}] /. ({m_, n_} → c_) :→ c D[λ, {x, m}, {y, n}]]
```

DeclareAlgebra

QLImplementation

```
Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z__] := (x ** y) ** z;
0 ** _ = _ ** 0 = 0;
(x_Plus) ** y_ := (# ** y) & /@ x; x_ ** (y_Plus) := (x ** #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ** y - y ** x;
```

QLImplementation

```
DeclareAlgebra[U_Symbol, opts__Rule] := Module[{gp, sr, cp, CE, pow,
    gs = Generators /. {opts}, cs = Centrals /. {opts} },
   (#_U = U@#) & /@ gs;
   gp = Alternatives @@ gs; gp = gp | gp_; (* gen's pattern *)
   sr = Thread[gs → Range@Length@gs]; (* sorting rule *)
   cp = Alternatives @@ cs; (* cent's pattern *)
   CE[ℰ_] := Collect[ℰ, _U, (Expand[#] /. ħ^d_ /; d > $TħD :> 0) &];
   U_i_[ℰ_] := ℰ /. {t : cp :> t_i, u_U :> Replace[u, x_ :> x_i, 1]};
   U_i_[NCM[]] := U[];
   B[U@(x_)_i_, U@(y_)_i_] := B[U@x_i, U@y_i] = U_i@B[U@x, U@y];
   B[U@(x_)_i_, U@(y_)_j_] /; i =!= j := 0;
   B[U@y_, U@x_] := CE[-B[U@x, U@y]];
   x_ ** U[] := x; U[] ** x_ := x;
   (a_. * x_U) ** (b_. * y_U) := If[a b === 0, 0, CE[a b (x ** y)]];
   U[xx___, x_] ** U[y_, yy___] := If[OrderedQ[{x, y} /. sr],
     U[xx, x, y, yy], U@xx ** (U@y ** U@x + B[U@x, U@y]) ** U@yy];
   U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ** U@{r}];
   U@{c_. * l : gp, r___} := CE[c U[l] ** U@{r}];
   U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
   U@{} = U[];
   U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
   U@{l_, r___} := U@{Expand[l], r};
   U[ℰ_NonCommutativeMultiply] := U /@ ℰ;
   𝕆_U[poly_, specs___] := Module[{sp, null, vs, us},
     sp = Replace[{specs}, l_List :> l_null, {1}];
     vs = Join @@ (First /@ sp);
     us = Join @@ (sp /. l_s_ :> (l /. x_i_ :> x_s));
     CE[Total[
        CoefficientRules[poly, vs] /. (p_ → c_) :> c U@(us^p)
       ]] /. x_null :> x
    ];
   pow[ℰ_, 0] = U[]; pow[ℰ_, n_] := pow[ℰ, n - 1] ** ℰ;
   𝕊_U[ℰ_, ss___Rule] := CE@Total[
     CoefficientRules[ℰ, First /@ {ss}] /.
      (p_ → c_) :> c NCM @@ MapThread[pow, {Last /@ {ss}, p}]];
   S_i_[c_. * u_U] := CE[(c /. S_i[U, Centrals]) DeleteCases[u, _i] **
      U_i[NCM @@ Reverse@Cases[u, x_i :> S@U@x]]];
  ]
```

DeclareMorphism

QLImplementation

```
DeclareMorphism[m_, U_ → V_, ongs_List, oncs_List: {}] := (
  Replace[ongs, (g_ → img_) ⧴ (m[U[g]] = img), {1}];
  m[U[]] = V[];
  m[U[g_i_]] := V_i[m[U@g]];
  m[U[vs__]] := NCM @@ (m /@ U /@ {vs});
  m[ℰ_] := Simp[ℰ /. oncs /. u_U ⧴ m[u]]; )
```

Meta-Operations

QLImplementation

```
S_i_[ℰ_Plus] := Simp[S_i /@ ℰ];
```

Implementing $sl_2^{\gamma\epsilon}$

CU

```
DeclareAlgebra[CU, Generators → {y, a, x}, Centrals → {t}];
B[a_CU, y_CU] = -γ y_CU; B[x_CU, a_CU] = -γ x_CU;
B[x_CU, y_CU] = 2 ϵ a_CU - t CU[];
(S@CU@y = -y_CU; S@a_CU = -a_CU; S@x_CU = -x_CU;)
S_i_[CU, Centrals] = {t_i → -t_i};
```

Verifying associativity on triples of generators:

```
With[{bas = CU /@ {y, a, x}},
 Table[z1 ** (z2 ** z3) - (z1 ** z2) ** z3 // Simp // HL,
  {z1, bas}, {z2, bas}, {z3, bas} ] ]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

Verifying associativity on a “random” triple:

```
With[{z1 = CU[y, y, a, a, x, x], z2 = CU[y, a, x], z3 = CU[y, y, a, x]}, {
   (rhs = (z1 ** z2) ** z3 // Simp) // Short,
   z1 ** (z2 ** z3) - rhs // Simp // HL
  }] // Timing
```

$\{0.828125, \{(28\, t^2 \gamma^4 + 116\, t\, \gamma^5 \epsilon + 120\, \gamma^6 \epsilon^2)\, CU[y, y, y, x, x] + \ll 24 \gg, 0\}\}$

Verifying that *S* is an anti-homomorphism on CU:

```
With[{bas = CU /@ {y_1, a_1, x_1}},
 Table[HL@Simp[S_1[z1 ** z2] - S_1[z2] ** S_1[z1]],
  {z1, bas}, {z2, bas} ] ]
```

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

Verifying the involutivity of S on products of triples:

```
With[{bas = CU /@ {y1, a1, x1}},
 Table[HL@Simp[z1 ** z2 ** z3 - S1@S1[z1 ** z2 ** z3]],
  {z1, bas}, {z2, bas}, {z3, bas} ] ]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

Implementing $\mathcal{U}_{\gamma\epsilon;\hbar}$

With $q = e^{\hbar\gamma\epsilon}$, $A = e^{-\hbar\epsilon a}$, $T = e^{\hbar t/2}$, and $[f, g]_q := fg - qgf$, our algebra is $\mathcal{U}_{\gamma\epsilon,\hbar} = \langle t, y, a, x\rangle / \mathcal{R}$, where $\mathcal{R} = \left([t, *] = 0,\ [a, y] = -\gamma y,\ [x, y]_q = \left(1 - T^2 A^2\right) / \hbar,\ [x, a] = -\gamma x\right)$.

QU

```
DeclareAlgebra[QU, Generators → {y, a, x}, Centrals → {t, T}];
q = SS[e^(γ ϵ ħ)]; (*T=SS[e^(ħ t/2)];*)
B[a_QU, y_QU] = -γ y_QU; B[x_QU, a_QU] = -γ QU@x;
B[x_QU, y_QU] = (q - 1) QU@{y, x} + 𝕆_QU[SS[(1 - T^2 e^(-2 ϵ a ħ)) / ħ], {a}];
(S@y_QU = 𝕆_QU[SS[-T^-2 e^(ħ ϵ a) y], {a, y}]; S@a_QU = -a_QU; S@x_QU = 𝕆_QU[SS[-e^(ħ ϵ a) x], {a, x}];)
S_i_[QU, Centrals] = {t_i → -t_i, T_i → T_i^-1};
```

```
With[{bas = QU /@ {y, a, x}}, Table[{z1, z2} → Simp[z1 ** z2 - z2 ** z1], {z1, bas}, {z2, bas}] ]
```

$$\{\{\{QU[y], QU[y]\} \to 0, \{QU[y], QU[a]\} \to \gamma\, QU[y], \{QU[y], QU[x]\} \to \frac{(-1 + T^2)\, QU[\,]}{\hbar} - 2 T^2 \epsilon\, QU[a] + 2 T^2 \epsilon^2 \hbar\, QU[a, a] + \left(-\gamma \epsilon \hbar - \frac{1}{2} \gamma^2 \epsilon^2 \hbar^2\right) QU[y, x]\},$$
$$\{\{QU[a], QU[y]\} \to -\gamma\, QU[y], \{QU[a], QU[a]\} \to 0, \{QU[a], QU[x]\} \to \gamma\, QU[x]\},$$
$$\{\{QU[x], QU[y]\} \to \frac{(1 - T^2)\, QU[\,]}{\hbar} + 2 T^2 \epsilon\, QU[a] - 2 T^2 \epsilon^2 \hbar\, QU[a, a] + \left(\gamma \epsilon \hbar + \frac{1}{2} \gamma^2 \epsilon^2 \hbar^2\right) QU[y, x],$$
$$\{QU[x], QU[a]\} \to -\gamma\, QU[x], \{QU[x], QU[x]\} \to 0\}\}$$

Verifying associativity on triples of generators:

```
With[{bas = QU /@ {y, a, x}},
 Table[HL[z1 ** (z2 ** z3) - (z1 ** z2) ** z3] // Simp,
  {z1, bas}, {z2, bas}, {z3, bas} ] ]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

Verifying associativity on a “random” triple (~34 secs @ $TħD=5, $TϵD=2):

```
With[{z1 = QU[y, y, a, a, x, x], z2 = QU[y, a, x], z3 = QU[y, y, a, x]}, {
   (rhs = (z1 ** z2) ** z3 // Simp) // Short,
   HL[z1 ** (z2 ** z3) - rhs // Simp]
  }] // Timing
```

```
{32.625, {<<1>>, 0}}
```

Verifying that S is an anti-homomorphism on QU:

```
With[{bas = QU /@ {y1, a1, x1}},
 Table[{z1, z2} → HL@Simp[S1[z1 ** z2] - S1[z2] ** S1[z1]],
  {z1, bas}, {z2, bas} ] ]
```

```
{{{QU[y1], QU[y1]} → 0, {QU[y1], QU[a1]} → 0, {QU[y1], QU[x1]} → 0},
 {{QU[a1], QU[y1]} → 0, {QU[a1], QU[a1]} → 0, {QU[a1], QU[x1]} → 0},
 {{QU[x1], QU[y1]} → 0, {QU[x1], QU[a1]} → 0, {QU[x1], QU[x1]} → 0}}
```

Verifying that $\text{Lim}_{\hbar\to 0}$ QU = CU using a "random" product (~23 secs @ \$Tℏ D=5, \$Tϵ D=2):

```
With[{z1 = CU[y, y, a, a, x, x], z2 = CU[y, a, x], z3 = CU[y, y, a, x]}, {
   Short[lhs = z1 ** (z2 ** z3)],
   Short[rhs = (QU @@ z1) ** ((QU @@ z2) ** (QU @@ z3))],
   Expand[Limit[rhs /. {QU → CU, T → ⅇ^(ℏ t/2)}, ℏ → 0] - lhs] // HL
  }] // Timing
```

```
{34., {2 (8 t^2 γ^4 + 16 t γ^5 ϵ) CU[y, y, y, x, x] + <<107>> + CU[y, y, y, y, <<5>>, x, x, x, x],
  (-8 T^2 γ^6 ϵ^2 + 8 T^4 γ^6 ϵ^2) QU[y, y, y, x, x] + <<489>> + (γ ϵ ℏ + <<1>>) QU[<<1>>], 0}}
```

Implementing θ

theta

```
DeclareMorphism[Cθ, CU → CU, {y → -x_CU, a → -a_CU, x → -y_CU}, {t → -t, T → T^-1}];
DeclareMorphism[Qθ, QU → QU, {y → 𝕆_QU[SS[-T^-1 ⅇ^(ℏ ϵ a) x], {a, x}],
   a → -a_QU, x → 𝕆_QU[SS[-T^-1 ⅇ^(ℏ ϵ a) y], {a, y}]}, {t → -t, T → T^-1}]
```

Verifying involutivity on CU:

```
With[{bas = CU /@ {y, a, x}},
 Table[z → Cθ[z] → HL[Cθ[Cθ[z]]], {z, bas}] ]
```

```
{CU[y] → -CU[x] → CU[y], CU[a] → -CU[a] → CU[a], CU[x] → -CU[y] → CU[x]}
```

