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 $\Lambda[\epsilon_-, b_-, c_-, u_-, w_-, \alpha_-, \beta_-, \delta_-] := \text{With}[\{v = (1 + b \delta)^{-1}\}, \text{Simplify}[$ 

$$\epsilon \left( 2 c w \alpha \delta v + 2 c u \beta \delta v + 2 c u w \delta^2 v + u w^2 \alpha \delta^2 v^3 - \frac{1}{2} b u^2 w^2 \delta^4 v^3 + \frac{1}{2} w^2 \alpha^2 \delta (2 + b \delta) v^3 - \right.$$


$$u^2 w \beta \delta^2 (1 + 2 b \delta) v^3 - \frac{1}{2} u^2 \beta^2 \delta (2 + 3 b \delta) v^3 + 2 c (\delta + \alpha \beta v) - 2 b u w \delta^2 v^2 (\delta + \alpha \beta v) +$$


$$\left. w \alpha v^2 (2 \delta + \alpha \beta v) - u \beta (1 + 2 b \delta) v^2 (2 \delta + \alpha \beta v) - \frac{1}{2} b v (2 \delta^2 + 4 \alpha \beta \delta v + \alpha^2 \beta^2 v^2) \right)]$$


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logos =  $\Lambda[1, b, c, u, w, \alpha, \beta, \delta]$ 
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$$- \frac{1}{2 (1 + b \delta)^3} \left( 2 u \alpha \beta^2 + b \alpha^2 \beta^2 + 4 u \beta \delta + 4 b \alpha \beta \delta + 2 u^2 \beta^2 \delta + 4 b u \alpha \beta^2 \delta + \right.$$


$$2 b \delta^2 + 12 b u \beta \delta^2 + 4 b^2 \alpha \beta \delta^2 + 3 b u^2 \beta^2 \delta^2 + 4 b^2 \delta^3 + 8 b^2 u \beta \delta^3 + 2 b^3 \delta^4 +$$


$$w^2 \delta (\alpha + u \delta) (b u \delta^2 - \alpha (2 + b \delta)) - 4 c (1 + b \delta)^2 (\alpha (\beta + w \delta) + \delta (1 + u \beta + b \delta + u w \delta)) +$$


$$\left. w (-2 \alpha^2 \beta + 4 \alpha \delta (-1 + b (-1 + u \beta) \delta) + 2 u \delta^2 (2 b \delta (1 + b \delta) + u (\beta + 2 b \beta \delta))) \right)$$


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Solve[v == (1 + b  $\delta$ )-1, b]
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{ {b ->  $\frac{1 - v}{\delta v}$  } }
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(Collect[Simplify[logos /. b ->  $\frac{1 - v}{\delta v}$ ], {c, u, w}, C[Simplify[#]] &] // Expand) /. C[x_] -> x
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$$2 c w \alpha \delta v + 2 c u \beta \delta v + 2 c u w \delta^2 v + \frac{1}{2} u^2 \beta^2 \delta (-3 + v) v^2 +$$


$$u^2 w \beta \delta^2 (-2 + v) v^2 + \frac{1}{2} u^2 w^2 \delta^3 (-1 + v) v^2 + u w^2 \alpha \delta^2 v^3 + \frac{1}{2} w^2 \alpha^2 \delta v^2 (1 + v) +$$


$$2 c (\delta + \alpha \beta v) + 2 u w \delta (-1 + v) v (\delta + \alpha \beta v) + u \beta (-2 + v) v (2 \delta + \alpha \beta v) +$$


$$w \alpha v^2 (2 \delta + \alpha \beta v) + \frac{(-1 + v) (2 \delta^2 + 4 \alpha \beta \delta v + \alpha^2 \beta^2 v^2)}{2 \delta}$$


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(Collect[Simplify[(1 + b  $\delta$ )-1 logos /. b ->  $\frac{1 - v}{\delta v}$ ], {c, u, w}, C[Simplify[#]] &] // Expand) /. C[x_] -> x
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$$2 c w \alpha \delta v^2 + 2 c u \beta \delta v^2 + 2 c u w \delta^2 v^2 + \frac{1}{2} u^2 \beta^2 \delta (-3 + v) v^3 +$$


$$u^2 w \beta \delta^2 (-2 + v) v^3 + \frac{1}{2} u^2 w^2 \delta^3 (-1 + v) v^3 + u w^2 \alpha \delta^2 v^4 + \frac{1}{2} w^2 \alpha^2 \delta v^3 (1 + v) +$$


$$2 c v (\delta + \alpha \beta v) + 2 u w \delta (-1 + v) v^2 (\delta + \alpha \beta v) + u \beta (-2 + v) v^2 (2 \delta + \alpha \beta v) +$$


$$w \alpha v^3 (2 \delta + \alpha \beta v) + \frac{(-1 + v) v (2 \delta^2 + 4 \alpha \beta \delta v + \alpha^2 \beta^2 v^2)}{2 \delta}$$


