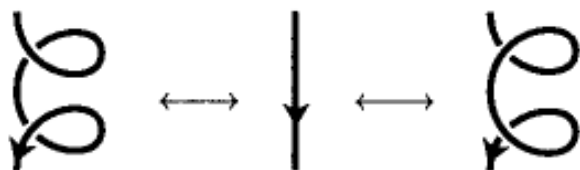
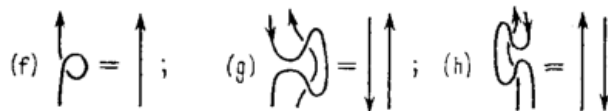
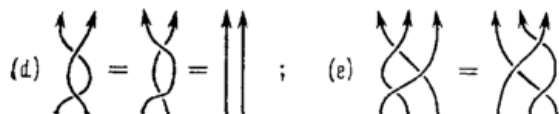
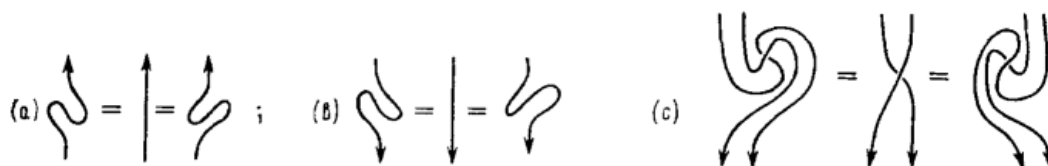


Pensieve header: Solving for caps and cups for NOE-1, t variables.

From Turaev 's "Operator Invariants..." and Ohtsuki 's "Quantum Invariants" :



```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1606"];
<< NOE-1t.m
```

Solution to all-but-one equations, as found on 160817 at YB4NOE-1t.nb:

$$\begin{aligned}
\text{EC0} = & \left\{ f_1[x_, z_] \Rightarrow \frac{1}{2} \left( -2 f_6[x, z] + 2 x f_6[x, z] + \right. \right. \\
& 2 f_{20}[x, z] - g_5[x] + 4 g_6[x] - 12 x g_6[x] + 12 x^2 g_6[x] - 4 x^3 g_6[x] + g_8[z] \left. \right), \\
& f_2[x_, y_] \Rightarrow -f_9[x, y] + y f_9[x, y] + f_{26}[x, y] - 2 y f_{26}[x, y] + y^2 f_{26}[x, y] + g_7[x], \\
& f_3[x_, z_] \Rightarrow f_{18}[x, z] - 2 x f_{18}[x, z] + x^2 f_{18}[x, z], \\
& f_4[x_, z_] \Rightarrow -f_{15}[x, z] - g_1[z], f_5[x_, y_] \Rightarrow f_{26}[x, y] - 2 y f_{26}[x, y] + y^2 f_{26}[x, y], \\
& f_7[x_, z_] \Rightarrow f_6[x, z] - x f_6[x, z] + g_9[x] + g_{10}[z], \\
& f_8[x_, z_] \Rightarrow 0, f_{10}[x_, z_] \Rightarrow 2 \left( -f_{18}[x, z] + x f_{18}[x, z] \right), \\
& f_{11}[x_, y_] \Rightarrow -2 f_{18}[x, y] + 4 x f_{18}[x, y] - 2 x^2 f_{18}[x, y] - g_5[x], f_{12}[x_, z_] \Rightarrow 0, \\
& f_{13}[x_, z_] \Rightarrow -f_{23}[x, z] - g_2[z], f_{14}[x_, z_] \Rightarrow \frac{-f_{15}[x, z] + g_3[z]}{-1 + x}, \\
& f_{15}[y_, z_] \Rightarrow \frac{1}{y} \left( 2 - y f_{23}[y, z] + y z f_{23}[y, z] - g_1[y] - y g_1[z] - y g_2[z] + y z g_2[z] - \right. \\
& \left. y g_3[y] - g_5[y] + y g_5[y] \right), f_{16}[x_, z_] \Rightarrow 0, f_{17}[x_, z_] \Rightarrow 2 \left( -f_{26}[x, z] + z f_{26}[x, z] \right), \\
& f_{18}[x_, y_] \Rightarrow \frac{1}{2 \left( -1 + x \right)} \left( -f_{19}[x, y] - 4 g_6[x] + 8 x g_6[x] - 4 x^2 g_6[x] \right), \\
& f_{19}[x_, y_] \Rightarrow \frac{1}{\left( -1 + x \right) \left( -1 + y \right)} \left( 1 + 2 f_{20}[x, y] - 2 y f_{20}[x, y] - g_5[x] + y g_5[x] - g_5[y] + \right. \\
& \left. y g_5[y] - 4 g_6[y] + 16 y g_6[y] - 24 y^2 g_6[y] + 16 y^3 g_6[y] - 4 y^4 g_6[y] + y h_1 \right), \\
& f_{21}[x_, z_] \Rightarrow 0, f_{22}[x_, z_] \Rightarrow \frac{-f_{23}[x, z] + g_4[z]}{-1 + x}, f_{24}[x_, z_] \Rightarrow 0, \\
& f_{25}[x_, z_] \Rightarrow 0, g_1[y_] \Rightarrow -g_2[y] + y g_2[y] - g_3[y] - g_4[y] + y g_4[y] - h_1[], \\
& g_3[y_] \Rightarrow \frac{1}{2 \left( -1 + y \right)} \left( 2 + 2 g_4[y] - 4 y g_4[y] + 2 y^2 g_4[y] - 2 g_5[y] + 2 y g_5[y] - \right. \\
& \left. 8 g_6[y] + 32 y g_6[y] - 48 y^2 g_6[y] + 32 y^3 g_6[y] - 8 y^4 g_6[y] + h_1[] + y h_1[] \right), \\
& g_{10}[y_] \Rightarrow \frac{1}{2 \left( 1 - 2 y + y^2 \right)} \left( 1 + g_5[y] - 3 y g_5[y] + 2 y^2 g_5[y] - 4 g_6[y] + 16 y g_6[y] - \right. \\
& \left. 24 y^2 g_6[y] + 16 y^3 g_6[y] - 4 y^4 g_6[y] - 2 y g_7[y] + 2 y^2 g_7[y] - \right. \\
& \left. g_8[y] + y g_8[y] - 2 g_9[y] + 4 y g_9[y] - 2 y^2 g_9[y] + y h_1[] \right) \left. \right\};
\end{aligned}$$

Length [EC0]