Verifying that θ is a multiplicative homomorphism on CU:

```
With[{bas = CU /@ {y, a, x}},
 Table[Cθ[z1 ** z2] - Cθ[z1] ** Cθ[z2] // HL, {z1, bas}, {z2, bas}] ]
```

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

Verifying involutivity on QU:

```
With[{bas = QU /@ {y, a, x}},
 Table[z → Qθ[z] → HL[Qθ[Qθ[z]]], {z, bas}] ]
```

$$\left\{QU[y]\to -\frac{QU[x]}{T}-\frac{\epsilon\hbar\, QU[a,x]}{T}-\frac{\epsilon^2\hbar^2\, QU[a,a,x]}{2T}\to QU[y],\ QU[a]\to -QU[a]\to QU[a],\right.$$
$$\left. QU[x]\to\left(-\frac{1}{T}+\frac{\gamma\epsilon\hbar}{T}-\frac{\gamma^2\epsilon^2\hbar^2}{2T}\right)QU[y]+\left(-\frac{\epsilon\hbar}{T}+\frac{\gamma\epsilon^2\hbar^2}{T}\right)QU[y,a]-\frac{\epsilon^2\hbar^2\, QU[y,a,a]}{2T}\to QU[x]\right\}$$

Verifying that θ is a multiplicative homomorphism on QU:

```
With[{bas = QU /@ {y, a, x}},
 Table[{z1, z2} → HL[Simp[Qθ[z1 ** z2] - Qθ[z1] ** Qθ[z2]]], {z1, bas}, {z2, bas}] ]
{{{QU[y], QU[y]} → 0, {QU[y], QU[a]} → 0, {QU[y], QU[x]} → 0},
 {{QU[a], QU[y]} → 0, {QU[a], QU[a]} → 0, {QU[a], QU[x]} → 0},
 {{QU[x], QU[y]} → 0, {QU[x], QU[a]} → 0, {QU[x], QU[x]} → 0}}
```

The Asymmetric Dequantizator

Following pensieve://People/VanDerVeen/Dequant1.pdf.

ADeq

$$\mathbb{AD}\$f = \frac{\gamma}{\hbar} e^{\hbar\left(\frac{t}{2}-(a+\gamma)\,\epsilon\right)} \left(\left(\mathrm{Cosh}\left[\hbar\left(a\,\epsilon + \frac{\gamma\,\epsilon}{2} - \frac{t}{2}\right)\right] - \mathrm{Cosh}\left[\hbar\sqrt{\left(\frac{t-\gamma\,\epsilon}{2}\right)^2 + \epsilon\,\omega}\,\right]\right) \Big/ \left(\mathrm{Sinh}\left[\frac{\gamma\,\epsilon\,\hbar}{2}\right]\left(a^2\,\epsilon + a\,\gamma\,\epsilon - a\,t - \omega\right)\right)\right);$$

Scaling behaviour of AD$f:

```
HL@Simplify[AD$f == ((AD$f /. γ → 1) /. {ϵ → γ ϵ, a → γ^-1 a, ω → γ^-1 ω})]
True
```

```
HL@FullSimplify[
  AD$f == ((AD$f /. γ → 1) /. {ħ → γ^2 ħ, ϵ → ϵ / γ, a → a / γ, t → γ^-2 t, ω → γ^-3 ω})]
True
```

ADeq

```
AD$ω = γ CU[y, x] + ϵ CU[a, a] - (t - γ ϵ) CU[a];
```

ADeq

```
DeclareMorphism[AD, QU → CU,
 {a → a_CU, x → CU@x, y → S_CU[SS[AD$f], a → a_CU, ω → AD$ω] ** y_CU}]
```

Verifying that the asymmetric dequantizator is a homomorphism:

```
With[{bas = QU /@ {y, a, x}},
 Table[{z1, z2} → HL[SimpT[AD[z1 ** z2] - AD[z1] ** AD[z2]]], {z1, bas}, {z2, bas}] ]
{{{QU[y], QU[y]} → 0, {QU[y], QU[a]} → 0, {QU[y], QU[x]} → 0},
 {{QU[a], QU[y]} → 0, {QU[a], QU[a]} → 0, {QU[a], QU[x]} → 0},
 {{QU[x], QU[y]} → 0, {QU[x], QU[a]} → 0, {QU[x], QU[x]} → 0}}
```

The Symmetric Dequantizator

Following pensieve://People/VanDerVeen/Dequant1.pdf.

SDeq

```
SD$g = Sqrt[((Cosh[ħ/2 Sqrt[t^2 + γ^2 ϵ^2 + 4 ϵ ϖ]] - Cosh[ħ/2 (t - (2 a + γ) ϵ)])/
    (Sinh[γ ϵ ħ/2] (t (2 a + γ) - 2 a (a + γ) ϵ + 2 ϖ) ħ/(2 γ)))];
```

Verify agreement with the formulas in pensieve://People/VanDerVeen/Dequant1.pdf:

```
{SD$P = (Cosh[ħ ((ϵ-t)/2 + ϵ a)] - Cosh[ħ Sqrt[(t^2+ϵ^2)/4 + ϵ ϖ]])/(ħ Sinh[-ϵ ħ/2] (ϖ - ϵ a^2 + (t - ϵ) a + t/2)),
 Simplify[SD$P == (SD$P /. {a → -a - 1, t → -t})] // HL,
 PowerExpand@Simplify[(SD$P /. {ħ → γ^2 ħ, ϵ → ϵ/γ, a → a/γ, t → γ^-2 t, ϖ → γ^-3 ϖ}) ==
    SD$g (SD$g /. {a → -a - γ, t → -t})] // HL,
 SD$Q = Simplify[SD$P /. {a → c - 1/2}],
 Simplify[SD$Q == (SD$Q /. {c → -c, t → -t})] // HL,
 Simplify[SD$g == FullSimplify[
    Sqrt[SD$Q] /. c → a + 1/2 /. {ħ → γ^2 ħ, ϵ → ϵ/γ, a → a/γ, t → γ^-2 t, ϖ → γ^-3 ϖ}]] // HL
}
```

$$\left\{-\left(\left(\left(\mathrm{Cosh}\left[\left(a\,\epsilon+\frac{1}{2}(-t+\epsilon)\right)\hbar\right]-\mathrm{Cosh}\left[\sqrt{\frac{1}{4}(t^2+\epsilon^2)+\epsilon\varpi}\;\hbar\right]\right)\mathrm{Csch}\left[\frac{\epsilon\hbar}{2}\right]\right)\Big/\left(\left(\frac{t}{2}+a(t-\epsilon)-a^2\epsilon+\varpi\right)\hbar\right)\right),\ \mathrm{True},\ \mathrm{True},\right.$$

$$\left.-\left(\left(4\left(\mathrm{Cosh}\left[\frac{1}{2}(t-2c\epsilon)\hbar\right]-\mathrm{Cosh}\left[\frac{1}{2}\sqrt{t^2+\epsilon^2+4\epsilon\varpi}\;\hbar\right]\right)\mathrm{Csch}\left[\frac{\epsilon\hbar}{2}\right]\right)\Big/\left((4ct+\epsilon-4c^2\epsilon+4\varpi)\hbar\right)\right),\ \mathrm{True},\ \mathrm{True}\right\}$$

SDeq

```
SD$f = FullSimplify[e^(ħ (t/2-ϵ a)) (SD$g /. {a → -a, t → -t})];
```

SDeq

```
SD$ϖ = γ CU[y, x] + ϵ CU[a, a] - (t - γ ϵ) CU[a] - t γ CU[]/2;
```

SDeq

```
DeclareMorphism[SD, QU → CU, {a → a_CU,
  x → 𝕊_CU[SS[SD$f], a → a_CU, ϖ → SD$ϖ] ** x_CU,
  y → 𝕊_CU[SS[SD$g], a → a_CU, ϖ → SD$ϖ] ** y_CU
 }]
```

Verifying the θ-symmetry:

```
Table[HL@SimpT[Cθ[SD[z]] == SD[Qθ[z]]], {z, QU /@ {y, a, x}}]
```

{True, True, True}

Verifying that the symmetric dequantizator is a homomorphism:

```
With[{bas = QU /@ {y, a, x}},
 Table[{z1, z2} → HL@SimpT[SD[z1 ** z2] - SD[z1] ** SD[z2]], {z1, bas}, {z2, bas}] ]
```

```
{{{QU[y], QU[y]} → 0, {QU[y], QU[a]} → 0, {QU[y], QU[x]} → 0},
 {{QU[a], QU[y]} → 0, {QU[a], QU[a]} → 0, {QU[a], QU[x]} → 0},
 {{QU[x], QU[y]} → 0, {QU[x], QU[a]} → 0, {QU[x], QU[x]} → 0}}
```

R in QU.

Quesne's formula:

Quesne

$$\mathbb{e}_{q\_,n\_}[x\_] := \mathbb{e}^\wedge\left(\sum_{k=1}^{n}\frac{(1-q)^k x^k}{k\,(1-q^k)}\right);\ \mathbb{e}_{q\_}[x\_] := \mathbb{e}_{q,\$T\epsilon D}[x]$$

```
Table[Together@SeriesCoefficient[𝕖ρ,5[x], {x, 0, n}], {n, 0, 5}]
```

$$\left\{1,1,\frac{1}{1+\rho},\frac{1}{(1+\rho)(1+\rho+\rho^2)},\frac{1}{(1+\rho)^2(1+\rho^2)(1+\rho+\rho^2)},\right.$$
$$\left.1\Big/\left((1+\rho)^2(1+\rho^2)(1+\rho+\rho^2)(1+\rho+\rho^2+\rho^3+\rho^4)\right)\right\}$$

```
Table[HL@FunctionExpand[QFactorial[n, ρ] SeriesCoefficient[𝕖ρ,5[x], {x, 0, n}]], {n, 0, 5}]
```

```
{1, 1, 1, 1, 1, 1}
```

R

$$QU[R_{i\_,j\_}] := \mathbb{O}_{QU}\left[SS\left[\mathbb{e}^{\hbar b_1 a_2}\,\mathbb{e}_q[\hbar y_1 x_2]\ /.\ b_1 \to \gamma^{-1}(\epsilon a_1 - t_i)\right], \{y_1, a_1\}_i, \{a_2, x_2\}_j\right];$$
$$QU\left[R^{-1}_{i\_,j\_}\right] := S_j@QU[R_{i,j}];$$

```
QU[R3,4] // Short
```

$$QU[\,] + \frac{\epsilon\,\hbar \ll 1\gg}{\gamma} + \ll 21\gg + \frac{\hbar^3 \ll 1\gg \ll 1\gg}{2\,\gamma^2} - \frac{\hbar^3\, QU[a_4, a_4, a_4]\, t_3^3}{6\,\gamma^3}$$

Verifying R2 (~2 secs @ \$TħD=4, \$TϵD=2):

$$QU\left[R_{1,2} ** R^{-1}_{1,2}\right]\ //\ Simp\ //\ HL\ //\ Timing$$

```
{0.546875, QU[]}
```

Verifying R3 (~156 secs @ \$TħD=4, \$TϵD=2):

$$\{Short[lhs = QU[R_{1,2} ** R_{1,3} ** R_{2,3}]],\ HL@SimpT[lhs - QU[R_{2,3} ** R_{1,3} ** R_{1,2}]]\}\ //\ Timing$$

$$\left\{11.3125, \left\{QU[\,] + \ll 456\gg + QU[y_1, y_1, y_1, x_3, x_3, x_3]\left(\frac{\hbar^3}{6} - \frac{1}{2}\hbar^3 T_2^2 + \frac{1}{2}\hbar^3 T_2^4 - \frac{1}{6}\hbar^3 T_2^6\right), 0\right\}\right\}$$

The representation ρ

rho

```
ρ@y_CU = ρ@y_QU = (0 0; ϵ 0); ρ@a_CU = ρ@a_QU = (γ 0; 0 0);
ρ@x_CU = (0 γ; 0 0); ρ@x_QU = SS@(0 (1 - e^(-γ ϵ ℏ)) / (ϵ ℏ); 0 0);
ρ[e^ℰ_] := MatrixExp[ρ[ℰ]];
ρ[ℰ_] :=
  (ℰ /. {t → γ ϵ, T → e^(ℏ γ ϵ/2)} /. (U : CU | QU) [u___] ⧴ Fold[Dot, (1 0; 0 1), ρ /@ U /@ {u}])
```

Verifying that ρ represents CU and QU:

```
Table[ρ[z1 ** z2] == ρ[z1].ρ[z2] // SS // HL,
 {U, {CU, QU}}, {z1, U /@ {y, a, x}}, {z2, U /@ {y, a, x}} ]
```

```
{{{True, True, True}, {True, True, True}, {True, True, True}},
 {{True, True, True}, {True, True, True}, {True, True, True}}}
```

The Classical Logos $C\Lambda$

Lemma 3C. To degree k,
$\mathbb{O}_{CU}\left(e^{\eta y+\xi x+\delta y x} \mid x y\right) = \mathbb{O}_{CU}\left(\nu e^{\nu(-t\xi\eta+\eta y+\xi x+\delta y x)} C\Lambda_k(\epsilon, \gamma, y, a, x, \eta, \xi, \delta) \mid y a x\right)$, with $\nu = (1+t\delta)^{-1}$ and where $C\Lambda_k(\epsilon, \gamma, y, a, x, \eta, \xi, \delta)$ is a fixed polynomial of degree at most $4k$ in y, $\sqrt{a}$, x, η, ξ, with scalar coefficients.

