

Pensieve header: Making table1.tex and table2.tex; continuing pensieve://Talks/QMUL-1908/.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\PG"];
Once[<< KnotTheory`];
Ribbons = {Knot[0, 1], Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27],
  Knot[9, 41], Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42],
  Knot[10, 48], Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123],
  Knot[10, 129], Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]};
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.  
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[ ]:= CF = Factor;
E[L_, Q_, P_]$_k := E[L, Q, Series[Normal@P, {e, 0, $k}]];
E_{d→r}[L_, Q_, P_]$_k := E_{d→r} @ E[L, Q, P]$_k;
E3@E[ω_, L_, Q_, Ps_] := CF /@ E[L, ω⁻¹ Q, ω⁻¹ (ω⁻⁴ e)⁻¹+Range@Length@Ps.Ps]$_k;
E4@E[L_, Q_, P_] := Module[
  {ω = Normal[P]⁻¹ /. e → 0, Ps = CoefficientList[P, e]},
  CF /@ E[ω, L, ω Q, ω⁻³+4 Range@Length@Ps Ps]];
E3@E_{sp___}[as___] := E3@E[as] /. E → E_{sp};
E4@E_{sp___}[as___] := E4@E[as] /. E → E_{sp};
```

Defining  $\{\omega, \rho_1, \rho_2\}$ .

```
In[ ]:= $k = 2;
Clear[QP, ω];
QP[Knot[n_, k_]] := QP[Knot[n, k]] = Collect[Module[{fname},
  fname = "Data/" <> ToString[n] <> "_" <> ToString[k] <> ".m";
  Collect[E3[Get[fname][[2, 2]][[3]] // Normal, e, Simplify]
], e, CF];
ω[K_Knot] := ω[K] = Factor[(QP@K /. e → 0)⁻¹];
c_{r,d}[K_Knot] :=
  Factor[SeriesCoefficient[QP[K], {y, 0, 0}, {e, 0, k}, {a, 0, d}] ω[K]¹+²k-d];
```

```
In[ ]:= ρ1[K_Knot] := ρ1[K] = Factor[
$$\frac{T(-c_{1,0}[K] + \omega[K] T \partial_T \omega[K])}{(T-1)^2}$$
];
ρ2[K_Knot] := ρ2[K] = CF[-2 c_{2,0}[K] + ω[K] c_{2,1}[K]];
plus[ε_] := Expand[ε] /. T^{n-} /; n < 0 => 0;
```

In[ ]:= **line** = { $\omega$ [#],  $\rho 1$ [#],  $\rho 2$ [#]} & /@ {**Knot**[3, 1]}  
**plus** /@ **line**

Out[ ]:=  $\left\{ \left\{ \frac{1 - T + T^2}{T}, \frac{1 + T^2}{T}, \frac{3 - 12 T + 26 T^2 - 38 T^3 + 26 T^4 - 12 T^5 + 3 T^6}{T^3} \right\} \right\}$

Out[ ]:=  $\left\{ \{-1 + T, T, -38 + 26 T - 12 T^2 + 3 T^3\} \right\}$

$\rho 2p$  from ExtractRhos of Roland's KnotTableWCRho2.nb:

```
In[ ]:= wc11 =  $\left( 1 + a + \frac{1}{2} \times (-1 - T) - a T + x y \right) \epsilon + \left( \frac{1}{2} (a + a^2 + a T + a^2 T) + a x y \right) \epsilon^2 + 0[\epsilon]^3$ ;
wc21 =  $\left( 1 + a + \frac{1}{2} \times (-1 - T) - a T + x y \right) \epsilon^2 + 0[\epsilon]^3$ ;
wc22 =  $\left( \frac{1}{4} \times (1 - 2 T + T^2) + a (1 - 2 T + T^2) + a^2 (1 - 2 T + T^2) + a (2 - 2 T) x y + x^2 y^2 \right) \epsilon^2 + 0[\epsilon]^3$ ;
 $\rho 2p$ [Knot[0, 1]] = 0;
 $\rho 2p$ [Knot[n_, k_]] := Module[{Z, Z0, Z1, Z2, Z3, Z4, Alex, rho2},
  Z = E3[Get["Data/" <> Tostring[n] <> "_" <> Tostring[k] <> ".m"]][[2, 2]];
  Z0 = Z[[3]];
  Alex = (Normal[Z0] /.  $\epsilon \rightarrow 0$ )-1;
  Z1 = Z0 * Alex // CF;
  Z2 = Z1 - Coefficient[Z1, y x  $\epsilon$ ] wc11 // CF;
  Z3 = (Z2 - Coefficient[Z2, y2 x2  $\epsilon^2$ ] wc22) // CF;
  Z4 = Z3 - Coefficient[Z3, y x  $\epsilon^2$ ] wc21 // CF;
  rho2 =  $4 \left( \frac{(-1 + T)^2}{T} \right)^{-1} \text{Alex}^4 \left( \text{Z4}[[3, 3]] - \frac{\text{Z4}[[3, 2]]^2}{2} \right)$  // Together // Expand;
  Last@Last@CoefficientRules[rho2, T-1]
```

In[ ]:=  **$\rho 2p$ [Knot[3, 1]]**

Out[ ]:=  $-9 - 4 T - T^2 - T^3$

In[ ]:= **Column**[ **$\rho 2p$**  /@ **AllKnots**[{0, 6}]]

0  
 $-9 - 4 T - T^2 - T^3$   
 $-23 + 2 T + 5 T^2 - T^3$   
 $-61 - 4 T - 21 T^2 - 22 T^3 - 7 T^5 - T^6 - 2 T^7$   
Out[ ]:=  $-102 + 16 T + 4 T^2 - 14 T^3$   
 $-298 + 108 T + 36 T^2 - 14 T^3$   
 $-529 + 326 T - 25 T^2 - 82 T^3 + 62 T^4 - 33 T^5 + 13 T^6 - 2 T^7$   
 $-671 + 498 T - 155 T^2 - 68 T^3 + 96 T^4 - 49 T^5 + 13 T^6 - 2 T^7$

Recovering  $Z$  from  $\{\omega, \rho_1, \rho_2\}$ .