24

$$\begin{aligned}
\text{Rp}[i_, j_] := & \\
& \mathbb{E} \left[ 1, b_i c_j, u_i w_j, c_i f_1[t_i, t_j] + c_j f_2[t_i, t_j] + c_i^2 f_3[t_i, t_j] + c_i c_j f_4[t_i, t_j] + c_j^2 f_5[t_i, t_j] + \right. \\
& u_i w_i f_6[t_i, t_j] + u_i w_j f_7[t_i, t_j] + u_j w_i f_8[t_i, t_j] + u_j w_j f_9[t_i, t_j] + c_i u_i w_i f_{10}[t_i, t_j] + \\
& c_i u_i w_j f_{11}[t_i, t_j] + c_i u_j w_i f_{12}[t_i, t_j] + c_i u_j w_j f_{13}[t_i, t_j] + c_j u_i w_i f_{14}[t_i, t_j] + \\
& c_j u_i w_j f_{15}[t_i, t_j] + c_j u_j w_i f_{16}[t_i, t_j] + c_j u_j w_j f_{17}[t_i, t_j] + u_i^2 w_i^2 f_{18}[t_i, t_j] + \\
& u_i^2 w_i w_j f_{19}[t_i, t_j] + u_i^2 w_j^2 f_{20}[t_i, t_j] + u_i u_j w_i^2 f_{21}[t_i, t_j] + u_i u_j w_i w_j f_{22}[t_i, t_j] + \\
& u_i u_j w_j^2 f_{23}[t_i, t_j] + u_j^2 w_i^2 f_{24}[t_i, t_j] + u_j^2 w_i w_j f_{25}[t_i, t_j] + u_j^2 w_j^2 f_{26}[t_i, t_j] \left. \right]
\end{aligned}$$

**Rp[1, 2]**

$$\mathbb{E} \left[ 1, b_1 c_2, u_1 w_2, c_1 f_1[t_1, t_2] + c_2 f_2[t_1, t_2] + c_1^2 f_3[t_1, t_2] + c_1 c_2 f_4[t_1, t_2] + c_2^2 f_5[t_1, t_2] + u_1 w_1 f_6[t_1, t_2] + u_1 w_2 f_7[t_1, t_2] + u_2 w_1 f_8[t_1, t_2] + u_2 w_2 f_9[t_1, t_2] + c_1 u_1 w_1 f_{10}[t_1, t_2] + c_1 u_1 w_2 f_{11}[t_1, t_2] + c_1 u_2 w_1 f_{12}[t_1, t_2] + c_1 u_2 w_2 f_{13}[t_1, t_2] + c_2 u_1 w_1 f_{14}[t_1, t_2] + c_2 u_1 w_2 f_{15}[t_1, t_2] + c_2 u_2 w_1 f_{16}[t_1, t_2] + c_2 u_2 w_2 f_{17}[t_1, t_2] + u_1^2 w_1^2 f_{18}[t_1, t_2] + u_1^2 w_1 w_2 f_{19}[t_1, t_2] + u_1^2 w_2^2 f_{20}[t_1, t_2] + u_1 u_2 w_1^2 f_{21}[t_1, t_2] + u_1 u_2 w_1 w_2 f_{22}[t_1, t_2] + u_1 u_2 w_2^2 f_{23}[t_1, t_2] + u_2^2 w_1^2 f_{24}[t_1, t_2] + u_2^2 w_1 w_2 f_{25}[t_1, t_2] + u_2^2 w_2^2 f_{26}[t_1, t_2] \right]$$

**Short** [t1 = Rp[1, 2] (Rp[3, 4] Rp[5, 6] // m[3, 5, x]) // m[1, 6, y] // m[2, 4, z]]

$$\mathbb{E} \left[ 1, \langle\langle 1 \rangle\rangle, \langle\langle 1 \rangle\rangle, c_x f_1[t_x, t_y] - u_x w_z f_1[t_x, t_y] + \langle\langle 332 \rangle\rangle + u_x^2 w_z^2 f_{26}[t_x, t_z] + u_x^2 w_z^2 f_{26}[t_y, t_z] \right]$$

**Short** [t2 = (Rp[1, 2] Rp[3, 4] // m[1, 3, x]) Rp[5, 6] // m[2, 5, y] // m[4, 6, z], 5]

$$\mathbb{E} \left[ 1, \langle\langle 1 \rangle\rangle, \langle\langle 1 \rangle\rangle, 2 c_y u_x w_z + u_x^2 w_y w_z + \frac{1}{2} u_x^2 w_z^2 - \frac{1}{2} t_y u_x^2 w_z^2 - u_x u_y w_z^2 + c_x f_1[t_x, t_y] + c_x f_1[t_x, t_z] - u_x w_y f_1[t_x, t_z] - u_x w_z f_1[t_x, t_z] + \langle\langle 446 \rangle\rangle + t_y^2 u_y^2 w_z^2 f_{26}[t_x, t_y] + u_x^2 w_z^2 f_{26}[t_x, t_z] + t_y^2 u_x^2 w_z^2 f_{26}[t_y, t_z] - 2 t_y^2 t_z u_x^2 w_z^2 f_{26}[t_y, t_z] + t_y^2 t_z^2 u_x^2 w_z^2 f_{26}[t_y, t_z] + 2 t_y u_x u_z w_z^2 f_{26}[t_y, t_z] - 2 t_y t_z u_x u_z w_z^2 f_{26}[t_y, t_z] + u_x^2 w_z^2 f_{26}[t_y, t_z] \right]$$

**t3 = (t1 ≡ t2) // EC0**

$$u_x w_x f_6[t_x, t_y] + u_x w_x f_6[t_x, t_z] + \dots 151 \dots + c_x u_y w_z (\dots 1 \dots) + \frac{c_x \dots 2 \dots}{-1+t_y} \left( \frac{\dots 1 \dots}{2 \dots 1 \dots} - \dots 1 \dots \right) + c_y c_z \left( g_2[t_z] - t_z g_2[t_z] + g_4[t_z] - t_z g_4[t_z] + h_1[] + \frac{1}{2(-1+t_z)} (2 + 2 g_4[t_z] - 4 t_z g_4[t_z] + \dots 9 \dots + 32 t_z^3 g_6[t_z] - 8 t_z^4 g_6[t_z] + h_1[] + t_z h_1[]) \right) - \frac{2-t_y \dots 1 \dots + \dots 19 \dots}{t_y} \right) == \dots 1 \dots$$

large output | show less | show more | show all | set size limit...

**Together [t3]**

True

## Finding Rm

$$\text{Rm}[i_, j_] := \mathbb{E} \left[ 1, -b_i c_j, -t_i^{-1} u_i w_j, c_i g_1[t_i, t_j] + c_j g_2[t_i, t_j] + c_i^2 g_3[t_i, t_j] + c_i c_j g_4[t_i, t_j] + c_j^2 g_5[t_i, t_j] + u_i w_i g_6[t_i, t_j] + u_i w_j g_7[t_i, t_j] + u_j w_i g_8[t_i, t_j] + u_j w_j g_9[t_i, t_j] + c_i u_i w_i g_{10}[t_i, t_j] + c_i u_i w_j g_{11}[t_i, t_j] + c_i u_j w_i g_{12}[t_i, t_j] + c_i u_j w_j g_{13}[t_i, t_j] + c_j u_i w_i g_{14}[t_i, t_j] + c_j u_i w_j g_{15}[t_i, t_j] + c_j u_j w_i g_{16}[t_i, t_j] + c_j u_j w_j g_{17}[t_i, t_j] + u_i^2 w_i^2 g_{18}[t_i, t_j] + u_i^2 w_i w_j g_{19}[t_i, t_j] + u_i^2 w_j^2 g_{20}[t_i, t_j] + u_i u_j w_i^2 g_{21}[t_i, t_j] + u_i u_j w_i w_j g_{22}[t_i, t_j] + u_i u_j w_j^2 g_{23}[t_i, t_j] + u_j^2 w_i^2 g_{24}[t_i, t_j] + u_j^2 w_i w_j g_{25}[t_i, t_j] + u_j^2 w_j^2 g_{26}[t_i, t_j] \right]$$