Comment. Even better, $\log(C\Lambda_k)$ is of degree at most $2k+2$ in said variables.

```
eqn = ρ[e^(ξ x_CU)].ρ[e^(η y_CU)] == ρ[e^(d y_CU)].ρ[e^(c (t CU[] - 2 ϵ a_CU))].ρ[e^(b x_CU)]
```

$$\{\{1+\gamma\epsilon\eta\xi, \gamma\xi\}, \{\epsilon\eta, 1\}\} == \left\{\left\{e^{-c\gamma\epsilon}, b\,e^{-c\gamma\epsilon}\gamma\right\}, \left\{d\,e^{-c\gamma\epsilon}\epsilon, e^{c\gamma\epsilon}+b\,d\,e^{-c\gamma\epsilon}\gamma\epsilon\right\}\right\}$$

```
sol = Solve[Thread[Flatten /@ eqn], {d, b, c}][[1]] /. C[1] → 0
```

$$\left\{d\to\frac{\eta}{1+\gamma\epsilon\eta\xi}, b\to\frac{\xi}{1+\gamma\epsilon\eta\xi}, c\to\frac{\mathrm{Log}\left[\frac{1}{1+\gamma\epsilon\eta\xi}\right]}{\gamma\epsilon}\right\}$$

Proof of Lemma 3C. We know that $\mathbb{O}_{CU}\left(e^{\xi x+\eta y} \mid x y\right) = \mathbb{O}_{CU}\left(e^{c t+a y-2\epsilon c a+b x} \mid y a x\right)$, with $\left\{d\to\frac{\eta}{1+\gamma\epsilon\eta\xi}, b\to\frac{\xi}{1+\gamma\epsilon\eta\xi}, c\to\frac{\mathrm{Log}[1+\gamma\epsilon\eta\xi]}{-\gamma\epsilon}\right\}$. Expanding in ϵ we get
$\mathbb{O}_{CU}\left(e^{\xi x+\eta y} \mid x y\right) = \mathbb{O}_{CU}\left(\lambda_\epsilon(\xi,\eta)\,e^{\eta y+\xi x-\eta\xi t} \mid y a x\right) = \mathbb{O}_{CU}\left(\lambda_\epsilon(\partial_x,\partial_y)\,e^{\eta y+\xi x-\eta\xi t} \mid y a x\right)$ and so
$\mathbb{O}_{CU}\left(e^{\eta y+\xi x+\delta y x} \mid x y\right) = \mathbb{O}\left(\lambda_\epsilon(\partial_x,\partial_y)\,e^{\delta\partial_\eta\partial_\xi}\,e^{\eta y+\xi x-\eta\xi t} \mid y a x\right) = \mathbb{O}\left(\lambda_\epsilon(\partial_x,\partial_y)\,\nu\,e^{\nu(-t\xi\eta+\eta y+\xi x+\delta y x)} \mid y a x\right)$.

Logos

```
SSϵ[ℰ_] := Block[{ϵ}, Collect[Normal@Series[ℰ, {ϵ, 0, $TϵD}], ϵ, Together]];
(* Shielded ϵ-Series *)
CΛ[t1_, y1_, a1_, x1_, ξ1_, η1_, δ_] := Module[
  {eqn, d, b, c, sol, λ, q, ν, ξ, η},
  eqn = ρ[ⅇ^(ξ x_CU)].ρ[ⅇ^(η y_CU)] == ρ[ⅇ^(d y_CU)].ρ[ⅇ^(c (t CU[] - 2 ϵ a_CU))].ρ[ⅇ^(b x_CU)];
  sol = Solve[Thread[Flatten /@ eqn], {d, b, c}][[1]] /. C[1] → 0;
  λ = Simplify[ⅇ^(-η y-ξ x+η ξ t) SSϵ[ⅇ^(c t + d y - 2 ϵ c a + b x) /. sol]];
  q = ⅇ^(ν (-t ξ η+η y+ξ x+δ y x));
  Collect[ν q^-1 DP_{ξ→D_x, η→D_y}[λ][q] /. ν → (1 + t δ)^-1, ϵ, Simplify] /.
   {t → t1, y → y1, a → a1, x → x1, ξ → ξ1, η → η1}
 ];
```

```
CΛ[t, y, a, x, ξ, η, δ]
```

$$
\begin{aligned}
&\frac{1}{1+t\,\delta}+\frac{1}{24\left(1+t\,\delta\right)^9}\\
&\epsilon^2\Big(48\,a^2\left(1+t\,\delta\right)^4\left(2\,\delta^2\left(1+t\,\delta\right)^2+4\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)-\\
&\quad 24\,a\,\gamma\left(1+t\,\delta\right)^4\left(2\,\delta^2\left(1+t\,\delta\right)^2+4\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)-\\
&\quad 48\,a\,y\,\gamma\left(1+t\,\delta\right)^3(x\,\delta+\eta)\\
&\quad\left(6\,\delta^2\left(1+t\,\delta\right)^2+6\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)+24\,y\,\gamma^2\left(1+t\,\delta\right)^3\\
&\quad(x\,\delta+\eta)\left(6\,\delta^2\left(1+t\,\delta\right)^2+6\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)-48\,a\,x\,\gamma\\
&\quad\left(1+t\,\delta\right)^3(y\,\delta+\xi)\left(6\,\delta^2\left(1+t\,\delta\right)^2+6\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)+\\
&\quad 24\,x\,\gamma^2\left(1+t\,\delta\right)^3(y\,\delta+\xi)\left(6\,\delta^2\left(1+t\,\delta\right)^2+6\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+\right.\\
&\qquad\left.(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)+12\,y^2\,\gamma^2\left(1+t\,\delta\right)^2(x\,\delta+\eta)^2\\
&\quad\left(12\,\delta^2\left(1+t\,\delta\right)^2+8\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)+12\,x^2\,\gamma^2\\
&\quad\left(1+t\,\delta\right)^2(y\,\delta+\xi)^2\left(12\,\delta^2\left(1+t\,\delta\right)^2+8\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)\,(y\,\delta+\xi)+(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\right)+\\
&\quad 24\,a\,t\,\gamma\left(1+t\,\delta\right)^2\left(6\,\delta^3\left(1+t\,\delta\right)^3+18\,\delta^2\left(1+t\,\delta\right)^2(x\,\delta+\eta)\,(y\,\delta+\xi)+\right.\\
&\qquad\left.9\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2+(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\right)-\\
&\quad 8\,t\left(\gamma+t\,\gamma\,\delta\right)^2\left(6\,\delta^3\left(1+t\,\delta\right)^3+18\,\delta^2\left(1+t\,\delta\right)^2(x\,\delta+\eta)\,(y\,\delta+\xi)+\right.\\
&\qquad\left.9\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2+(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\right)+\\
&\quad 24\,x\,y\left(\gamma+t\,\gamma\,\delta\right)^2\left(6\,\delta^3\left(1+t\,\delta\right)^3+18\,\delta^2\left(1+t\,\delta\right)^2(x\,\delta+\eta)\,(y\,\delta+\xi)+\right.\\
&\qquad\left.9\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2+(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\right)-\\
&\quad 12\,t\,y\,\gamma^2\left(1+t\,\delta\right)(x\,\delta+\eta)\left(24\,\delta^3\left(1+t\,\delta\right)^3+36\,\delta^2\left(1+t\,\delta\right)^2(x\,\delta+\eta)\,(y\,\delta+\xi)+\right.\\
&\qquad\left.12\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2+(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\right)-\\
&\quad 12\,t\,x\,\gamma^2\left(1+t\,\delta\right)(y\,\delta+\xi)\left(24\,\delta^3\left(1+t\,\delta\right)^3+36\,\delta^2\left(1+t\,\delta\right)^2(x\,\delta+\eta)\,(y\,\delta+\xi)+\right.\\
&\qquad\left.12\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2+(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\right)+\\
&\quad 3\,t^2\,\gamma^2\left(24\,\delta^4\left(1+t\,\delta\right)^4+96\,\delta^3\left(1+t\,\delta\right)^3(x\,\delta+\eta)\,(y\,\delta+\xi)+72\,\delta^2\left(1+t\,\delta\right)^2\right.\\
&\qquad\left.(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2+16\,\delta\left(1+t\,\delta\right)(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3+(x\,\delta+\eta)^4\,(y\,\delta+\xi)^4\right)\Big)+\\
&\frac{1}{2\left(1+t\,\delta\right)^5}\,\epsilon\left(4\,a\left(1+t\,\delta\right)^2\left(\left(t+x\,y\right)\delta^2+\eta\,\xi+\delta\left(1+y\,\eta+x\,\xi\right)\right)+\right.\\
&\quad\gamma\left(2\,t^3\,\delta^4+4\,t^2\,\delta^2\left(\delta-x\,y\,\delta^2+\eta\,\xi\right)-2\left(y\,\eta\left(\delta\left(2+y\,\eta\right)+\eta\,\xi\right)+x^2\,\delta\left(2\,y^2\,\delta^2+3\,y\,\delta\,\xi+\xi^2\right)+\right.\right.\\
&\qquad\left.x\left(3\,y^2\,\delta^2\,\eta+4\,y\,\delta\,(\delta+\eta\,\xi)+\xi\left(2\,\delta+\eta\,\xi\right)\right)\right)-t\left(3\,x^2\,y^2\,\delta^4-4\,\delta\,\eta\,\xi-\eta^2\,\xi^2+\right.\\
&\qquad\left.\left.\left.4\,x\,y\,\delta^3\left(3+y\,\eta+x\,\xi\right)+\delta^2\left(-2+y^2\,\eta^2+4\,x\,\xi+x^2\,\xi^2+4\,y\,(\eta+x\,\eta\,\xi)\right)\right)\right)\right)
\end{aligned}
$$

```
{Short[lhs = 𝕆CU[SS[ℯ^(ℏ (ξ x+η y+δ x y))], {x, y}], 5], HL[lhs ==
   𝕆CU[SS[ℯ^(ℏ ν (ξ x+η y+δ x y-t ℏ ξ η)) CΛ[t, y, a, x, ℏ ξ, ℏ η, ℏ δ] /. ν → (1 + ℏ t δ)^-1], {y, a, x}]]}
```