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$$\text{ncore} = \text{With}\left[\left\{\xi = \frac{b}{t-1}\right\}, -b v \alpha \beta + v \beta u + v \delta u w + v \alpha w + \theta \right].$$

$$\left\{\beta \rightarrow \xi^{-1} \beta, \delta \rightarrow \xi^{-1} \delta, v \rightarrow (1+b \delta)^{-1}\right\} /. b \rightarrow \frac{1-v}{\delta v} // \text{Simplify}$$

$$\frac{1}{-1+v} \left(\theta (-1+v) + v \left(-(-1+t) \beta (\alpha (-1+v) + u \delta v) + w (\alpha (-1+v) - (-1+t) u \delta^2 v) \right) \right)$$

$$\text{nlogos} = \text{With}\left[\left\{\xi = \frac{b}{t-1}\right\}, \Delta[\xi \epsilon, b, c, \xi u, w, \alpha, \xi^{-1} \beta, \xi^{-1} \delta]\right]$$

$$\frac{1}{2 (1 + (-1+t) \delta)^3} \left(-2 u \alpha \beta^2 + \alpha^2 \beta^2 - t \alpha^2 \beta^2 - 4 u \beta \delta + 4 \alpha \beta \delta - 4 t \alpha \beta \delta - 2 u^2 \beta^2 \delta + 4 u \alpha \beta^2 \delta - 4 t u \alpha \beta^2 \delta + 2 \delta^2 - 2 t \delta^2 + 12 u \beta \delta^2 - 12 t u \beta \delta^2 - 4 \alpha \beta \delta^2 + 8 t \alpha \beta \delta^2 - 4 t^2 \alpha \beta \delta^2 + 3 u^2 \beta^2 \delta^2 - 3 t u^2 \beta^2 \delta^2 - 4 \delta^3 + 8 t \delta^3 - 4 t^2 \delta^3 - 8 u \beta \delta^3 + 16 t u \beta \delta^3 - 8 t^2 u \beta \delta^3 + 2 \delta^4 - 6 t \delta^4 + 6 t^2 \delta^4 - 2 t^3 \delta^4 + w^2 \delta (\alpha + u \delta) (-(-1+t) u \delta^2 + \alpha (2 + (-1+t) \delta)) + 2 w (\alpha^2 \beta - 2 \alpha \delta (-1 + (-1+t) (-1+u \beta) \delta) - u \delta^2 (2 (-1+t) \delta (1 + (-1+t) \delta) + u \beta (1 + 2 (-1+t) \delta))) + 4 c (1 + (-1+t) \delta)^2 (\alpha (\beta + w \delta) + \delta (1 + (-1+t) \delta + u (\beta + w \delta))) \right) \epsilon$$

FreeQ[nlogos, b]

True

(Collect[ϵ^{-1} Simplify[nlogos], {c, u, w}, C[Simplify[#]] &] // Expand) /. C[x_] => x

$$\frac{u w^2 \alpha \delta^2}{(1 + (-1+t) \delta)^3} - \frac{(-1+t) u^2 w^2 \delta^4}{2 (1 + (-1+t) \delta)^3} + \frac{2 c w \alpha \delta}{1 + (-1+t) \delta} + \frac{2 c u \beta \delta}{1 + (-1+t) \delta} + \frac{2 c u w \delta^2}{1 + (-1+t) \delta} + \frac{w^2 \alpha^2 \delta (2 + (-1+t) \delta)}{2 (1 + (-1+t) \delta)^3} - \frac{u^2 w \beta \delta^2 (1 + 2 (-1+t) \delta)}{(1 + (-1+t) \delta)^3} - \frac{u^2 \beta^2 \delta (2 + 3 (-1+t) \delta)}{2 (1 + (-1+t) \delta)^3} - \frac{2 (-1+t) u w \delta^2 (\alpha \beta + \delta (1 + (-1+t) \delta))}{(1 + (-1+t) \delta)^3} + \frac{2 c (\alpha \beta + \delta (1 + (-1+t) \delta))}{1 + (-1+t) \delta} + \frac{w \alpha (\alpha \beta + 2 \delta (1 + (-1+t) \delta))}{(1 + (-1+t) \delta)^3} - \frac{u \beta (1 + 2 (-1+t) \delta) (\alpha \beta + 2 \delta (1 + (-1+t) \delta))}{(1 + (-1+t) \delta)^3} - \frac{(-1+t) (\alpha^2 \beta^2 + 4 \alpha \beta \delta (1 + (-1+t) \delta) + 2 \delta^2 (1 + (-1+t) \delta)^2)}{2 (1 + (-1+t) \delta)^3}$$