**Short** [t1 = Rp[1, 2] Rm[3, 4] // m[1, 3, 1] // m[2, 4, 2] // Last]

$$c_1 f_1[t_1, t_2] + c_2 f_2[t_1, t_2] + \langle\langle 198 \rangle\rangle + u_x^2 w_z^2 g_{26}[t_1, t_2]$$

**Short**[**t2 = CoefficientRules**[**Expand**[**t1**], {**c1**, **u1**, **w1**, **c2**, **u2**, **w2**}] /. {(**\_** → **c\_**) ⇒ **c**} /. {**t1** → **x**, **t2** → **y**}

{**f3**[**x**, **y**] + **g3**[**x**, **y**], <<24>>, -**f8**[**x**, **y**] + **x f8**[**x**, **y**] + **f9**[**x**, **y**] + **g9**[**x**, **y**] }

**Short**[{**sol**} = **Solve**[**# = 0 & /@ t2**, **Table**[**gi**[**x**, **y**], {**i**, **26**}]]]

{**g1**[**x**, **y**] → -**f1**[**x**, **y**], <<24>>, **g26**[**x**, **y**] → -**f24**[**x**, **y**] + 2 **x f24**[**x**, **y**] - <<1>> <<1>> + <<1>> - **x f25**[**x**, **y**] - **f26**[**x**, **y**] }

**EC1 = EC0** ∪ (**sol** /. {**Rule** → **RuleDelayed**} /. **gi\_**[**x**, **y**] ⇒ **gi**[**x\_**, **y\_**])

{**f1**[**x\_**, **z\_**] ⇒  $\frac{1}{2} (-2 f_6[x, z] + 2 x f_6[x, z] + 2 f_{20}[x, z] - g_5[x] + 4 g_6[x] - 12 x g_6[x] + 12 x^2 g_6[x] - 4 x^3 g_6[x] + g_8[z])$ ,  
**f2**[**x\_**, **y\_**] ⇒ -**f9**[**x**, **y**] + **y f9**[**x**, **y**] + **f26**[**x**, **y**] - 2 **y f26**[**x**, **y**] + **y<sup>2</sup> f26**[**x**, **y**] + **g7**[**x**],  
**f3**[**x\_**, **z\_**] ⇒ **f18**[**x**, **z**] - 2 **x f18**[**x**, **z**] + **x<sup>2</sup> f18**[**x**, **z**],  
**f4**[**x\_**, **z\_**] ⇒ -**f15**[**x**, **z**] - **g1**[**z**],  
**f5**[**x\_**, **y\_**] ⇒ **f26**[**x**, **y**] - 2 **y f26**[**x**, **y**] + **y<sup>2</sup> f26**[**x**, **y**],  
**f7**[**x\_**, **z\_**] ⇒ **f6**[**x**, **z**] - **x f6**[**x**, **z**] + **g9**[**x**] + **g10**[**z**],  
**f8**[**x\_**, **z\_**] ⇒ 0, **f10**[**x\_**, **z\_**] ⇒ 2 (-**f18**[**x**, **z**] + **x f18**[**x**, **z**]),  
**f11**[**x\_**, **y\_**] ⇒ -2 **f18**[**x**, **y**] + 4 **x f18**[**x**, **y**] - 2 **x<sup>2</sup> f18**[**x**, **y**] - **g5**[**x**], **f12**[**x\_**, **z\_**] ⇒ 0,  
**f13**[**x\_**, **z\_**] ⇒ -**f23**[**x**, **z**] - **g2**[**z**], **f14**[**x\_**, **z\_**] ⇒  $\frac{-f_{15}[x, z] + g_3[z]}{-1 + x}$ , **f15**[**y\_**, **z\_**] ⇒  $\frac{1}{y}$   
(2 - **y f23**[**y**, **z**] + **y z f23**[**y**, **z**] - **g1**[**y**] - **y g1**[**z**] - **y g2**[**z**] + **y z g2**[**z**] - **y g3**[**y**] - **g5**[**y**] + **y g5**[**y**]),  
**f16**[**x\_**, **z\_**] ⇒ 0, **f17**[**x\_**, **z\_**] ⇒ 2 (-**f26**[**x**, **z**] + **z f26**[**x**, **z**]),  
**f18**[**x\_**, **y\_**] ⇒  $\frac{-f_{19}[x, y] - 4 g_6[x] + 8 x g_6[x] - 4 x^2 g_6[x]}{2(-1 + x)}$ ,  
**f19**[**x\_**, **y\_**] ⇒  $\frac{1}{(-1 + x)(-1 + y)} (1 + 2 f_{20}[x, y] - 2 y f_{20}[x, y] - g_5[x] + y g_5[x] - g_5[y] + y g_5[y] - 4 g_6[y] + 16 y g_6[y] - 24 y^2 g_6[y] + 16 y^3 g_6[y] - 4 y^4 g_6[y] + y h_1[])$ ,  
**f21**[**x\_**, **z\_**] ⇒ 0, **f22**[**x\_**, **z\_**] ⇒  $\frac{-f_{23}[x, z] + g_4[z]}{-1 + x}$ , **f24**[**x\_**, **z\_**] ⇒ 0, **f25**[**x\_**, **z\_**] ⇒ 0,  
**g1**[**y\_**] ⇒ -**g2**[**y**] + **y g2**[**y**] - **g3**[**y**] - **g4**[**y**] + **y g4**[**y**] - **h1**[],  
**g1**[**x\_**, **y\_**] ⇒ -**f1**[**x**, **y**], **g2**[**x\_**, **y\_**] ⇒ -**f2**[**x**, **y**],  
**g3**[**y\_**] ⇒  $\frac{1}{2(-1 + y)} (2 + 2 g_4[y] - 4 y g_4[y] + 2 y^2 g_4[y] - 2 g_5[y] + 2 y g_5[y] - 8 g_6[y] + 32 y g_6[y] - 48 y^2 g_6[y] + 32 y^3 g_6[y] - 8 y^4 g_6[y] + h_1[] + y h_1[])$ ,  
**g3**[**x\_**, **y\_**] ⇒ -**f3**[**x**, **y**], **g4**[**x\_**, **y\_**] ⇒ -**f4**[**x**, **y**], **g5**[**x\_**, **y\_**] ⇒ -**f5**[**x**, **y**],  
**g6**[**x\_**, **y\_**] ⇒ -**f6**[**x**, **y**] + **f8**[**x**, **y**] - **y f8**[**x**, **y**] - **f12**[**x**, **y**] + **y f12**[**x**, **y**] + **f16**[**x**, **y**] - **y f16**[**x**, **y**],  
**g7**[**x\_**, **y\_**] ⇒  
 $-\frac{f_4[x, y]}{x} - \frac{1}{x} (f_1[x, y] - f_2[x, y] - f_3[x, y] - f_5[x, y] - f_6[x, y] + x f_6[x, y] + f_7[x, y]) +$   
 $\frac{(1 - y)(-f_8[x, y] + x f_8[x, y] + f_9[x, y])}{x} + \frac{1}{x}$   
 $(-1 + y)(-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y]) + \frac{(1 - y)(-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y])}{x}$ ,  
**g8**[**x\_**, **y\_**] ⇒ -**x f8**[**x**, **y**], **g9**[**x\_**, **y\_**] ⇒ **f8**[**x**, **y**] - **x f8**[**x**, **y**] - **f9**[**x**, **y**], **g10**[**y\_**] ⇒