$\{(1 - t\,\delta\,\hbar + t^2\,\delta^2\,\hbar^2 + t\,\gamma\,\delta^2\,\epsilon\,\hbar^2 - t\,\eta\,\xi\,\hbar^2 - t^3\,\delta^3\,\hbar^3 - 3\,t^2\,\gamma\,\delta^3\,\epsilon\,\hbar^3 - 2\,t\,\gamma^2\,\delta^3\,\epsilon^2\,\hbar^3 + 2\,t^2\,\delta\,\eta\,\xi\,\hbar^3 + 2\,t\,\gamma\,\delta\,\epsilon\,\eta\,\xi\,\hbar^3)\,\mathrm{CU}[\,] + (2\,\delta\,\epsilon\,\hbar - 4\,t\,\delta^2\,\epsilon\,\hbar^2 - 2\,\gamma\,\delta^2\,\epsilon^2\,\hbar^2 + 2\,\epsilon\,\eta\,\xi\,\hbar^2 + 6\,t^2\,\delta^3\,\epsilon\,\hbar^3 + 12\,t\,\gamma\,\delta^3\,\epsilon^2\,\hbar^3 - 8\,t\,\delta\,\epsilon\,\eta\,\xi\,\hbar^3 - 4\,\gamma\,\delta\,\epsilon^2\,\eta\,\xi\,\hbar^3)\,\mathrm{CU}[a] + (\xi\,\hbar - 2\,t\,\delta\,\xi\,\hbar^2 - 2\,\gamma\,\delta\,\epsilon\,\xi\,\hbar^2 + 3\,t^2\,\delta^2\,\xi\,\hbar^3 + 9\,t\,\gamma\,\delta^2\,\epsilon\,\xi\,\hbar^3 + 6\,\gamma^2\,\delta^2\,\epsilon^2\,\xi\,\hbar^3 - t\,\eta\,\xi^2\,\hbar^3 - \gamma\,\epsilon\,\eta\,\xi^2\,\hbar^3)\,\mathrm{CU}[x] + \ll 24 \gg + \frac{1}{2}\,\delta^2\,\eta\,\hbar^3\,\mathrm{CU}[y, y, y, x, x] + \frac{1}{6}\,\delta^3\,\hbar^3\,\mathrm{CU}[y, y, y, x, x, x],\ \mathrm{True}\}$

C𝕆 and Swaps

Swaps from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

CdsO

```
SetAttributes[C𝕆, Orderless];
CU@C𝕆[specs___, 𝔼[L_, Q_, P_]] := 𝕆CU[SS[ℯ^(L+Q) P], specs]
```

```
CU@C𝕆[𝔼[ℏ t1 a2, ℏ t1^-1 (ℯ^t1 - 1) y1 x2, 1 + ϵ x1 y2], {y1, x1}1, {x2, a2, y2}2] // Short
```

$\mathrm{CU}[\,] + \ll 27 \gg + \mathrm{CU}[y_1, x_1]\left(-\gamma\,\epsilon\,\hbar^2\,t_2 + e^{t_1}\,\gamma\,\epsilon\,\hbar^2\,t_2 + \frac{\ll 1 \gg}{t_1} - \frac{\ll 1 \gg}{\ll 1 \gg} + \frac{1}{2}\,\gamma^2\,\epsilon\,\hbar^3\,t_1\,t_2 - \frac{1}{2}\,e^{t_1}\,\gamma^2\,\epsilon\,\hbar^3\,t_1\,t_2\right)$

```
HL[ρ[ℯ^(ξ CU@x)].ρ[ℯ^(α CU@a)] == ρ[ℯ^(α CU@a)].ρ[ℯ^(ℯ^(-γ α) ξ CU@x)]]
```

True

SW

```
SW[x_i_, a_j_][C𝕆[{lh___, x_i_, a_j_, rh___}_s_, more___, 𝔼[L_, Q_, P_]]] :=
 C𝕆[{lh, a_j, x_i, rh}_s, more,
  With[{q = ℯ^(-γ α) ξ x_i + α a_j},
   𝔼[L, ℯ^(-γ α) ξ x_i + (Q /. x_i → 0), ℯ^-q DP[x_i→D_ξ, a_j→D_α][P][ℯ^q]] /. {α → ∂_(a_j) L, ξ → ∂_(x_i) Q}]]
```

```
c𝕆 = C𝕆[𝔼[ℏ t1 a2, ℏ t1^-1 (ℯ^t1 - 1) y1 x2, 1 + ϵ x1 y2], {y1, x1}1, {x2, a2, y2}2]
```

$C\mathbb{O}\left[\{y_1, x_1\}_1, \{x_2, a_2, y_2\}_2, \mathbb{E}\left[\hbar\,a_2\,t_1, \frac{(-1 + e^{t_1})\,\hbar\,x_2\,y_1}{t_1}, 1 + \epsilon\,x_1\,y_2\right]\right]$

```
SW[x2, a2][c𝕆]
```

$C\mathbb{O}\left[\{y_1, x_1\}_1, \{a_2, x_2, y_2\}_2, \mathbb{E}\left[\hbar\,a_2\,t_1, \frac{e^{-\gamma\,\hbar\,t_1}\,(-1 + e^{t_1})\,\hbar\,x_2\,y_1}{t_1}, 1 + \epsilon\,x_1\,y_2\right]\right]$

```
With[{c𝕆 = C𝕆[{y1, x1}1, {x2, a2, y2}2, 𝔼[ℏ t1 a2, ℏ t1^-1 (ℯ^t1 - 1) y1 x2, 1 + ϵ x1 y2]]},
 HL[CU[c𝕆] == CU[c𝕆 // SW[x2, a2]]]]
```

True

```
With[{cO = CO[{y1, a1, x1}1, {x2, a2, y2}2,
    E[ħ (l11 t1 a1 + l12 t1 a2 + l21 t2 a1 + l22 t2 a2), ħ (γ11 x1 y1 + γ12 x1 y2 + γ21 x2 y1 + γ22 x2 y2),
     1 + ϵ (l1 a1 + l2 a2 + p11 x1 y1 + p12 x1 y2 + p21 x2 y1 + p22 x2 y2)]]},
 {CU[cO] // Short, HL[CU[cO] == CU[cO // SW_{x2,a2}]]}
]
```

$\{CU[a_1, a_1, a_1, a_1] \left(\frac{1}{6} \epsilon \hbar^3 l_1 l_{11}^3 t_1^3 + \frac{1}{2} \epsilon \hbar^3 l_1 l_{11}^2 l_{21} t_1^2 t_2 + \frac{1}{2} \ll 6 \gg \ll 1 \gg + \frac{1}{6} \epsilon \hbar^3 l_1 l_{21}^3 t_2^3\right) + \ll 180 \gg + \ll 1 \gg, \text{True}\}$

SW

```
SW_{x_i_,y_j_→k_}[CO[{lh___, x_i_, y_j_, rh___}_s_, more___, E[L_, Q_, P_]]] :=
 CO[{lh, y_k, a_k, x_k, rh}_s, more,
  With[{q = ν (ξ x_k + η y_k + δ x_k y_k - t_k ξ η)},
   E[L, q + (Q /. x_i | y_j → 0), e^-q DP_{x_i→D_ξ, y_j→D_η}[P][CΛ[t_k, y_k, a_k, x_k, ξ, η, δ] e^q]] /.
    ν → (1 + t_k δ)^-1 /. {ξ → (∂_{x_i} Q /. y_j → 0), η → (∂_{y_j} Q /. x_i → 0), δ → ∂_{x_i,y_j} Q}]]
```

```
With[{cO = CO[{x1, y1}1, {x2, a2, y2}2,
    E[ħ (l12 t1 a2 + l22 t2 a2), ħ (γ11 x1 y1 + γ12 x1 y2 + γ21 x2 y1 + γ22 x2 y2),
     1 + ϵ (l2 a2 + p11 x1 y1 + p12 x1 y2 + p21 x2 y1 + p22 x2 y2)]]},
 {CU[cO] // Short, HL[CU[cO] == CU[cO // SW_{x1,y1→1}]]}
]
```

$\{12 \epsilon^2 \hbar^3 CU[y_1, a_1, a_1, x_1] \gamma_{11}^3 + \frac{16}{3} \epsilon^2 \ll 3 \gg \gamma_{11}^3 + \ll 158 \gg + CU[] \left(\ll 301 \gg + \epsilon \hbar^3 p_{11} t_1 t_2^3 \gamma_{22}^3 + 4 \epsilon \hbar^3 p_{22} t_2^4 \gamma_{22}^3\right), \text{True}\}$

The Quantum Logos QΛ

Goal 1: In QU, compute $F = e^{-\eta y} e^{\xi x} e^{\eta y} e^{-\xi x}$.

First compute $G = e^{\xi x} y e^{-\xi x}$, a finite sum.

```
adx[ℰ_] := Simp[QU@x ** ℰ - ℰ ** QU@x];
G = Simp[NestList[adx, QU@y, $TϵD + 1].Table[ξ^k / k!, {k, 0, $TϵD + 1}]]
```

$\frac{(\xi - T^2 \xi) QU[]}{\hbar} + 2 T^2 \epsilon \xi QU[a] + \left(\frac{1}{2} \gamma \epsilon \xi^2 - \frac{3}{2} T^2 \gamma \epsilon \xi^2 + \left(\frac{1}{4} \gamma^2 \epsilon^2 \xi^2 - \frac{5}{4} T^2 \gamma^2 \epsilon^2 \xi^2\right) \hbar\right) QU[x] + QU[y] - 2 T^2 \epsilon^2 \xi \hbar QU[a, a] + 3 T^2 \gamma \epsilon^2 \xi^2 \hbar QU[a, x] + \left(\frac{1}{6} \gamma^2 \epsilon^2 \xi^3 - \frac{7}{6} T^2 \gamma^2 \epsilon^2 \xi^3\right) \hbar QU[x, x] + \left(\gamma \epsilon \xi \hbar + \frac{1}{2} \gamma^2 \epsilon^2 \xi \hbar^2\right) QU[y, x] + \frac{1}{2} \gamma^2 \epsilon^2 \xi^2 \hbar^2 QU[y, x, x]$

```
G /. ϵ → 0
```

$\frac{(\xi - T^2 \xi) QU[]}{\hbar} + QU[y]$

Now F satisfies the ODE $\partial_\eta F = \partial_\eta \left(e^{-\eta y} e^{\eta G}\right) = -yF + FG$ with initial conditions $F(\eta = 0) = 1$. We set it up and solve:

```
F = Sum[f_{l,i,j,k}[η] ϵ^l QU@{y^i, a^j, x^k},
  {l, 0, $TϵD}, {i, 0, l}, {j, 0, l}, {k, 0, Min[l, 2 l - i - j]}]
```

$$\begin{aligned}&QU[]\, f_{0,0,0,0}[\eta] + \epsilon\, QU[]\, f_{1,0,0,0}[\eta] + \epsilon\, QU[x]\, f_{1,0,0,1}[\eta] + \epsilon\, QU[a]\, f_{1,0,1,0}[\eta] +\\ &\epsilon\, QU[a, x]\, f_{1,0,1,1}[\eta] + \epsilon\, QU[y]\, f_{1,1,0,0}[\eta] + \epsilon\, QU[y, x]\, f_{1,1,0,1}[\eta] +\\ &\epsilon\, QU[y, a]\, f_{1,1,1,0}[\eta] + \epsilon^2 QU[]\, f_{2,0,0,0}[\eta] + \epsilon^2 QU[x]\, f_{2,0,0,1}[\eta] +\\ &\epsilon^2 QU[x, x]\, f_{2,0,0,2}[\eta] + \epsilon^2 QU[a]\, f_{2,0,1,0}[\eta] + \epsilon^2 QU[a, x]\, f_{2,0,1,1}[\eta] +\\ &\epsilon^2 QU[a, x, x]\, f_{2,0,1,2}[\eta] + \epsilon^2 QU[a, a]\, f_{2,0,2,0}[\eta] + \epsilon^2 QU[a, a, x]\, f_{2,0,2,1}[\eta] +\\ &\epsilon^2 QU[a, a, x, x]\, f_{2,0,2,2}[\eta] + \epsilon^2 QU[y]\, f_{2,1,0,0}[\eta] + \epsilon^2 QU[y, x]\, f_{2,1,0,1}[\eta] +\\ &\epsilon^2 QU[y, x, x]\, f_{2,1,0,2}[\eta] + \epsilon^2 QU[y, a]\, f_{2,1,1,0}[\eta] + \epsilon^2 QU[y, a, x]\, f_{2,1,1,1}[\eta] +\\ &\epsilon^2 QU[y, a, x, x]\, f_{2,1,1,2}[\eta] + \epsilon^2 QU[y, a, a]\, f_{2,1,2,0}[\eta] + \epsilon^2 QU[y, a, a, x]\, f_{2,1,2,1}[\eta] +\\ &\epsilon^2 QU[y, y]\, f_{2,2,0,0}[\eta] + \epsilon^2 QU[y, y, x]\, f_{2,2,0,1}[\eta] + \epsilon^2 QU[y, y, x, x]\, f_{2,2,0,2}[\eta] +\\ &\epsilon^2 QU[y, y, a]\, f_{2,2,1,0}[\eta] + \epsilon^2 QU[y, y, a, x]\, f_{2,2,1,1}[\eta] + \epsilon^2 QU[y, y, a, a]\, f_{2,2,2,0}[\eta]\end{aligned}$$

```
unowns = Cases[F, f___[η], ∞]
```