```

In[ ]:= Clear[LogP];
LogP[Knot[n_, k_]] := LogP[Knot[n, k]] = Collect[Module[{fname},
  fname = ".../Projects/SL2Invariant/k=2/Data/" <>
  ToString[n] <> "-" <> ToString[k] <> ".m";
  Collect[ω[Knot[n, k]] * E3[Get[fname][[2, 2]][[3]] // Log // Normal, ε, Simplify]
], ε, CF];

```

```

In[ ]:= LogP[Knot[3, 1]]

```

$$\frac{(-2 - 2a + 3T + 2aT - 2T^2 + T^3 - 2aT^3 + 2aT^4 - 2xy - 2T^3xy) \in}{(1 - T + T^2)^2} -$$

$$\frac{1}{2(1 - T + T^2)^4} (-T - 4aT - 4a^2T + 11T^2 + 32aT^2 + 24a^2T^2 - 16T^3 - 56aT^3 -$$

$$48a^2T^3 + 12T^4 + 64aT^4 + 64a^2T^4 - 8T^5 - 40aT^5 - 48a^2T^5 + 3T^6 + 16aT^6 + 24a^2T^6 -$$

$$T^7 - 4aT^7 - 4a^2T^7 + 4xy - 4Txy - 16aTxy + 8T^2xy + 48aT^2xy + 8T^3xy -$$

$$72aT^3xy - 8T^4xy + 64aT^4xy + 8T^5xy - 24aT^5xy + 4T^6xy - 4T^7xy + 8aT^7xy -$$

$$2x^2y^2 - 6Tx^2y^2 + 12T^2x^2y^2 - 22T^3x^2y^2 + 12T^4x^2y^2 - 6T^5x^2y^2 - 2T^6x^2y^2) \in^2$$

## Generating table1.tex and table2.tex

```

In[ ]:= pmrule = {"+" -> "\\!+\\!", "-" -> "\\!-\\!"};
KnotLine[K_] :=
StringReplace["\\rolcell{n_k}{n^t_{k}}{ω}{ρ1p}{ρ2p}{G}{U}{R}{C}", {
  "n" -> ToString@K[[1]],
  "k" -> ToString@K[[2]],
  "t" -> If[AlternatingQ[K], "a", "n"],
  "ω" -> StringReplace[ToString[plus@ω[K], FormatType -> TeXForm], pmrule],
  "ρ1p" -> StringReplace[ToString[plus@ρ1[K], FormatType -> TeXForm], pmrule],
  "ρ2p" -> StringReplace[ToString[ρ2p[K], FormatType -> TeXForm], pmrule],
  "G" -> ToString@ThreeGenus@K,
  "U" -> ToString@UnknottingNumber@K,
  "R" -> If[MemberQ[Ribbons, K], "\\gY", "\\N"],
  "C" -> If[MemberQ[
    {FullyAmphicheiral, NegativeAmphicheiral, ""}, SymmetryType@K], "\\oY", "\\N"]
}]

```