$$\begin{aligned}
& \frac{1}{2(1-2y+y^2)} \left( 1 + g_5[y] - 3y g_5[y] + 2y^2 g_5[y] - 4g_6[y] + 16y g_6[y] - 24y^2 g_6[y] + 16y^3 g_6[y] - \right. \\
& \quad \left. 4y^4 g_6[y] - 2y g_7[y] + 2y^2 g_7[y] - g_8[y] + y g_8[y] - 2g_9[y] + 4y g_9[y] - 2y^2 g_9[y] + y h_1[] \right), \\
g_{10}[x_-, y_-] & \Rightarrow -f_{10}[x, y] + f_{12}[x, y] - y f_{12}[x, y], \quad g_{11}[x_-, y_-] \Rightarrow \\
& -\frac{1}{x} \left( 2f_3[x, y] - f_4[x, y] - f_{10}[x, y] + x f_{10}[x, y] + f_{11}[x, y] \right) + \\
& \quad \frac{(1-y) \left( -f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y] \right)}{x}, \\
g_{12}[x_-, y_-] & \Rightarrow -x f_{12}[x, y], \quad g_{13}[x_-, y_-] \Rightarrow f_{12}[x, y] - x f_{12}[x, y] - f_{13}[x, y], \\
g_{14}[x_-, y_-] & \Rightarrow -f_{14}[x, y] + f_{16}[x, y] - y f_{16}[x, y], \\
g_{15}[x_-, y_-] & \Rightarrow -\frac{1}{x} \left( f_4[x, y] - 2f_5[x, y] - f_{14}[x, y] + x f_{14}[x, y] + f_{15}[x, y] \right) + \\
& \quad \frac{(1-y) \left( -f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y] \right)}{x}, \\
g_{16}[x_-, y_-] & \Rightarrow -x f_{16}[x, y], \quad g_{17}[x_-, y_-] \Rightarrow f_{16}[x, y] - x f_{16}[x, y] - f_{17}[x, y], \\
g_{18}[x_-, y_-] & \Rightarrow -f_{18}[x, y] + f_{21}[x, y] - y f_{21}[x, y] + (-1+2y-y^2) f_{24}[x, y], \\
g_{19}[x_-, y_-] & \Rightarrow -\frac{1}{x} \left( f_{10}[x, y] - f_{12}[x, y] + y f_{12}[x, y] - f_{14}[x, y] + f_{16}[x, y] - y f_{16}[x, y] - \right. \\
& \quad \left. 2f_{18}[x, y] + 2x f_{18}[x, y] + f_{19}[x, y] + 2f_{21}[x, y] - 2x f_{21}[x, y] - 2y f_{21}[x, y] + \right. \\
& \quad \left. 2xy f_{21}[x, y] - f_{22}[x, y] + y f_{22}[x, y] - 2f_{24}[x, y] + 2x f_{24}[x, y] + 4y f_{24}[x, y] - \right. \\
& \quad \left. 4xy f_{24}[x, y] - 2y^2 f_{24}[x, y] + 2xy^2 f_{24}[x, y] + f_{25}[x, y] - 2y f_{25}[x, y] + y^2 f_{25}[x, y] \right), \\
g_{20}[x_-, y_-] & \Rightarrow \frac{f_4[x, y]}{x^2} + \frac{(1-y) \left( -f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y] \right)}{x^2} + \frac{1}{x^2} \\
& \quad (-1+y) \left( -f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y] \right) - \\
& \quad \frac{1}{x^2} \left( f_3[x, y] + f_5[x, y] - f_{10}[x, y] + x f_{10}[x, y] + f_{11}[x, y] + f_{14}[x, y] - x f_{14}[x, y] - f_{15}[x, y] + \right. \\
& \quad \left. f_{18}[x, y] - 2x f_{18}[x, y] + x^2 f_{18}[x, y] - f_{19}[x, y] + x f_{19}[x, y] + f_{20}[x, y] \right) + \frac{1}{x^2} \\
& \quad (1-y) \left( f_{21}[x, y] - 2x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y] \right) + \\
& \quad \frac{1}{x^2} (-1+2y-y^2) \left( f_{24}[x, y] - 2x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y] \right), \\
g_{21}[x_-, y_-] & \Rightarrow -x f_{21}[x, y] + 2x f_{24}[x, y] - 2xy f_{24}[x, y], \\
g_{22}[x_-, y_-] & \Rightarrow -f_{12}[x, y] + f_{16}[x, y] + 2f_{21}[x, y] - 2x f_{21}[x, y] - f_{22}[x, y] - \\
& \quad 4f_{24}[x, y] + 4x f_{24}[x, y] + 4y f_{24}[x, y] - 4xy f_{24}[x, y] + 2f_{25}[x, y] - 2y f_{25}[x, y], \\
g_{23}[x_-, y_-] & \Rightarrow -\frac{-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y]}{x} + \frac{-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y]}{x} - \\
& \quad \frac{1}{x} \left( f_{21}[x, y] - 2x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y] \right) + \\
& \quad \left( \frac{2}{x} - \frac{2y}{x} \right) \left( f_{24}[x, y] - 2x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y] \right), \\
g_{24}[x_-, y_-] & \Rightarrow -x^2 f_{24}[x, y], \quad g_{25}[x_-, y_-] \Rightarrow 2x f_{24}[x, y] - 2x^2 f_{24}[x, y] - x f_{25}[x, y], \\
g_{26}[x_-, y_-] & \Rightarrow -f_{24}[x, y] + 2x f_{24}[x, y] - x^2 f_{24}[x, y] + f_{25}[x, y] - x f_{25}[x, y] - f_{26}[x, y] \}
\end{aligned}$$

```

ur[i_] := E[t_i^{-1/4}, 0, 0,
  t_i^{-1} (c_i ur_1[t_i] + c_i^2 ur_2[t_i] + u_i w_i ur_3[t_i] + c_i u_i w_i ur_4[t_i] + u_i^2 w_i^2 ur_5[t_i]) // Expand];
nr[i_] := E[t_i^{1/4}, 0, 0,
  t_i (-c_i ur_1[t_i] - c_i^2 ur_2[t_i] - u_i w_i ur_3[t_i] - c_i u_i w_i ur_4[t_i] - u_i^2 w_i^2 ur_5[t_i]) // Expand];
ul[i_] := E[t_i^{1/4}, 0, 0,
  t_i (c_i ul_1[t_i] + c_i^2 ul_2[t_i] + u_i w_i ul_3[t_i] + c_i u_i w_i ul_4[t_i] + u_i^2 w_i^2 ul_5[t_i]) // Expand];
nl[i_] := E[t_i^{-1/4}, 0, 0,
  t_i^{-1} (-c_i ul_1[t_i] - c_i^2 ul_2[t_i] - u_i w_i ul_3[t_i] - c_i u_i w_i ul_4[t_i] - u_i^2 w_i^2 ul_5[t_i]) // Expand];

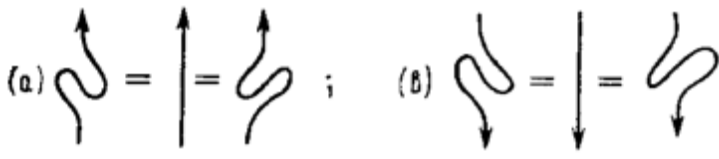
```