$$\begin{aligned}&\{f_{0,0,0,0}[\eta], f_{1,0,0,0}[\eta], f_{1,0,0,1}[\eta], f_{1,0,1,0}[\eta], f_{1,0,1,1}[\eta], f_{1,1,0,0}[\eta], f_{1,1,0,1}[\eta], f_{1,1,1,0}[\eta],\\ &f_{2,0,0,0}[\eta], f_{2,0,0,1}[\eta], f_{2,0,0,2}[\eta], f_{2,0,1,0}[\eta], f_{2,0,1,1}[\eta], f_{2,0,1,2}[\eta], f_{2,0,2,0}[\eta], f_{2,0,2,1}[\eta],\\ &f_{2,0,2,2}[\eta], f_{2,1,0,0}[\eta], f_{2,1,0,1}[\eta], f_{2,1,0,2}[\eta], f_{2,1,1,0}[\eta], f_{2,1,1,1}[\eta], f_{2,1,1,2}[\eta], f_{2,1,2,0}[\eta],\\ &f_{2,1,2,1}[\eta], f_{2,2,0,0}[\eta], f_{2,2,0,1}[\eta], f_{2,2,0,2}[\eta], f_{2,2,1,0}[\eta], f_{2,2,1,1}[\eta], f_{2,2,2,0}[\eta]\}\end{aligned}$$

```
bas = Union @@ Table[ϵ^l Cases[Coefficient[F, ϵ, l], _QU, ∞], {l, 0, $TϵD}]
```

$$\begin{aligned}&\{QU[], \epsilon\, QU[], \epsilon^2 QU[], \epsilon\, QU[a], \epsilon^2 QU[a], \epsilon\, QU[x], \epsilon^2 QU[x], \epsilon\, QU[y], \epsilon^2 QU[y],\\ &\epsilon^2 QU[a, a], \epsilon\, QU[a, x], \epsilon^2 QU[a, x], \epsilon^2 QU[x, x], \epsilon\, QU[y, a], \epsilon^2 QU[y, a], \epsilon\, QU[y, x],\\ &\epsilon^2 QU[y, x], \epsilon^2 QU[y, y], \epsilon^2 QU[a, a, x], \epsilon^2 QU[a, x, x], \epsilon^2 QU[y, a, a], \epsilon^2 QU[y, a, x],\\ &\epsilon^2 QU[y, x, x], \epsilon^2 QU[y, y, a], \epsilon^2 QU[y, y, x], \epsilon^2 QU[a, a, x, x], \epsilon^2 QU[y, a, a, x],\\ &\epsilon^2 QU[y, a, x, x], \epsilon^2 QU[y, y, a, a], \epsilon^2 QU[y, y, a, x], \epsilon^2 QU[y, y, x, x]\}\end{aligned}$$

```
Short[eqns = Flatten[{(Coefficient[F - QU[], #] /. η → 0) == 0,
      Expand[Coefficient[Simp[F ** G - QU[y] ** F - ∂_η F], #]] == 0} & /@ bas], 8]
```

$$\begin{aligned}&\{-1 + f_{0,0,0,0}[0] + \epsilon\, f_{1,0,0,0}[0] + \epsilon^2 f_{2,0,0,0}[0] == 0,\\ &\frac{\xi f_{0,0,0,0}[\eta]}{\hbar} - \frac{T^2 \xi f_{0,0,0,0}[\eta]}{\hbar} + \frac{\epsilon \xi f_{1,0,0,0}[\eta]}{\hbar} - \frac{T^2 \epsilon \xi f_{1,0,0,0}[\eta]}{\hbar} + \frac{\epsilon f_{1,0,0,1}[\eta]}{\hbar} -\\ &\quad \frac{T^2 \epsilon f_{1,0,0,1}[\eta]}{\hbar} + \frac{\epsilon^2 \xi f_{2,0,0,0}[\eta]}{\hbar} - \frac{T^2 \epsilon^2 \xi f_{2,0,0,0}[\eta]}{\hbar} + \frac{\epsilon^2 f_{2,0,0,1}[\eta]}{\hbar} -\\ &\quad \frac{T^2 \epsilon^2 f_{2,0,0,1}[\eta]}{\hbar} - f_{0,0,0,0}'[\eta] - \epsilon f_{1,0,0,0}'[\eta] - \epsilon^2 f_{2,0,0,0}'[\eta] == 0,\ f_{1,0,0,0}[0] == 0,\\ &\frac{\xi f_{1,0,0,0}[\eta]}{\hbar} - \frac{T^2 \xi f_{1,0,0,0}[\eta]}{\hbar} + \frac{f_{1,0,0,1}[\eta]}{\hbar} - \frac{T^2 f_{1,0,0,1}[\eta]}{\hbar} - f_{1,0,0,0}'[\eta] == 0,\\ &f_{2,0,0,0}[0] == 0, \ll 53 \gg, f_{2,2,1,1}[0] == 0,\\ &\gamma \xi \hbar f_{1,1,1,0}[\eta] - 2 \gamma f_{2,1,2,1}[\eta] + \frac{\xi f_{2,2,1,1}[\eta]}{\hbar} - \frac{T^2 \xi f_{2,2,1,1}[\eta]}{\hbar} - f_{2,2,1,1}'[\eta] == 0,\\ &f_{2,2,0,2}[0] == 0, \gamma \xi \hbar f_{1,1,0,1}[\eta] - \gamma f_{2,1,1,2}[\eta] + \frac{\xi f_{2,2,0,2}[\eta]}{\hbar} - \frac{T^2 \xi f_{2,2,0,2}[\eta]}{\hbar} - f_{2,2,0,2}'[\eta] == 0\}\end{aligned}$$

```
Short[{sol} = DSolve[eqns, unowns, η], 8]
```

$$\Big\{\Big\{f_{0,0,0,0}[\eta]\to e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}},\ f_{1,0,0,0}[\eta]\to\frac{e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}(-1+T^2)(-1+3T^2)\gamma\eta^2\xi^2}{4\hbar},$$

$$f_{1,0,0,1}[\eta]\to-\frac{1}{2}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}(-1+3T^2)\gamma\eta\xi^2,\ f_{1,0,1,0}[\eta]\to 2e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}T^2\eta\xi,$$

$$f_{1,0,1,1}[\eta]\to 0,\ f_{1,1,0,0}[\eta]\to-\frac{1}{2}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}(-1+3T^2)\gamma\eta^2\xi,\ f_{1,1,0,1}[\eta]\to e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma\eta\xi\hbar,$$

$$f_{1,1,1,0}[\eta]\to 0,\ f_{2,0,0,0}[\eta]\to\frac{1}{288\hbar^2}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}(-1+T^2)\gamma^2\eta^2\xi^2\big(-9\eta^2\xi^2+63T^2\eta^2\xi^2-$$
$$135T^4\eta^2\xi^2+81T^6\eta^2\xi^2-40\eta\xi\hbar+272T^2\eta\xi\hbar-328T^4\eta\xi\hbar-36\hbar^2+180T^2\hbar^2\big),$$

$$f_{2,0,0,1}[\eta]\to-\frac{1}{24\hbar}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta\xi^2\big(-3\eta^2\xi^2+21T^2\eta^2\xi^2-45T^4\eta^2\xi^2+$$
$$27T^6\eta^2\xi^2-10\eta\xi\hbar+68T^2\eta\xi\hbar-82T^4\eta\xi\hbar-6\hbar^2+30T^2\hbar^2\big),$$

$$f_{2,0,0,2}[\eta]\to\frac{1}{24}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta\xi^3\big(3\eta\xi-18T^2\eta\xi+27T^4\eta\xi+4\hbar-28T^2\hbar\big),$$

$$f_{2,0,1,0}[\eta]\to\frac{1}{2\hbar}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}T^2\gamma\eta^2\xi^2\big(\eta\xi-4T^2\eta\xi+3T^4\eta\xi+4\hbar-6T^2\hbar\big),$$

$$f_{2,0,1,1}[\eta]\to-e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}T^2\gamma\eta\xi^2\big(-\eta\xi+3T^2\eta\xi-3\hbar\big),\ f_{2,0,1,2}[\eta]\to 0,$$

$$f_{2,0,2,0}[\eta]\to 2e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}T^2\eta\xi\big(T^2\eta\xi-\hbar\big),\ f_{2,0,2,1}[\eta]\to 0,\ f_{2,0,2,2}[\eta]\to 0,$$

$$f_{2,1,0,0}[\eta]\to-\frac{1}{24\hbar}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta^2\xi\big(-3\eta^2\xi^2+21T^2\eta^2\xi^2-$$
$$45T^4\eta^2\xi^2+27T^6\eta^2\xi^2-10\eta\xi\hbar+68T^2\eta\xi\hbar-82T^4\eta\xi\hbar-6\hbar^2+30T^2\hbar^2\big),$$

$$f_{2,1,0,1}[\eta]\to\frac{1}{4}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta\xi\big(2\eta^2\xi^2-10T^2\eta^2\xi^2+12T^4\eta^2\xi^2+5\eta\xi\hbar-21T^2\eta\xi\hbar+2\hbar^2\big),$$

$$f_{2,1,0,2}[\eta]\to-\frac{1}{2}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta\xi^2\big(-\eta\xi+3T^2\eta\xi-\hbar\big)\hbar,$$

$$f_{2,1,1,0}[\eta]\to-e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}T^2\gamma\eta^2\xi\big(-\eta\xi+3T^2\eta\xi-3\hbar\big),$$

$$f_{2,1,1,1}[\eta]\to 2e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}T^2\gamma\eta^2\xi^2\hbar,\ f_{2,1,1,2}[\eta]\to 0,\ f_{2,1,2,0}[\eta]\to 0,\ f_{2,1,2,1}[\eta]\to 0,$$

$$f_{2,2,0,0}[\eta]\to\frac{1}{24}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta^3\xi\big(3\eta\xi-18T^2\eta\xi+27T^4\eta\xi+4\hbar-28T^2\hbar\big),$$

$$f_{2,2,0,1}[\eta]\to-\frac{1}{2}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta^2\xi\big(-\eta\xi+3T^2\eta\xi-\hbar\big)\hbar,$$

$$f_{2,2,0,2}[\eta]\to\frac{1}{2}e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\gamma^2\eta^2\xi^2\hbar^2,\ f_{2,2,1,0}[\eta]\to 0,\ f_{2,2,1,1}[\eta]\to 0,\ f_{2,2,2,0}[\eta]\to 0\Big\}\Big\}$$

```
Union@Cases[sol, e-, ∞]
```