```
In[ ]:= KnotLine /@ AllKnots[{0, 6}]
```

```
Out[ ]:= {\rolcell{0_1}{0^a_1}{1}{0}{0}{0}{0}{\gY}{\oY},
  \rolcell{3_1}{3^a_1}{T\!-\!1}{T}{\!-\!T^3\!-\!T^2\!-\!4 T\!-\!9}{1}{1}{\N}{\N},
  \rolcell{4_1}{4^a_1}{3\!-\!T}{0}{\!-\!T^3\!+\!5 T^2\!+\!2 T\!-\!23}{1}{1}{\N}{\oY},
  \rolcell{5_1}{5^a_1}{T^2\!-\!T\!+\!1}{2 T^3\!+\!3 T}{\!-\!2
    T^7\!-\!T^6\!-\!7 T^5\!-\!22 T^3\!-\!21 T^2\!-\!4 T\!-\!61}{2}{2}{\N}{\N},
  \rolcell{5_2}{5^a_2}{2 T\!-\!3}{5 T\!-\!4}{\!-\!14 T^3\!+\!4 T^2\!+\!16
    T\!-\!102}{1}{1}{\N}{\N}, \rolcell{6_1}{6^a_1}{5\!-\!2
    T}{T\!-\!4}{\!-\!14 T^3\!+\!36 T^2\!+\!108 T\!-\!298}{1}{1}{\gY}{\N},
  \rolcell{6_2}{6^a_2}{\!-\!T^2\!+\!3 T\!-\!3}{T^3\!-\!4 T^2\!+\!4
    T\!-\!4}{\!-\!2 T^7\!+\!13 T^6\!-\!33 T^5\!+\!62
    T^4\!-\!82 T^3\!-\!25 T^2\!+\!326 T\!-\!529}{2}{1}{\N}{\N},
  \rolcell{6_3}{6^a_3}{T^2\!-\!3 T\!+\!5}{0}{\!-\!2 T^7\!+\!13 T^6\!-\!49
    T^5\!+\!96 T^4\!-\!68 T^3\!-\!155 T^2\!+\!498 T\!-\!671}{2}{1}{\N}{\oY}}
```

```
In[ ]:= Make[range_] := Make[range, 2];
Make[range_, col_] := StringJoin@@Table[
  StringJoin[StringJoin@@Riffle[L, " &n"], " \\\n\\hline\n"],
  {L, Partition[KnotLine /@ AllKnots[range], UpTo@col]}
];
Make[{0, 6}, 3]
```

```
Out[ ]:= \rolcell{0_1}{0^a_1}{1}{0}{0}{0}{0}{\gY}{\oY} &
  \rolcell{3_1}{3^a_1}{T\!-\!1}{T}{\!-\!T^3\!-\!T^2\!-\!4 T\!-\!9}{1}{1}{\N}{\N} &
  \rolcell{4_1}{4^a_1}{3\!-\!T}{0}{\!-\!T^3\!+\!5
    T^2\!+\!2 T\!-\!23}{1}{1}{\N}{\oY} \\
  \hline
  \rolcell{5_1}{5^a_1}{T^2\!-\!T\!+\!1}{2 T^3\!+\!3 T}{\!-\!2
    T^7\!-\!T^6\!-\!7 T^5\!-\!22 T^3\!-\!21 T^2\!-\!4 T\!-\!61}{2}{2}{\N}{\N} &
  \rolcell{5_2}{5^a_2}{2 T\!-\!3}{5 T\!-\!4}{\!-\!14
    T^3\!+\!4 T^2\!+\!16 T\!-\!102}{1}{1}{\N}{\N} &
  \rolcell{6_1}{6^a_1}{5\!-\!2 T}{T\!-\!4}{\!-\!14
    T^3\!+\!36 T^2\!+\!108 T\!-\!298}{1}{1}{\gY}{\N} \\
  \hline
  \rolcell{6_2}{6^a_2}{\!-\!T^2\!+\!3 T\!-\!3}{T^3\!-\!4
    T^2\!+\!4 T\!-\!4}{\!-\!2 T^7\!+\!13 T^6\!-\!33 T^5\!+\!62
    T^4\!-\!82 T^3\!-\!25 T^2\!+\!326 T\!-\!529}{2}{1}{\N}{\N} &
  \rolcell{6_3}{6^a_3}{T^2\!-\!3 T\!+\!5}{0}{\!-\!2 T^7\!+\!13 T^6\!-\!49
    T^5\!+\!96 T^4\!-\!68 T^3\!-\!155 T^2\!+\!498 T\!-\!671}{2}{1}{\N}{\oY} \\
  \hline
```

```
In[ ]:= DeleteFile["table1.tex"];
WriteString["table1.tex", Make[{0, 8}, 3]];
Close["table1.tex"]
```

```
Out[ ]:= table1.tex
```

```
In[ ]:= DeleteFile["table2.tex"];  
WriteString["table2.tex", Make[{9, 10}, 2]];  
Close["table2.tex"]
```

```
Out[ ]:= table2.tex
```