ur[1]

$$E\left[\frac{1}{t_1^{1/4}}, 0, 0, \frac{c_1 ur_1[t_1]}{t_1} + \frac{c_1^2 ur_2[t_1]}{t_1} + \frac{u_1 w_1 ur_3[t_1]}{t_1} + \frac{c_1 u_1 w_1 ur_4[t_1]}{t_1} + \frac{u_1^2 w_1^2 ur_5[t_1]}{t_1}\right]$$

### The Serpents

From Turaev 's "Operator Invariants...":



```

ur[1] nr[2] // m[1, 2, 1]
E[1, 0, 0, 0]

ul[1] nl[2] // m[1, 2, 1]
E[1, 0, 0, 0]

```

### Centrality of (some) Kinks

```

(* kpr for "kink positive right", etc. *)
kpr[i_] := Module[{s1, s2, s3},
  Rp[i, s3] nr[s1] ul[s2] // m[i, s1, i] // m[i, s2, i] // m[i, s3, i];
kpl[i_] := Module[{s1, s2, s3},
  Rp[s3, i] nl[s1] ur[s2] // m[i, s1, i] // m[i, s2, i] // m[i, s3, i];
kmr[i_] := Module[{s1, s2, s3},
  Rm[s3, i] nr[s1] ul[s2] // m[i, s1, i] // m[i, s2, i] // m[i, s3, i];
kml[i_] := Module[{s1, s2, s3},
  Rm[i, s3] nl[s1] ur[s2] // m[i, s1, i] // m[i, s2, i] // m[i, s3, i];

```

**Most** /@ {kpr[1], kpl[1], kmr[1], kml[1]}

$$\left\{ \mathbb{E} \left[ \sqrt{t_1}, b_1 c_1, \frac{u_1 w_1}{\sqrt{t_1}} \right], \mathbb{E} \left[ \sqrt{t_1}, b_1 c_1, \frac{u_1 w_1}{\sqrt{t_1}} \right], \right. \\ \left. \mathbb{E} \left[ \frac{1}{\sqrt{t_1}}, -b_1 c_1, -\frac{u_1 w_1}{\sqrt{t_1}} \right], \mathbb{E} \left[ \frac{1}{\sqrt{t_1}}, -b_1 c_1, -\frac{u_1 w_1}{\sqrt{t_1}} \right] \right\}$$

**Most** /@ {kpr[1] kmr[2] // m[1, 2, 1], kpl[1] kml[2] // m[1, 2, 1]}

$$\{ \mathbb{E}[1, 0, 0], \mathbb{E}[1, 0, 0] \}$$

**E1a** = ((Rp[x, y] kpr[3] // m[x, 3, x]) ≡ (Rp[x, y] kpr[3] // m[3, x, x])) /.  
(a\_ = b\_) => a - b // EC1

$$2 t_x u_x w_y - 2 c_x t_x u_x w_y + u_x^2 w_x w_y + \frac{3}{2} u_x^2 w_y^2 - \frac{1}{2} t_x u_x^2 w_y^2 - t_x^2 u_x w_y f_6[t_x, t_x] + t_x^3 u_x w_y f_6[t_x, t_x] - \\ t_x^2 u_x w_y f_9[t_x, t_x] + \dots 177 \dots + 2 t_x^2 u_x^2 w_y^2 ur_4[t_x] - t_x^3 u_x^2 w_y^2 ur_4[t_x] + 2 t_x^2 u_x^2 w_x w_y ur_5[t_x] - \\ 2 t_x^3 u_x^2 w_x w_y ur_5[t_x] + t_x^2 u_x^2 w_y^2 ur_5[t_x] - 2 t_x^3 u_x^2 w_y^2 ur_5[t_x] + t_x^4 u_x^2 w_y^2 ur_5[t_x]$$

large output    show less    show more    show all    set size limit...

**Short** [E1b = CoefficientRules[Expand[E1a], {Cx, ux, wx, Cy, uy, wy}] /. {(\_ -> c\_) => c} /.  
{tx -> x, ty -> y}]

$$\{ 2x + \ll 86 \gg + 2x^2 ur_2[x] + x^2 ur_4[x] - x^3 ur_4[x], \\ \ll 2 \gg, \frac{x}{2(\ll 1 \gg)^3} - \frac{x^2}{\ll 1 \gg^3} + \ll 114 \gg + x^2 \ll 2 \gg_3[x] - x^3 ur_3[x] \}$$

**Short** [E1c = DeleteCases[Simplify[E1b], 0] // SortBy[LeafCount], 9]

$$\left\{ x^2 (-2 ul_2[x] + (-1+x) ul_4[x] + 2 ur_2[x] + ur_4[x] - x ur_4[x]), \right. \\ x^2 (-h_1[] - ul_1[x] + ul_2[x] - ul_3[x] + x ul_3[x] + ur_1[x] - ur_2[x] + ur_3[x] - x ur_3[x]), \\ x (2 ul_2[x] + (1-2x) ul_4[x] - 2x ul_5[x] + \\ 2x^2 ul_5[x] - 2ur_2[x] - ur_4[x] + 2x ur_4[x] + 2x ur_5[x] - 2x^2 ur_5[x]), \\ x (-(-2+x) ul_2[x] + (-1+x)^2 ul_4[x] - x ul_5[x] + 2x^2 ul_5[x] - x^3 ul_5[x] - 2ur_2[x] + \\ x ur_2[x] - ur_4[x] + 2x ur_4[x] - x^2 ur_4[x] + x ur_5[x] - 2x^2 ur_5[x] + x^3 ur_5[x]) \}$$