$$\Big\{e^{-\frac{\eta(-\xi+T^2\xi)}{\hbar}}\Big\}$$

```
FF = Collect[F /. sol /. {e- → 1, QU → Times}, ϵ, Simplify]
```

$$1+\frac{1}{4\hbar}\epsilon\,\eta\,\xi\left(8\,a\,T^2\,\hbar+\left(-1+3\,T^2\right)\gamma\,\eta\left(\left(-1+T^2\right)\xi-2\,y\,\hbar\right)+2\,x\,\gamma\,\hbar\left(\xi-3\,T^2\,\xi+2\,y\,\hbar\right)\right)+$$
$$\frac{1}{288\,\hbar^2}\epsilon^2\,\eta\,\xi\left(576\,a^2\,T^2\left(T^2\,\eta\,\xi-\hbar\right)\hbar^2+144\,a\,T^2\,\gamma\,\hbar\left(6\,x\,\xi\,\hbar^2+\left(-1+3\,T^2\right)\eta^2\,\xi\left(\left(-1+T^2\right)\xi-2\,y\,\hbar\right)+2\,\eta\,\hbar\left(\left(x-3\,T^2\,x\right)\xi^2+3\,y\,\hbar+\xi\left(2-3\,T^2+2\,x\,y\,\hbar\right)\right)\right)+\gamma^2\left(9\left(1-3\,T^2\right)^2\eta^3\,\xi\left(\xi-T^2\,\xi+2\,y\,\hbar\right)^2+24\,x\,\hbar^3\left(2\left(1-7\,T^2\right)x\,\xi^2+6\,y\,\hbar+\xi\left(3-15\,T^2+6\,x\,y\,\hbar\right)\right)+12\,\eta\,\hbar^2\left(3\left(1-3\,T^2\right)^2x^2\,\xi^3+6\,y\,\hbar\left(1-5\,T^2+2\,x\,y\,\hbar\right)+3\,\xi\left(1+5\,T^4+10\,x\,y\,\hbar+4\,x^2\,y^2\,\hbar^2-6\,T^2\left(1+7\,x\,y\,\hbar\right)\right)+2\,x\,\xi^2\left(5+41\,T^4+6\,x\,y\,\hbar-2\,T^2\left(17+9\,x\,y\,\hbar\right)\right)\right)-4\,\eta^2\,\hbar\left(9\left(1-3\,T^2\right)^2\left(-1+T^2\right)x\,\xi^3+12\left(-1+7\,T^2\right)y^2\,\hbar^2+6\,y\,\xi\,\hbar\left(-5-41\,T^4-6\,x\,y\,\hbar+2\,T^2\left(17+9\,x\,y\,\hbar\right)\right)+2\,\xi^2\left(-5+41\,T^6-18\,x\,y\,\hbar-3\,T^4\left(25+36\,x\,y\,\hbar\right)+T^2\left(39+90\,x\,y\,\hbar\right)\right)\right)\right)\right)$$

```
{Short[lhs = SimpT@𝕆_QU[SS[e^(ℏ (ξ x+η y))], {x, y}], 5],
 HL[lhs == SimpT@𝕆_QU[SS[e^(ℏ (ξ x+η y+(1-T^2) ξ η)) (FF /. {ξ → ℏ ξ, η → ℏ η})], {y, a, x}]]}
```

$$\left\{\left(1-t\,\eta\,\xi\,\hbar^2-\frac{1}{2}t^2\,\eta\,\xi\,\hbar^3\right)QU[\,]+\left(2\,\epsilon\,\eta\,\xi\,\hbar^2+2\,t\,\epsilon\,\eta\,\xi\,\hbar^3\right)QU[a]+\left(\xi\,\hbar+\left(-t\,\eta\,\xi^2-\gamma\,\epsilon\,\eta\,\xi^2\right)\hbar^3\right)QU[x]+\left(\eta\,\hbar+\left(-t\,\eta^2\,\xi-\gamma\,\epsilon\,\eta^2\,\xi\right)\hbar^3\right)QU[y]-2\,\epsilon^2\,\eta\,\xi\,\hbar^3\,QU[a,a]+2\,\epsilon\,\eta\,\xi^2\,\hbar^3\,QU[a,x]+\frac{1}{2}\xi^2\,\hbar^2\,QU[x,x]+2\,\epsilon\,\eta^2\,\xi\,\hbar^3\,QU[y,a]+\left(\eta\,\xi\,\hbar^2+\gamma\,\epsilon\,\eta\,\xi\,\hbar^3\right)QU[y,x]+\frac{1}{2}\eta^2\,\hbar^2\,QU[y,y]+\frac{1}{6}\xi^3\,\hbar^3\,QU[x,x,x]+\frac{1}{2}\eta\,\xi^2\,\hbar^3\,QU[y,x,x]+\frac{1}{2}\eta^2\,\xi\,\hbar^3\,QU[y,y,x]+\frac{1}{6}\eta^3\,\hbar^3\,QU[y,y,y],\ \text{True}\right\}$$

Logos

```
QΛ[T_, y1_, a1_, x1_, ξ1_, η1_, δ_] := Module[
   {adx, G, F, f, unowns, bas, eqns, sol, λ, q, ν, ξ, η, t},
   adx[ℰ_] := Simp[x_QU ** ℰ - ℰ ** x_QU];
   G = Simp[NestList[adx, y_QU, $TϵD + 1].Table[ξ^k / k!, {k, 0, $TϵD + 1}]];
   F = Sum[f_{l,i,j,k}[η] ϵ^l QU@{y^i, a^j, x^k},
     {l, 0, $TϵD}, {i, 0, l}, {j, 0, l}, {k, 0, Min[l, 2 l - i - j]}];
   unowns = Cases[F, f___[η], ∞];
   bas = Union @@ Table[ϵ^l Cases[Coefficient[F, ϵ, l], _QU, ∞], {l, 0, $TϵD}];
   eqns = Flatten[{(Coefficient[F - QU[], #] /. η → 0) == 0,
        Expand[Coefficient[Simp[F ** G - y_QU ** F - ∂_η F], #]] == 0} & /@ bas];
   {sol} = DSolve[eqns, unowns, η];
   λ = Collect[F /. sol /. {e- → 1, QU → Times}, ϵ, Simplify];
   q = e^(ν (-t ξ η+η y+ξ x+δ y x));
   Collect[ν q^-1 DP_{ξ→D_x,η→D_y}[λ][q] /. ν → (1 + t δ)^-1 /. t → (T^2 - 1) / ℏ, ϵ, Simplify] /.
    {y → y1, a → a1, x → x1, ξ → ξ1, η → η1}
  ];
```

```
QΛ[T, y, a, x, ξ, η, δ]
```

$$\frac{\hbar}{\left(-1+T^2\right)\delta+\hbar}+\frac{1}{288\left(\left(-1+T^2\right)\delta+\hbar\right)^9}$$

$$\epsilon^2 \hbar^3 \left(-48\, y^2 \gamma^2 (x\delta+\eta)^2 \hbar^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 3\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) + \right.$$
$$336\, T^2 y^2 \gamma^2 (x\delta+\eta)^2 \hbar^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 3\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) -$$
$$48\, x^2 \gamma^2 (y\delta+\xi)^2 \hbar^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 3\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) +$$
$$336\, T^2 x^2 \gamma^2 (y\delta+\xi)^2 \hbar^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 3\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) +$$
$$144\, x\, y^2 \gamma^2 (x\delta+\eta) \left(\delta - T^2\delta - \hbar\right)^5 \hbar^2 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) +$$
$$144\, x^2 y\, \gamma^2 (y\delta+\xi) \left(\delta - T^2\delta - \hbar\right)^5 \hbar^2 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) -$$
$$864\, a\, T^2 y\, \gamma\, (x\delta+\eta)\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^5 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) -$$
$$72\, y\, \gamma^2 (x\delta+\eta)\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^5 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) +$$
$$360\, T^2 y\, \gamma^2 (x\delta+\eta)\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^5 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) -$$
$$864\, a\, T^2 x\, \gamma\, (y\delta+\xi)\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^5 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) -$$
$$72\, x\, \gamma^2 (y\delta+\xi)\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^5 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) +$$
$$360\, T^2 x\, \gamma^2 (y\delta+\xi)\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^5 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - 2\delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) +$$
$$576\, a^2 T^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^6 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - \delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) - 144\, x\, y\, \gamma^2 \hbar$$
$$\left(\left(-1+T^2\right)\delta+\hbar\right)^6 \left(-(x\delta+\eta)(y\delta+\xi)\hbar - \delta\left(\left(-1+T^2\right)\delta+\hbar\right)\right) + 576\, a^2 T^4 \left(\left(-1+T^2\right)\delta+\hbar\right)^4$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$
$$576\, a\, T^2 \gamma \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + \right.$$
$$\left. 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) - 864\, a\, T^4 \gamma \left(\left(-1+T^2\right)\delta+\hbar\right)^4$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$
$$36\, \gamma^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + \right.$$
$$\left. 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) - 216\, T^2 \gamma^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$
$$180\, T^4 \gamma^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + \right.$$
$$\left. 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) + 576\, a\, T^2 x\, y\, \gamma\, \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^4$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$
$$360\, x\, y\, \gamma^2 \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar \right.$$
$$\left. \left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) - 1512\, T^2 x\, y\, \gamma^2 \hbar \left(\left(-1+T^2\right)\delta+\hbar\right)^4$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$
$$144\, x^2 y^2 \gamma^2 \hbar^2 \left(\left(-1+T^2\right)\delta+\hbar\right)^4 \left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 4\delta(x\delta+\eta)(y\delta+\xi)\hbar \right.$$
$$\left. \left(\left(-1+T^2\right)\delta+\hbar\right) + 2\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) - 288\, a\, T^2 y\, \gamma\, (x\delta+\eta) \left(\delta - T^2\delta - \hbar\right)^3 \hbar$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 6\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 6\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$
$$864\, a\, T^4 y\, \gamma\, (x\delta+\eta) \left(\delta - T^2\delta - \hbar\right)^3 \hbar$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 6\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 6\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) -$$
$$120\, y\, \gamma^2 (x\delta+\eta) \left(\delta - T^2\delta - \hbar\right)^3 \hbar \left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 6\delta(x\delta+\eta)(y\delta+\xi)\hbar \right.$$
$$\left. \left(\left(-1+T^2\right)\delta+\hbar\right) + 6\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) + 816\, T^2 y\, \gamma^2 (x\delta+\eta) \left(\delta - T^2\delta - \hbar\right)^3 \hbar$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 6\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 6\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) -$$
$$984\, T^4 y\, \gamma^2 (x\delta+\eta) \left(\delta - T^2\delta - \hbar\right)^3 \hbar$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 6\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 6\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) -$$
$$288\, a\, T^2 x\, \gamma\, (y\delta+\xi) \left(\delta - T^2\delta - \hbar\right)^3 \hbar$$
$$\left((x\delta+\eta)^2 (y\delta+\xi)^2 \hbar^2 + 6\delta(x\delta+\eta)(y\delta+\xi)\hbar\left(\left(-1+T^2\right)\delta+\hbar\right) + 6\delta^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\right) +$$