(Collect[Simplify[$\epsilon^{-1} 2 (1 + (-1+t) \delta)^3$ nlogos], {c, u, w}, C[Simplify[#]] &] // Expand) /. C[x_] => x

$$2 u w^2 \alpha \delta^2 - (-1+t) u^2 w^2 \delta^4 + u^2 \beta^2 \delta (-2 - 3 (-1+t) \delta) + 4 c w \alpha \delta (1 + (-1+t) \delta)^2 + 4 c u \beta \delta (1 + (-1+t) \delta)^2 + 4 c u w \delta^2 (1 + (-1+t) \delta)^2 + w^2 \alpha^2 \delta (2 + (-1+t) \delta) - 2 u^2 w \beta \delta^2 (1 + 2 (-1+t) \delta) + 4 c (1 + (-1+t) \delta)^2 (\alpha \beta + \delta + (-1+t) \delta^2) - 4 (-1+t) u w \delta^2 (\alpha \beta + \delta (1 + (-1+t) \delta)) + 2 w \alpha (\alpha \beta + 2 \delta (1 + (-1+t) \delta)) - 2 u \beta (1 + 2 (-1+t) \delta) (\alpha \beta + 2 \delta (1 + (-1+t) \delta)) - (-1+t) (\alpha^2 \beta^2 + 4 \alpha \beta \delta (1 + (-1+t) \delta) + 2 \delta^2 (1 + (-1+t) \delta)^2)$$

Solve $[(1 + (-1 + t) \delta)^{-1} == \mu, t]$

$$\left\{ \left\{ t \rightarrow \frac{1 - \mu + \delta \mu}{\delta \mu} \right\} \right\}$$

$(\text{Collect}[\text{Simplify}[\epsilon^{-1} \text{nlogos} /. t \rightarrow \frac{1 - \mu + \delta \mu}{\delta \mu}], \{c, u, w\}, \text{C}[\text{Simplify}[\#]] \&] // \text{Expand}) /.$

$\text{C}[x_] \Rightarrow x$

$$2 c w \alpha \delta \mu + 2 c u \beta \delta \mu + 2 c u w \delta^2 \mu + \frac{1}{2} u^2 \beta^2 \delta (-3 + \mu) \mu^2 +$$

$$u^2 w \beta \delta^2 (-2 + \mu) \mu^2 + \frac{1}{2} u^2 w^2 \delta^3 (-1 + \mu) \mu^2 + u w^2 \alpha \delta^2 \mu^3 + \frac{1}{2} w^2 \alpha^2 \delta \mu^2 (1 + \mu) +$$

$$2 c (\delta + \alpha \beta \mu) + 2 u w \delta (-1 + \mu) \mu (\delta + \alpha \beta \mu) + u \beta (-2 + \mu) \mu (2 \delta + \alpha \beta \mu) +$$

$$w \alpha \mu^2 (2 \delta + \alpha \beta \mu) + \frac{(-1 + \mu) (2 \delta^2 + 4 \alpha \beta \delta \mu + \alpha^2 \beta^2 \mu^2)}{2 \delta}$$

$\text{Simplify}[(1 + (-1 + t) \delta)^3 \text{nlogos} / \epsilon /. \{t \rightarrow t_k, c \rightarrow c_k, u \rightarrow u_k, w \rightarrow w_k\}]$

$$\frac{1}{2} \left(\alpha^2 \beta^2 + 4 \alpha \beta \delta + 2 \delta^2 - 4 \alpha \beta \delta^2 - 4 \delta^3 + 2 \delta^4 - \alpha^2 \beta^2 t_k - 4 \alpha \beta \delta t_k - 2 \delta^2 t_k + 8 \alpha \beta \delta^2 t_k + 8 \delta^3 t_k - \right.$$

$$6 \delta^4 t_k - 4 \alpha \beta \delta^2 t_k^2 - 4 \delta^3 t_k^2 + 6 \delta^4 t_k^2 - 2 \delta^4 t_k^3 - 2 \alpha \beta^2 u_k - 4 \beta \delta u_k + 4 \alpha \beta^2 \delta u_k + 12 \beta \delta^2 u_k -$$