**Solve** [(# == 0) & /@ E1c, {ul1[x], ul2[x], ul4[x]}]

$$\left\{ \left\{ \begin{aligned} ul_1[x] &\rightarrow -h_1[] - ul_3[x] + x ul_3[x] + ul_5[x] - 2x ul_5[x] + \\ &x^2 ul_5[x] + ur_1[x] + ur_3[x] - x ur_3[x] - ur_5[x] + 2x ur_5[x] - x^2 ur_5[x], \\ ul_2[x] &\rightarrow ul_5[x] - 2x ul_5[x] + x^2 ul_5[x] + ur_2[x] - ur_5[x] + 2x ur_5[x] - x^2 ur_5[x], \\ ul_4[x] &\rightarrow -2 ul_5[x] + 2x ul_5[x] + ur_4[x] + 2 ur_5[x] - 2x ur_5[x] \end{aligned} \right\} \right\}$$

**EC2** = EC1 ∪ {ul1[x\_] => -h1[] - ul3[x] + x ul3[x] + ul5[x] - 2x ul5[x] +  
x^2 ul5[x] + ur1[x] + ur3[x] - x ur3[x] - ur5[x] + 2x ur5[x] - x^2 ur5[x],  
ul2[x\_] => ul5[x] - 2x ul5[x] + x^2 ul5[x] + ur2[x] - ur5[x] + 2x ur5[x] - x^2 ur5[x],  
ul4[x\_] => -2 ul5[x] + 2x ul5[x] + ur4[x] + 2 ur5[x] - 2x ur5[x]}

$$\{ f_1[x_, z_] => \frac{1}{2} (-2 f_6[x, z] + 2x f_6[x, z] +$$

$$\begin{aligned}
& 2 f_{20}[x, z] - g_5[x] + 4 g_6[x] - 12 x g_6[x] + 12 x^2 g_6[x] - 4 x^3 g_6[x] + g_8[z], \\
f_2[x_-, y_-] & \Rightarrow -f_9[x, y] + y f_9[x, y] + f_{26}[x, y] - 2 y f_{26}[x, y] + y^2 f_{26}[x, y] + g_7[x], \\
f_3[x_-, z_-] & \Rightarrow f_{18}[x, z] - 2 x f_{18}[x, z] + x^2 f_{18}[x, z], \\
f_4[x_-, z_-] & \Rightarrow -f_{15}[x, z] - g_1[z], \\
f_5[x_-, y_-] & \Rightarrow f_{26}[x, y] - 2 y f_{26}[x, y] + y^2 f_{26}[x, y], \\
f_7[x_-, z_-] & \Rightarrow f_6[x, z] - x f_6[x, z] + g_9[x] + g_{10}[z], \\
f_8[x_-, z_-] & \Rightarrow 0, f_{10}[x_-, z_-] \Rightarrow 2(-f_{18}[x, z] + x f_{18}[x, z]), \\
f_{11}[x_-, y_-] & \Rightarrow -2 f_{18}[x, y] + 4 x f_{18}[x, y] - 2 x^2 f_{18}[x, y] - g_5[x], f_{12}[x_-, z_-] \Rightarrow 0, \\
f_{13}[x_-, z_-] & \Rightarrow -f_{23}[x, z] - g_2[z], f_{14}[x_-, z_-] \Rightarrow \frac{-f_{15}[x, z] + g_3[z]}{-1+x}, f_{15}[y_-, z_-] \Rightarrow \frac{1}{y} \\
& (2 - y f_{23}[y, z] + y z f_{23}[y, z] - g_1[y] - y g_1[z] - y g_2[z] + y z g_2[z] - y g_3[y] - g_5[y] + y g_5[y]), \\
f_{16}[x_-, z_-] & \Rightarrow 0, f_{17}[x_-, z_-] \Rightarrow 2(-f_{26}[x, z] + z f_{26}[x, z]), \\
f_{18}[x_-, y_-] & \Rightarrow \frac{-f_{19}[x, y] - 4 g_6[x] + 8 x g_6[x] - 4 x^2 g_6[x]}{2(-1+x)}, \\
f_{19}[x_-, y_-] & \Rightarrow \frac{1}{(-1+x)(-1+y)} (1 + 2 f_{20}[x, y] - 2 y f_{20}[x, y] - g_5[x] + y g_5[x] - g_5[y] + \\
& y g_5[y] - 4 g_6[y] + 16 y g_6[y] - 24 y^2 g_6[y] + 16 y^3 g_6[y] - 4 y^4 g_6[y] + y h_1[]), \\
f_{21}[x_-, z_-] & \Rightarrow 0, f_{22}[x_-, z_-] \Rightarrow \frac{-f_{23}[x, z] + g_4[z]}{-1+x}, f_{24}[x_-, z_-] \Rightarrow 0, f_{25}[x_-, z_-] \Rightarrow 0, \\
g_1[y_-] & \Rightarrow -g_2[y] + y g_2[y] - g_3[y] - g_4[y] + y g_4[y] - h_1[], \\
g_1[x_-, y_-] & \Rightarrow -f_1[x, y], g_2[x_-, y_-] \Rightarrow -f_2[x, y], \\
g_3[y_-] & \Rightarrow \frac{1}{2(-1+y)} (2 + 2 g_4[y] - 4 y g_4[y] + 2 y^2 g_4[y] - 2 g_5[y] + 2 y g_5[y] - \\
& 8 g_6[y] + 32 y g_6[y] - 48 y^2 g_6[y] + 32 y^3 g_6[y] - 8 y^4 g_6[y] + h_1[] + y h_1[]), \\
g_3[x_-, y_-] & \Rightarrow -f_3[x, y], g_4[x_-, y_-] \Rightarrow -f_4[x, y], g_5[x_-, y_-] \Rightarrow -f_5[x, y], \\
g_6[x_-, y_-] & \Rightarrow -f_6[x, y] + f_8[x, y] - y f_8[x, y] - f_{12}[x, y] + y f_{12}[x, y] + f_{16}[x, y] - y f_{16}[x, y], \\
g_7[x_-, y_-] & \Rightarrow \\
& -\frac{f_4[x, y]}{x} - \frac{1}{x} (f_1[x, y] - f_2[x, y] - f_3[x, y] - f_5[x, y] - f_6[x, y] + x f_6[x, y] + f_7[x, y]) + \\
& \frac{(1-y)(-f_8[x, y] + x f_8[x, y] + f_9[x, y])}{x} + \frac{1}{x} \\
& (-1+y)(-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y]) + \frac{(1-y)(-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y])}{x}, \\
g_8[x_-, y_-] & \Rightarrow -x f_8[x, y], g_9[x_-, y_-] \Rightarrow f_8[x, y] - x f_8[x, y] - f_9[x, y], g_{10}[y_-] \Rightarrow \\
& \frac{1}{2(1-2y+y^2)} (1 + g_5[y] - 3 y g_5[y] + 2 y^2 g_5[y] - 4 g_6[y] + 16 y g_6[y] - 24 y^2 g_6[y] + 16 y^3 g_6[y] - \\
& 4 y^4 g_6[y] - 2 y g_7[y] + 2 y^2 g_7[y] - g_8[y] + y g_8[y] - 2 g_9[y] + 4 y g_9[y] - 2 y^2 g_9[y] + y h_1[]), \\
g_{10}[x_-, y_-] & \Rightarrow -f_{10}[x, y] + f_{12}[x, y] - y f_{12}[x, y], g_{11}[x_-, y_-] \Rightarrow \\
& -\frac{1}{x} (2 f_3[x, y] - f_4[x, y] - f_{10}[x, y] + x f_{10}[x, y] + f_{11}[x, y]) + \\
& \frac{(1-y)(-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y])}{x}, \\
g_{12}[x_-, y_-] & \Rightarrow -x f_{12}[x, y], g_{13}[x_-, y_-] \Rightarrow f_{12}[x, y] - x f_{12}[x, y] - f_{13}[x, y], \\
g_{14}[x_-, y_-] & \Rightarrow -f_{14}[x, y] + f_{16}[x, y] - y f_{16}[x, y], \\
g_{15}[x_-, y_-] & \Rightarrow -\frac{1}{x} (f_4[x, y] - 2 f_5[x, y] - f_{14}[x, y] + x f_{14}[x, y] + f_{15}[x, y]) +
\end{aligned}$$