$$864\, a\, T^4\, x\, \gamma\, (y\,\delta+\xi)\, (\delta-T^2\,\delta-\hbar)^3\,\hbar$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,-$$
$$120\, x\,\gamma^2\,(y\,\delta+\xi)\,(\delta-T^2\,\delta-\hbar)^3\,\hbar$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$816\, T^2\, x\,\gamma^2\,(y\,\delta+\xi)\,(\delta-T^2\,\delta-\hbar)^3\,\hbar\,((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar$$
$$((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)-984\, T^4\, x\,\gamma^2\,(y\,\delta+\xi)\,(\delta-T^2\,\delta-\hbar)^3\,\hbar$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$144\, x\, y^2\,\gamma^2\,(x\,\delta+\eta)\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^3$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,-$$
$$432\, T^2\, x\, y^2\,\gamma^2\,(x\,\delta+\eta)\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^3$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$144\, x^2\, y\,\gamma^2\,(y\,\delta+\xi)\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^3$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,-$$
$$432\, T^2\, x^2\, y\,\gamma^2\,(y\,\delta+\xi)\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^3$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+6\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+6\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$36\, y^2\,\gamma^2\,(x\,\delta+\eta)^2\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^2$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+8\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+12\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,-$$
$$216\, T^2\, y^2\,\gamma^2\,(x\,\delta+\eta)^2\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^2$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+8\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+12\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$324\, T^4\, y^2\,\gamma^2\,(x\,\delta+\eta)^2\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^2$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+8\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+12\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$36\, x^2\,\gamma^2\,(y\,\delta+\xi)^2\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^2$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+8\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+12\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,-$$
$$216\, T^2\, x^2\,\gamma^2\,(y\,\delta+\xi)^2\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^2$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+8\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+12\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$324\, T^4\, x^2\,\gamma^2\,(y\,\delta+\xi)^2\,\hbar^2\,((-1+T^2)\,\delta+\hbar)^2$$
$$((x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2+8\,\delta\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)+12\,\delta^2\,((-1+T^2)\,\delta+\hbar)^2)\,+$$
$$36\, y\,\gamma^2\,(x\,\delta+\eta)\,(\delta-T^2\,\delta-\hbar)\,\hbar\,(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2$$
$$((-1+T^2)\,\delta+\hbar)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)^2-24\,\delta^3\,((-1+T^2)\,\delta+\hbar)^3)\,-$$
$$252\, T^2\, y\,\gamma^2\,(x\,\delta+\eta)\,(\delta-T^2\,\delta-\hbar)\,\hbar\,(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2$$
$$((-1+T^2)\,\delta+\hbar)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)^2-24\,\delta^3\,((-1+T^2)\,\delta+\hbar)^3)\,+$$
$$540\, T^4\, y\,\gamma^2\,(x\,\delta+\eta)\,(\delta-T^2\,\delta-\hbar)\,\hbar\,(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2$$
$$((-1+T^2)\,\delta+\hbar)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)^2-24\,\delta^3\,((-1+T^2)\,\delta+\hbar)^3)\,-$$
$$324\, T^6\, y\,\gamma^2\,(x\,\delta+\eta)\,(\delta-T^2\,\delta-\hbar)\,\hbar\,(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2$$
$$((-1+T^2)\,\delta+\hbar)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)^2-24\,\delta^3\,((-1+T^2)\,\delta+\hbar)^3)\,+$$
$$36\, x\,\gamma^2\,(y\,\delta+\xi)\,(\delta-T^2\,\delta-\hbar)\,\hbar\,(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2$$
$$((-1+T^2)\,\delta+\hbar)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)^2-24\,\delta^3\,((-1+T^2)\,\delta+\hbar)^3)\,-$$
$$252\, T^2\, x\,\gamma^2\,(y\,\delta+\xi)\,(\delta-T^2\,\delta-\hbar)\,\hbar\,(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2$$
$$((-1+T^2)\,\delta+\hbar)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,((-1+T^2)\,\delta+\hbar)^2-24\,\delta^3\,((-1+T^2)\,\delta+\hbar)^3)\,+$$

$$540\,T^4\,x\,\gamma^2\,(y\,\delta+\xi)\,\left(\delta-T^2\,\delta-\hbar\right)\hbar\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-24\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$324\,T^6\,x\,\gamma^2\,(y\,\delta+\xi)\,\left(\delta-T^2\,\delta-\hbar\right)\hbar\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-12\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-36\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-24\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$144\,a\,T^2\,\gamma\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)+$$

$$576\,a\,T^4\,\gamma\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$432\,a\,T^6\,\gamma\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$40\,\gamma^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)+$$

$$312\,T^2\,\gamma^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$600\,T^4\,\gamma^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)+$$

$$328\,T^6\,\gamma^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$144\,x\,y\,\gamma^2\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)+$$

$$720\,T^2\,x\,y\,\gamma^2\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)-$$

$$864\,T^4\,x\,y\,\gamma^2\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2\left(-(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3-9\,\delta\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)-18\,\delta^2\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^2-6\,\delta^3\left(\left(-1+T^2\right)\delta+\hbar\right)^3\right)+$$

$$9\,\gamma^2\left((x\,\delta+\eta)^4\,(y\,\delta+\xi)^4\,\hbar^4+16\,\delta\,(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3\left(\left(-1+T^2\right)\delta+\hbar\right)+72\,\delta^2\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2+96\,\delta^3\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^3+24\,\delta^4\left(\left(-1+T^2\right)\delta+\hbar\right)^4\right)-$$

$$72\,T^2\,\gamma^2\left((x\,\delta+\eta)^4\,(y\,\delta+\xi)^4\,\hbar^4+16\,\delta\,(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3\left(\left(-1+T^2\right)\delta+\hbar\right)+72\,\delta^2\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2+96\,\delta^3\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^3+24\,\delta^4\left(\left(-1+T^2\right)\delta+\hbar\right)^4\right)+$$

$$198\,T^4\,\gamma^2\left((x\,\delta+\eta)^4\,(y\,\delta+\xi)^4\,\hbar^4+16\,\delta\,(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3\left(\left(-1+T^2\right)\delta+\hbar\right)+72\,\delta^2\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2+96\,\delta^3\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^3+24\,\delta^4\left(\left(-1+T^2\right)\delta+\hbar\right)^4\right)-$$

$$216\,T^6\,\gamma^2\left((x\,\delta+\eta)^4\,(y\,\delta+\xi)^4\,\hbar^4+16\,\delta\,(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3\left(\left(-1+T^2\right)\delta+\hbar\right)+72\,\delta^2\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\left(\left(-1+T^2\right)\delta+\hbar\right)^2+96\,\delta^3\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\left(\left(-1+T^2\right)\delta+\hbar\right)^3+24\,\delta^4\left(\left(-1+T^2\right)\delta+\hbar\right)^4\right)+$$

$$81\,T^8\,\gamma^2\left((x\,\delta+\eta)^4\,(y\,\delta+\xi)^4\,\hbar^4+16\,\delta\,(x\,\delta+\eta)^3\,(y\,\delta+\xi)^3\,\hbar^3\left(\left(-1+T^2\right)\delta+\hbar\right)+\right.$$

$$72\,\delta^2\,(x\,\delta+\eta)^2\,(y\,\delta+\xi)^2\,\hbar^2\,\left((-1+T^2)\,\delta+\hbar\right)^2+$$
$$96\,\delta^3\,(x\,\delta+\eta)\,(y\,\delta+\xi)\,\hbar\,\left((-1+T^2)\,\delta+\hbar\right)^3+24\,\delta^4\,\left((-1+T^2)\,\delta+\hbar\right)^4\Big)\Big)+$$
$$\frac{1}{4\,\left((-1+T^2)\,\delta+\hbar\right)^5}\in\hbar^2\,\Big(8\,a\,T^2\,\left((-1+T^2)\,\delta+\hbar\right)^2\,\left(\eta\,\xi\,\hbar+\delta\,(1+y\,\eta+x\,\xi)\,\hbar+\delta^2\,(-1+T^2+x\,y\,\hbar)\right)+$$
$$\gamma\,\Big(\eta\,\xi\,\hbar^2\,\left((-1+3\,T^2)\,\eta\,\left((-1+T^2)\,\xi-2\,y\,\hbar\right)+2\,x\,\hbar\,\left(\xi-3\,T^2\,\xi+2\,y\,\hbar\right)\right)+$$
$$(-1+T^2)\,\delta^4\,\left(-2+6\,T^6-x^2\,y^2\,\hbar^2-2\,T^4\,(7+4\,x\,y\,\hbar)+T^2\,(10+8\,x\,y\,\hbar-5\,x^2\,y^2\,\hbar^2)\right)-$$
$$4\,\delta^3\,\hbar\,\Big(1-3\,T^6+x^2\,y^2\,\hbar^2+T^4\,\left(7+2\,x\,y\,(3+y\,\eta)\,\hbar+2\,x^2\,y\,\xi\,\hbar\right)+$$
$$T^2\,\left(-5-2\,x\,y\,(3+y\,\eta)\,\hbar+x^2\,y\,\hbar\,(-2\,\xi+y\,\hbar)\right)\Big)+$$
$$2\,\delta\,\hbar^2\,\Big((1-3\,T^2)\,y^2\,\eta^2\,\hbar+2\,\eta\,\left(\xi+3\,T^4\,\xi-4\,T^2\,\xi\,(1+x\,y\,\hbar)+y\,\hbar\,(1-3\,T^2+x\,y\,\hbar)\right)+$$
$$x\,\hbar\,\left((x-3\,T^2\,x)\,\xi^2+2\,y\,\hbar+\xi\,(2-6\,T^2+2\,x\,y\,\hbar)\right)\Big)-$$
$$\delta^2\,\hbar\,\Big((1-4\,T^2+3\,T^4)\,y^2\,\eta^2\,\hbar+\hbar\,\Big(-2+3\,T^4\,(-2+4\,x\,\xi+x^2\,\xi^2)+4\,x\,(\xi+y\,\hbar)+$$
$$x^2\,(\xi^2+2\,y\,\xi\,\hbar-4\,y^2\,\hbar^2)-2\,T^2\,\left(-4+x\,(8\,\xi-6\,y\,\hbar)+x^2\,\xi\,(2\,\xi-5\,y\,\hbar)\right)\Big)+2\,\eta$$
$$\left(-2\,(-1+T^2)\,\xi\,(1+3\,T^4-2\,T^2\,(2+x\,y\,\hbar))+y\,\hbar\,(2+6\,T^4+x\,y\,\hbar+T^2\,(-8+5\,x\,y\,\hbar))\right)\Big)\Big)\Big)$$

```
{Short[lhs = SimpT@𝕆QU[SS[ℯ^(ħ (ξ x+η y+δ x y))], {x, y}], 5],
 rhs = SimpT@𝕆QU[SS[
      ℯ^(ħ ν (ξ x+η y+δ x y-(T^2-1) ξ η)) QΛ[T, y, a, x, ħ ξ, ħ η, ħ δ] /. ν → (1 + (T^2 - 1) δ)^-1], {y, a, x}];
 HL[Simplify[lhs == rhs]]}
```

$$\Big\{\Big(1-t\,\delta\,\hbar+\left(-\frac{t^2\,\delta}{2}+t^2\,\delta^2+t\,\gamma\,\delta^2\in-t\,\eta\,\xi\right)\hbar^2+\Big(-\frac{t^3\,\delta}{6}+t^3\,\delta^2-t^3\,\delta^3+2\,t^2\,\gamma\,\delta^2\in-$$
$$3\,t^2\,\gamma\,\delta^3\in+t\,\gamma^2\,\delta^2\in^2-2\,t\,\gamma^2\,\delta^3\in^2-\frac{1}{2}\,t^2\,\eta\,\xi+2\,t^2\,\delta\,\eta\,\xi+2\,t\,\gamma\,\delta\in\eta\,\xi\Big)\hbar^3\Big)$$
$$QU[\,]+\left(2\,\delta\in\hbar+\left(2\,t\,\delta\in-4\,t\,\delta^2\in-2\,\gamma\,\delta^2\in^2+2\in\eta\,\xi\right)\hbar^2+\right.$$
$$\left.\left(t^2\,\delta\in-6\,t^2\,\delta^2\in+6\,t^2\,\delta^3\in-8\,t\,\gamma\,\delta^2\in^2+12\,t\,\gamma\,\delta^3\in^2+2\,t\in\eta\,\xi-8\,t\,\delta\in\eta\,\xi-4\,\gamma\,\delta\in^2\,\eta\,\xi\right)\hbar^3\right)$$
$$QU[a]+\ll 25\gg+\frac{1}{2}\,\delta^2\,\eta\,\hbar^3\,QU[y,y,y,x,x]+\frac{1}{6}\,\delta^3\,\hbar^3\,QU[y,y,y,x,x,x],\ \text{True}\Big\}$$