$$8 \beta \delta^3 u_k - 4 \alpha \beta^2 \delta t_k u_k - 12 \beta \delta^2 t_k u_k + 16 \beta \delta^3 t_k u_k - 8 \beta \delta^3 t_k^2 u_k - 2 \beta^2 \delta u_k^2 + 3 \beta^2 \delta^2 u_k^2 -$$

$$3 \beta^2 \delta^2 t_k u_k^2 + 2 (\alpha^2 \beta - \delta^2 u_k (2 \delta (1 + \delta (-1 + t_k)) (-1 + t_k) + \beta (1 + 2 \delta (-1 + t_k)) u_k) -$$

$$2 \alpha \delta (-1 + \delta (-1 + t_k) (-1 + \beta u_k))) w_k + \delta (\alpha + \delta u_k) (\alpha (2 + \delta (-1 + t_k)) - \delta^2 (-1 + t_k) u_k) w_k^2 +$$

$$4 c_k (1 + \delta (-1 + t_k))^2 (\alpha (\beta + \delta w_k) + \delta (1 + \delta (-1 + t_k) + u_k (\beta + \delta w_k))) \left. \right)$$

$(\text{Collect}[\text{Simplify}[(1 + (-1 + t) \delta)^3 \text{nlogos} / \epsilon], \{c, u, w\}, \text{C}[\text{Simplify}[\#]] \&] // \text{Expand}) /.$

$\{C[x_] \Rightarrow x, t \rightarrow t_k, c \rightarrow c_k, u \rightarrow u_k, w \rightarrow w_k\}$

$$-\frac{1}{2} (-1 + t) \left(\alpha^2 \beta^2 + 4 \alpha \beta \delta (1 + (-1 + t) \delta) + 2 \delta^2 (1 + (-1 + t) \delta)^2 \right) +$$

$$2 (1 + (-1 + t) \delta)^2 (\alpha \beta + \delta + (-1 + t) \delta^2) c_k - \beta (1 + 2 (-1 + t) \delta) (\alpha \beta + 2 \delta (1 + (-1 + t) \delta)) u_k +$$

$$2 \beta \delta (1 + (-1 + t) \delta)^2 c_k u_k - \frac{1}{2} \beta^2 \delta (2 + 3 (-1 + t) \delta) u_k^2 +$$

$$\alpha (\alpha \beta + 2 \delta (1 + (-1 + t) \delta)) w_k + 2 \alpha \delta (1 + (-1 + t) \delta)^2 c_k w_k -$$

$$2 (-1 + t) \delta^2 (\alpha \beta + \delta (1 + (-1 + t) \delta)) u_k w_k + 2 \delta^2 (1 + (-1 + t) \delta)^2 c_k u_k w_k -$$

$$\beta \delta^2 (1 + 2 (-1 + t) \delta) u_k^2 w_k + \frac{1}{2} \alpha^2 \delta (2 + (-1 + t) \delta) w_k^2 + \alpha \delta^2 u_k w_k^2 - \frac{1}{2} (-1 + t) \delta^4 u_k^2 w_k^2$$

With $\left[\left\{\xi = \frac{b}{t-1}\right\}, \Lambda[\epsilon, b, c, \xi u, w, \alpha, \xi^{-1} \beta, \xi^{-1} \delta]\right]$

$$\frac{1}{2 b (1 + (-1 + t) \delta)^3} (1 - t) \left(-2 u w^2 \alpha \delta^2 + (-1 + t) u^2 w^2 \delta^4 - 4 c w \alpha \delta (1 + (-1 + t) \delta)^2 - 4 c u \beta \delta (1 + (-1 + t) \delta)^2 - 4 c u w \delta^2 (1 + (-1 + t) \delta)^2 - w^2 \alpha^2 \delta (2 + (-1 + t) \delta) + 2 u^2 w \beta \delta^2 (1 + 2 (-1 + t) \delta) + u^2 \beta^2 \delta (2 + 3 (-1 + t) \delta) + 4 (-1 + t) u w \delta^2 (\alpha \beta + \delta (1 + (-1 + t) \delta)) - 4 c (1 + (-1 + t) \delta)^2 (\alpha \beta + \delta (1 + (-1 + t) \delta)) - 2 w \alpha (\alpha \beta + 2 \delta (1 + (-1 + t) \delta)) + 2 u \beta (1 + 2 (-1 + t) \delta) (\alpha \beta + 2 \delta (1 + (-1 + t) \delta)) - (1 - t) (\alpha^2 \beta^2 + 4 \alpha \beta \delta (1 + (-1 + t) \delta) + 2 \delta^2 (1 + (-1 + t) \delta)^2) \right) \epsilon$$