$$\frac{(1-y)(-f_{16}[x,y] + x f_{16}[x,y] + f_{17}[x,y])}{x},$$

$$g_{16}[x_, y_] := -x f_{16}[x, y], g_{17}[x_, y_] := f_{16}[x, y] - x f_{16}[x, y] - f_{17}[x, y],$$

$$g_{18}[x_, y_] := -f_{18}[x, y] + f_{21}[x, y] - y f_{21}[x, y] + (-1 + 2y - y^2) f_{24}[x, y],$$

$$g_{19}[x_, y_] := -\frac{1}{x} (f_{10}[x, y] - f_{12}[x, y] + y f_{12}[x, y] - f_{14}[x, y] + f_{16}[x, y] - y f_{16}[x, y] -$$

$$2 f_{18}[x, y] + 2 x f_{18}[x, y] + f_{19}[x, y] + 2 f_{21}[x, y] - 2 x f_{21}[x, y] - 2 y f_{21}[x, y] +$$

$$2 x y f_{21}[x, y] - f_{22}[x, y] + y f_{22}[x, y] - 2 f_{24}[x, y] + 2 x f_{24}[x, y] + 4 y f_{24}[x, y] -$$

$$4 x y f_{24}[x, y] - 2 y^2 f_{24}[x, y] + 2 x y^2 f_{24}[x, y] + f_{25}[x, y] - 2 y f_{25}[x, y] + y^2 f_{25}[x, y]),$$

$$g_{20}[x_, y_] := \frac{f_4[x, y]}{x^2} + \frac{(1-y)(-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y])}{x^2} + \frac{1}{x^2}$$

$$(-1 + y)(-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y]) -$$

$$\frac{1}{x^2} (f_3[x, y] + f_5[x, y] - f_{10}[x, y] + x f_{10}[x, y] + f_{11}[x, y] + f_{14}[x, y] - x f_{14}[x, y] - f_{15}[x, y] +$$

$$f_{18}[x, y] - 2 x f_{18}[x, y] + x^2 f_{18}[x, y] - f_{19}[x, y] + x f_{19}[x, y] + f_{20}[x, y]) + \frac{1}{x^2}$$

$$(1-y)(f_{21}[x, y] - 2 x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y]) +$$

$$\frac{1}{x^2} (-1 + 2y - y^2)(f_{24}[x, y] - 2 x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y]),$$

$$g_{21}[x_, y_] := -x f_{21}[x, y] + 2 x f_{24}[x, y] - 2 x y f_{24}[x, y],$$

$$g_{22}[x_, y_] := -f_{12}[x, y] + f_{16}[x, y] + 2 f_{21}[x, y] - 2 x f_{21}[x, y] - f_{22}[x, y] -$$

$$4 f_{24}[x, y] + 4 x f_{24}[x, y] + 4 y f_{24}[x, y] - 4 x y f_{24}[x, y] + 2 f_{25}[x, y] - 2 y f_{25}[x, y],$$

$$g_{23}[x_, y_] := -\frac{-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y]}{x} + \frac{-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y]}{x} -$$

$$\frac{1}{x} (f_{21}[x, y] - 2 x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y]) +$$

$$\left(\frac{2}{x} - \frac{2y}{x}\right) (f_{24}[x, y] - 2 x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y]),$$

$$g_{24}[x_, y_] := -x^2 f_{24}[x, y], g_{25}[x_, y_] := 2 x f_{24}[x, y] - 2 x^2 f_{24}[x, y] - x f_{25}[x, y],$$

$$g_{26}[x_, y_] := -f_{24}[x, y] + 2 x f_{24}[x, y] - x^2 f_{24}[x, y] + f_{25}[x, y] - x f_{25}[x, y] - f_{26}[x, y],$$

$$ul_1[x_] := -h_1[] - ul_3[x] + x ul_3[x] + ul_5[x] - 2 x ul_5[x] +$$

$$x^2 ul_5[x] + ur_1[x] + ur_3[x] - x ur_3[x] - ur_5[x] + 2 x ur_5[x] - x^2 ur_5[x],$$

$$ul_2[x_] := ul_5[x] - 2 x ul_5[x] + x^2 ul_5[x] + ur_2[x] - ur_5[x] + 2 x ur_5[x] - x^2 ur_5[x],$$

$$ul_4[x_] := -2 ul_5[x] + 2 x ul_5[x] + ur_4[x] + 2 ur_5[x] - 2 x ur_5[x]$$

**t1** = ((Rp[x, y] kpr[3] // m[y, 3, y]) ≡ (Rp[x, y] kpr[3] // m[3, y, y])) /.  
 (a\_ == b\_) := a - b //. EC2

$$2 c_y t_y^2 u_x w_y + \frac{1}{2} t_y^2 u_x^2 w_y^2 + \dots 196 \dots +$$

$$t_y^3 u_x w_y (-h_1[] - ul_3[t_y] + t_y ul_3[t_y] + ul_5[t_y] - 2 t_y ul_5[t_y] + t_y^2 ul_5[t_y] +$$

$$ur_1[t_y] + ur_3[t_y] - t_y ur_3[t_y] - ur_5[t_y] + 2 t_y ur_5[t_y] - t_y^2 ur_5[t_y])$$

large output
show less
show more
show all
set size limit...

