```
y = ({{0, 0}, {1, 0}}); x = ({{0, 1}, {0, 0}}); a = ({{1, 0}, {0, -1}});
Column[
 First@Solve[Thread[Flatten /@ (MatrixExp[ξ x].MatrixExp[α a].MatrixExp[η y] ==
         MatrixExp[Y y].MatrixExp[A a].MatrixExp[X x])], {Y, A, X}] /.
   C[1] → 0 /. (v_ → ℰ_) :↦ (v → Simplify@PowerExpand[ℰ])
]
```

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is

$$\alpha - \text{Log}\left[e^{-A+\alpha}\right] - \text{Log}\left[e^{\alpha} + e^{-\alpha}\,\eta\,\xi\right] == 0.$$

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is

$$\alpha - \text{Log}\left[e^{A+\alpha}\right] + \text{Log}\left[e^{\alpha} + e^{-\alpha}\,\eta\,\xi\right] == 0.$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$Y \to \frac{\eta}{e^{2\alpha}+\eta\,\xi}$$

$$A \to \text{Log}\left[e^{\alpha} + e^{-\alpha}\,\eta\,\xi\right]$$

$$X \to \frac{\xi}{e^{2\alpha}+\eta\,\xi}$$

```
y = ({{0, 0}, {1, 0}}); x = ({{0, 1}, {0, 0}}); a = ({{1, 0}, {0, -1}});
Column[
 First@Solve[Thread[Flatten /@
       (MatrixExp[η1 y].MatrixExp[α1 a].MatrixExp[ξ1 x].MatrixExp[η2 y].MatrixExp[α2 a].
          MatrixExp[ξ2 x] == MatrixExp[Y y].MatrixExp[A a].MatrixExp[X x])], {Y, A, X}] /.
   C[1] → 0 /. (v_ → ℰ_) :↦ (v → Simplify@PowerExpand[ℰ])
]
```

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is

$$\text{Log}\left[e^{-A+\alpha_1+\alpha_2}\right] + \text{Log}\left[e^{\alpha_2}\left(e^{\alpha_1} + e^{\alpha_1}\,\eta_2\,\xi_1\right)\right] - \alpha_1 - \alpha_2 == 0.$$

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is

$$\text{Log}\left[e^{A+\alpha_1}\right] - \text{Log}\left[e^{\alpha_2}\left(e^{\alpha_1} + e^{\text{Subscript[≪2≫]}}\,\eta_2\,\xi_1\right)\right] - \alpha_1 == 0.$$

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is

$$\text{Log}\left[e^{A+\alpha_1+\alpha_2}\right] - \text{Log}\left[e^{\alpha_2}\left(e^{\alpha_1} + e^{\text{Subscript[≪2≫]}}\,\eta_2\,\xi_1\right)\right] - \alpha_1 - \alpha_2 == 0.$$

General: Further output of Solve::incnst will be suppressed during this calculation.

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$Y \to \eta_1 + \frac{e^{-2\alpha_1}\,\eta_2}{1+\eta_2\,\xi_1}$$

$$A \to \text{Log}\left[e^{\alpha_1}\left(1 + \eta_2\,\xi_1\right)\right] + \alpha_2$$

$$X \to \frac{\xi_2+\xi_1\left(e^{-2\alpha_2}+\eta_2\,\xi_2\right)}{1+\eta_2\,\xi_1}$$

```
y = {{0,0,0},{0,0,1},{0,0,0}}; x = {{0,1,0},{0,0,0},{0,0,0}}; a = {{0,0,1},{0,0,0},{0,0,0}};
Column[
 First@Solve[Thread[Flatten /@ (MatrixExp[ξ x].MatrixExp[α a].MatrixExp[η y] ==
         MatrixExp[Y y].MatrixExp[A a].MatrixExp[X x])], {Y, A, X}] /.
   C[1] → 0 /. (v_ → ℰ_) :↦ (v → Expand@PowerExpand[ℰ])
]
```

$Y \to \eta$
$A \to \alpha + \eta\, \xi$
$X \to \xi$

```
y = {{0,0,0},{0,0,1},{0,0,0}}; x = {{0,1,0},{0,0,0},{0,0,0}}; a = {{0,0,1},{0,0,0},{0,0,0}};
Column[
 First@Solve[Thread[Flatten /@
       (MatrixExp[η1 y].MatrixExp[α1 a].MatrixExp[ξ1 x].MatrixExp[η2 y].MatrixExp[α2 a].
          MatrixExp[ξ2 x] == MatrixExp[Y y].MatrixExp[A a].MatrixExp[X x])], {Y, A, X}] /.
   C[1] → 0 /. (v_ → ℰ_) :↦ (v → Expand@PowerExpand[ℰ])
]
```

$Y \to \eta_1 + \eta_2$
$A \to \alpha_1 + \alpha_2 + \eta_2\, \xi_1$
$X \to \xi_1 + \xi_2$

```
y = {{0,1,0},{0,0,0},{0,0,0}}; a = {{0,0,0},{0,1,0},{0,0,0}}; x = {{0,0,0},{0,0,1},{0,0,0}}; c = {{0,0,-1},{0,0,0},{0,0,0}};
Column[
 First@Solve[Thread[Flatten /@ (MatrixExp[η1 y].MatrixExp[α1 a].
          MatrixExp[ξ1 x].MatrixExp[η2 y].MatrixExp[α2 a].MatrixExp[ξ2 x] ==
         MatrixExp[Y y].MatrixExp[A a].MatrixExp[X x].MatrixExp[γ c])], {Y, A, X, γ}] /.
   C[1] → 0 /. (v_ → ℰ_) :↦ (v → Expand@PowerExpand[ℰ])
]
```

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is $\text{Log}[e^{A}] - \text{Log}[e^{\alpha_1+\alpha_2}] == 0$.

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$Y \to \eta_1 + e^{-\alpha_1}\, \eta_2$
$A \to \alpha_1 + \alpha_2$
$X \to e^{-\alpha_2}\, \xi_1 + \xi_2$
$\gamma \to \eta_2\, \xi_1$

```
y = {{0,0,0},{0,0,1},{0,0,0}}; a = {{0,0,1},{0,0,0},{0,0,0}}; x = {{0,1,0},{0,0,0},{0,0,0}};
{a.x == x.a, a.y == y.a, x.y - y.x == a}
```

{True, True, True}

```
𝔼[M_?MatrixQ] := MatrixExp[M];
Simplify[𝔼[η1 y].𝔼[α1 a].𝔼[ξ1 x].𝔼[η2 y].𝔼[α2 a].𝔼[ξ2 x] == 𝔼[η0 y].𝔼[α0 a].𝔼[ξ0 x] /.
  {η0 → η1 + η2, α0 → α1 + α2 + η2 ξ1, ξ0 → ξ1 + ξ2}]
True
```