Stitching Direct

```
MatrixExp[η1 ρ[CU@y]].MatrixExp[α1 ρ[CU@a]].MatrixExp[ξ1 ρ[CU@x]].MatrixExp[η2 ρ[CU@y]].
  MatrixExp[α2 ρ[CU@a]].MatrixExp[ξ2 ρ[CU@x]] // Simplify // MatrixForm
```

$$\begin{pmatrix} e^{\gamma(\alpha_1+\alpha_2)}\,(1+\gamma\in\eta_2\,\xi_1) & e^{\gamma\,\alpha_1}\,\gamma\,\left(e^{\gamma\,\alpha_2}\,\xi_2+\xi_1\,(1+e^{\gamma\,\alpha_2}\,\gamma\in\eta_2\,\xi_2)\right) \\ e^{\gamma\,\alpha_2}\in\left(\eta_2+e^{\gamma\,\alpha_1}\,\eta_1\,(1+\gamma\in\eta_2\,\xi_1)\right) & 1+e^{\gamma\,\alpha_1}\,\gamma\in\eta_1\,\xi_1+e^{\gamma\,\alpha_2}\,\gamma\in\left(\eta_2+e^{\gamma\,\alpha_1}\,\eta_1\,(1+\gamma\in\eta_2\,\xi_1)\right)\xi_2 \end{pmatrix}$$

```
eqn = MatrixExp[η1 ρ[CU@y]].MatrixExp[α1 ρ[CU@a]].MatrixExp[ξ1 ρ[CU@x]].
   MatrixExp[η2 ρ[CU@y]].MatrixExp[α2 ρ[CU@a]].MatrixExp[ξ2 ρ[CU@x]] ==
  ℯ^(τ0 ∈ γ) MatrixExp[η0 ρ[CU@y]].MatrixExp[α0 ρ[CU@a]].MatrixExp[ξ0 ρ[CU@x]]
```

$$\Big\{\Big\{e^{\gamma\,\alpha_2}\,(e^{\gamma\,\alpha_1}+e^{\gamma\,\alpha_1}\,\gamma\in\eta_2\,\xi_1),\ e^{\gamma\,\alpha_1}\,\gamma\,\xi_1+e^{\gamma\,\alpha_2}\,\gamma\,(e^{\gamma\,\alpha_1}+e^{\gamma\,\alpha_1}\,\gamma\in\eta_2\,\xi_1)\,\xi_2\Big\},$$
$$\Big\{e^{\gamma\,\alpha_2}\,\left(e^{\gamma\,\alpha_1}\in\eta_1+\in\eta_2\,(1+e^{\gamma\,\alpha_1}\,\gamma\in\eta_1\,\xi_1)\right),$$
$$1+e^{\gamma\,\alpha_1}\,\gamma\in\eta_1\,\xi_1+e^{\gamma\,\alpha_2}\,\gamma\,\left(e^{\gamma\,\alpha_1}\in\eta_1+\in\eta_2\,(1+e^{\gamma\,\alpha_1}\,\gamma\in\eta_1\,\xi_1)\right)\xi_2\Big\}\Big\}==$$
$$\Big\{\Big\{e^{\alpha0\,\gamma+\gamma\in\tau0},\ e^{\alpha0\,\gamma+\gamma\in\tau0}\,\gamma\,\xi0\Big\},\ \Big\{e^{\alpha0\,\gamma+\gamma\in\tau0}\in\eta0,\ e^{\gamma\in\tau0}\,(1+e^{\alpha0\,\gamma}\,\gamma\in\eta0\,\xi0)\Big\}\Big\}$$

```
sol = Block[{ϵ}, Solve[Thread[Flatten /@ eqn], {τ0, η0, α0, ξ0}]][[1]]
```

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is $\text{Log}[e^{\gamma(\alpha 0+\epsilon\tau 0)}] - \text{Log}[e^{\gamma\alpha_2}(e^{\gamma\,\text{Subscript}[\ll 2\gg]} + e^{\text{Times}[\ll 2\gg]}\gamma\epsilon\eta_2\xi_1)] == 0$.

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Solve: Equations may not give solutions for all "solve" variables.

$$\Big\{\tau 0 \to \frac{1}{\gamma\epsilon}\left(-\text{Log}\left[e^{\alpha 0\,\gamma}\right] + \text{Log}\left[e^{\gamma\alpha_1+\gamma\alpha_2} + e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_2\xi_1\right]\right),$$

$$\eta 0 \to \Bigg(e^{-\gamma\alpha_1}\Bigg(\frac{1}{2} + \frac{1}{2}e^{\gamma\alpha_1}\gamma\epsilon\eta_1\xi_1 + \frac{1}{2}e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_1\xi_2 + \frac{1}{2}e^{\gamma\alpha_2}\gamma\epsilon\eta_2\xi_2 + \frac{1}{2}e^{\gamma\alpha_1+\gamma\alpha_2}\gamma^2\epsilon^2\eta_1\eta_2\xi_1\xi_2 - \frac{1}{2}\sqrt{\Big(\left(-1 - e^{\gamma\alpha_1}\gamma\epsilon\eta_1\xi_1 - e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_1\xi_2 - e^{\gamma\alpha_2}\gamma\epsilon\eta_2\xi_2 - e^{\gamma\alpha_1+\gamma\alpha_2}\gamma^2\epsilon^2\eta_1\eta_2\xi_1\xi_2\right)^2 + 4e^{-\alpha 0\,\gamma+\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\left(-e^{\gamma\alpha_1}\eta_1\xi_1 - \eta_2\xi_1 - e^{\gamma\alpha_1}\gamma\epsilon\eta_1\eta_2\xi_1^2 - e^{\gamma\alpha_1+\gamma\alpha_2}\eta_1\xi_2 - e^{\gamma\alpha_2}\eta_2\xi_2 - 2e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_1\eta_2\xi_1\xi_2 - e^{\gamma\alpha_2}\gamma\epsilon\eta_2^2\xi_1\xi_2 - e^{\gamma\alpha_1+\gamma\alpha_2}\gamma^2\epsilon^2\eta_1\eta_2^2\xi_1^2\xi_2\right)\Big)}\Bigg)\Bigg)\Big/$$

$$\left(\gamma\epsilon\left(\xi_1 + e^{\gamma\alpha_2}\xi_2 + e^{\gamma\alpha_2}\gamma\epsilon\eta_2\xi_1\xi_2\right)\right),\ \xi 0 \to \Bigg(e^{-\gamma\alpha_2}\Bigg(\frac{1}{2\gamma\epsilon} + \frac{1}{2}e^{\gamma\alpha_1}\eta_1\xi_1 + \frac{1}{2}e^{\gamma\alpha_1+\gamma\alpha_2}\eta_1\xi_2 + \frac{1}{2}e^{\gamma\alpha_2}\eta_2\xi_2 + \frac{1}{2}e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_1\eta_2\xi_1\xi_2 - \frac{1}{2\gamma\epsilon}\Big(\sqrt{\Big(\left(-1 - e^{\gamma\alpha_1}\gamma\epsilon\eta_1\xi_1 - e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_1\xi_2 - e^{\gamma\alpha_2}\gamma\epsilon\eta_2\xi_2 - e^{\gamma\alpha_1+\gamma\alpha_2}\gamma^2\epsilon^2\eta_1\eta_2\xi_1\xi_2\right)^2 + 4e^{-\alpha 0\,\gamma+\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\left(-e^{\gamma\alpha_1}\eta_1\xi_1 - \eta_2\xi_1 - e^{\gamma\alpha_1}\gamma\epsilon\eta_1\eta_2\xi_1^2 - e^{\gamma\alpha_1+\gamma\alpha_2}\eta_1\xi_2 - e^{\gamma\alpha_2}\eta_2\xi_2 - 2e^{\gamma\alpha_1+\gamma\alpha_2}\gamma\epsilon\eta_1\eta_2\xi_1\xi_2 - e^{\gamma\alpha_2}\gamma\epsilon\eta_2^2\xi_1\xi_2 - e^{\gamma\alpha_1+\gamma\alpha_2}\gamma^2\epsilon^2\eta_1\eta_2^2\xi_1^2\xi_2\right)\Big)}\Big)\Bigg)\Bigg)\Big/\left(e^{\gamma\alpha_1}\eta_1 + \eta_2 + e^{\gamma\alpha_1}\gamma\epsilon\eta_1\eta_2\xi_1\right)\Big\}$$

```
eqn = MatrixExp[η1 ρ[CU@y]].MatrixExp[α1 ρ[CU@a]].MatrixExp[ξ1 ρ[CU@x]].
    MatrixExp[η2 ρ[CU@y]].MatrixExp[α2 ρ[CU@a]].MatrixExp[ξ2 ρ[CU@x]] ==
  T0 MatrixExp[η0 ρ[CU@y]].MatrixExp[α0 ρ[CU@a]].MatrixExp[ξ0 ρ[CU@x]]
```

$$\Big\{\Big\{e^{\gamma\alpha_2}\left(e^{\gamma\alpha_1} + e^{\gamma\alpha_1}\gamma\epsilon\eta_2\xi_1\right),\ e^{\gamma\alpha_1}\gamma\xi_1 + e^{\gamma\alpha_2}\gamma\left(e^{\gamma\alpha_1} + e^{\gamma\alpha_1}\gamma\epsilon\eta_2\xi_1\right)\xi_2\Big\},$$
$$\Big\{e^{\gamma\alpha_2}\left(e^{\gamma\alpha_1}\epsilon\eta_1 + \epsilon\eta_2\left(1 + e^{\gamma\alpha_1}\gamma\epsilon\eta_1\xi_1\right)\right),$$
$$1 + e^{\gamma\alpha_1}\gamma\epsilon\eta_1\xi_1 + e^{\gamma\alpha_2}\gamma\left(e^{\gamma\alpha_1}\epsilon\eta_1 + \epsilon\eta_2\left(1 + e^{\gamma\alpha_1}\gamma\epsilon\eta_1\xi_1\right)\right)\xi_2\Big\}\Big\} ==$$
$$\Big\{\Big\{e^{\alpha 0\,\gamma}\,T0,\ e^{\alpha 0\,\gamma}\,T0\,\gamma\,\xi 0\Big\},\ \Big\{e^{\alpha 0\,\gamma}\,T0\,\epsilon\,\eta 0,\ T0\left(1 + e^{\alpha 0\,\gamma}\gamma\epsilon\eta 0\,\xi 0\right)\Big\}\Big\}$$

```
sol = Block[{ϵ}, Solve[Thread[Flatten /@ eqn], {T0, η0, α0, ξ0}]][[1]]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\Big\{T0 \to \frac{1}{1 + \gamma\epsilon\eta_2\xi_1},\ \eta 0 \to \frac{\eta_1 + e^{-\gamma\alpha_1}\eta_2 + \gamma\epsilon\eta_1\eta_2\xi_1}{1 + \gamma\epsilon\eta_2\xi_1},$$
$$\alpha 0 \to \frac{\text{Log}\left[e^{\gamma\alpha_1+\gamma\alpha_2}\left(1 + \gamma\epsilon\eta_2\xi_1\right)^2\right]}{\gamma},\ \xi 0 \to \frac{e^{-\gamma\alpha_2}\xi_1 + \xi_2 + \gamma\epsilon\eta_2\xi_1\xi_2}{1 + \gamma\epsilon\eta_2\xi_1}\Big\}$$

E

$\mathbb{E}[L, Q, P]$ means $e^{\hbar(L+Q)} P$, where L is linear in the a's, Q is a combination of $x_i\, y_j$, and P is a perturbation polynomial. It should be interpreted via $\mathbb{CO}[\mathbb{E}[\,...],\ \{x_1,\ a_1,\ y_1\}_i,\ ...]$ (with some default for direct interpretation), or likewise via $\mathbb{QO}[\mathbb{E}[\,...],\ \{x_1,\ a_1,\ y_1\}_i,\ ...]$. In themselves, $\mathbb{CO}$ and $\mathbb{QO}$ should have an interpretation in CU/QU by casting.