**Simplify[t1]**

0

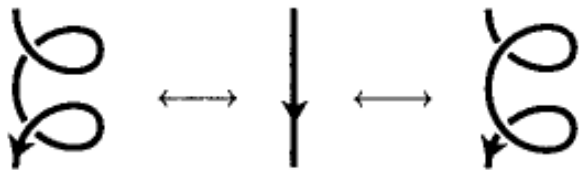
```
Simplify[{
  (Rp[x, y] kpr[3] // m[x, 3, x]) ≡ (Rp[x, y] kpr[3] // m[3, x, x]),
  (Rp[x, y] kpr[3] // m[y, 3, y]) ≡ (Rp[x, y] kpr[3] // m[3, y, y])
} /. (a_ == b_) => a - b // EC2]
{0, 0}
```

```
Simplify[{
  (Rp[x, y] kpl[3] // m[x, 3, x]) ≡ (Rp[x, y] kpl[3] // m[3, x, x]),
  (Rp[x, y] kpl[3] // m[y, 3, y]) ≡ (Rp[x, y] kpl[3] // m[3, y, y])
} /. (a_ == b_) => a - b // EC2]
{0, 0}
```

```
Simplify[{
  (Rp[x, y] kml[3] // m[x, 3, x]) ≡ (Rp[x, y] kml[3] // m[3, x, x]),
  (Rp[x, y] kml[3] // m[y, 3, y]) ≡ (Rp[x, y] kml[3] // m[3, y, y])
} /. (a_ == b_) => a - b // EC2]
{0, 0}
```

```
Simplify[{
  (Rp[x, y] kmr[3] // m[x, 3, x]) ≡ (Rp[x, y] kmr[3] // m[3, x, x]),
  (Rp[x, y] kmr[3] // m[y, 3, y]) ≡ (Rp[x, y] kmr[3] // m[3, y, y])
} /. (a_ == b_) => a - b // EC2]
{0, 0}
```

### The Framing Relations



**E2a = Last@Simplify[(kpr[1] kmr[2] // m[1, 2, 1]) // . EC2]**

$$\begin{aligned}
& \frac{1}{2(-1+t_1)} - \frac{c_1}{-1+t_1} + \frac{c_1}{(-1+t_1)t_1} - \frac{u_1 w_1}{(-1+t_1)t_1} - \frac{3c_1 g_2[t_1]}{-1+t_1} + \frac{c_1 g_2[t_1]}{(-1+t_1)t_1} + \frac{3c_1 t_1 g_2[t_1]}{-1+t_1} - \\
& \frac{c_1 t_1^2 g_2[t_1]}{-1+t_1} + \frac{2u_1 w_1 g_2[t_1]}{-1+t_1} - \frac{u_1 w_1 g_2[t_1]}{(-1+t_1)t_1} - \frac{t_1 u_1 w_1 g_2[t_1]}{-1+t_1} - \frac{g_5[t_1]}{2(-1+t_1)} + \frac{t_1 g_5[t_1]}{2(-1+t_1)} - \frac{2g_6[t_1]}{-1+t_1} - \\
& \frac{20c_1 g_6[t_1]}{-1+t_1} + \frac{4c_1 g_6[t_1]}{(-1+t_1)t_1} + \frac{8t_1 g_6[t_1]}{-1+t_1} + \frac{40c_1 t_1 g_6[t_1]}{-1+t_1} - \frac{12t_1^2 g_6[t_1]}{-1+t_1} - \frac{40c_1 t_1^2 g_6[t_1]}{-1+t_1} + \\
& \frac{8t_1^3 g_6[t_1]}{-1+t_1} + \frac{20c_1 t_1^3 g_6[t_1]}{-1+t_1} - \frac{2t_1^4 g_6[t_1]}{-1+t_1} - \frac{4c_1 t_1^4 g_6[t_1]}{-1+t_1} + \frac{16u_1 w_1 g_6[t_1]}{-1+t_1} - \frac{4u_1 w_1 g_6[t_1]}{(-1+t_1)t_1} - \\
& \frac{24t_1 u_1 w_1 g_6[t_1]}{-1+t_1} + \frac{16t_1^2 u_1 w_1 g_6[t_1]}{-1+t_1} - \frac{4t_1^3 u_1 w_1 g_6[t_1]}{-1+t_1} - \frac{g_7[t_1]}{-1+t_1} + \frac{t_1 g_7[t_1]}{-1+t_1} - \frac{g_8[t_1]}{2(-1+t_1)} + \\
& \frac{t_1 g_8[t_1]}{2(-1+t_1)} + \frac{c_1 h_1[]}{2(-1+t_1)t_1} + \frac{t_1 h_1[]}{2(-1+t_1)} - \frac{c_1 t_1 h_1[]}{2(-1+t_1)} - \frac{u_1 w_1 h_1[]}{2(-1+t_1)} - \frac{u_1 w_1 h_1[]}{2(-1+t_1)t_1} + \\
& \frac{2c_1 ul_3[t_1]}{-1+t_1} - \frac{4c_1 t_1 ul_3[t_1]}{-1+t_1} + \frac{2c_1 t_1^2 ul_3[t_1]}{-1+t_1} - \frac{2u_1 w_1 ul_3[t_1]}{-1+t_1} + \frac{2t_1 u_1 w_1 ul_3[t_1]}{-1+t_1} - \\
& \frac{2c_1 ul_5[t_1]}{-1+t_1} - \frac{2c_1^2 ul_5[t_1]}{-1+t_1} + \frac{6c_1 t_1 ul_5[t_1]}{-1+t_1} + \frac{6c_1^2 t_1 ul_5[t_1]}{-1+t_1} - \frac{6c_1 t_1^2 ul_5[t_1]}{-1+t_1} - \frac{6c_1^2 t_1^2 ul_5[t_1]}{-1+t_1} + \\
& \frac{2c_1 t_1^3 ul_5[t_1]}{-1+t_1} + \frac{2c_1^2 t_1^3 ul_5[t_1]}{-1+t_1} + \frac{4c_1 u_1 w_1 ul_5[t_1]}{-1+t_1} - \frac{8c_1 t_1 u_1 w_1 ul_5[t_1]}{-1+t_1} + \frac{4c_1 t_1^2 u_1 w_1 ul_5[t_1]}{-1+t_1} - \\
& \frac{2u_1^2 w_1^2 ul_5[t_1]}{-1+t_1} + \frac{2t_1 u_1^2 w_1^2 ul_5[t_1]}{-1+t_1} - \frac{2c_1 ur_3[t_1]}{-1+t_1} + \frac{4c_1 t_1 ur_3[t_1]}{-1+t_1} - \frac{2c_1 t_1^2 ur_3[t_1]}{-1+t_1} + \\
& \frac{2u_1 w_1 ur_3[t_1]}{-1+t_1} - \frac{2t_1 u_1 w_1 ur_3[t_1]}{-1+t_1} + \frac{2c_1 ur_5[t_1]}{-1+t_1} + \frac{2c_1^2 ur_5[t_1]}{-1+t_1} - \frac{6c_1 t_1 ur_5[t_1]}{-1+t_1} - \\
& \frac{6c_1^2 t_1 ur_5[t_1]}{-1+t_1} + \frac{6c_1 t_1^2 ur_5[t_1]}{-1+t_1} + \frac{6c_1^2 t_1^2 ur_5[t_1]}{-1+t_1} - \frac{2c_1 t_1^3 ur_5[t_1]}{-1+t_1} - \frac{2c_1^2 t_1^3 ur_5[t_1]}{-1+t_1} - \\
& \frac{4c_1 u_1 w_1 ur_5[t_1]}{-1+t_1} + \frac{8c_1 t_1 u_1 w_1 ur_5[t_1]}{-1+t_1} - \frac{4c_1 t_1^2 u_1 w_1 ur_5[t_1]}{-1+t_1} + \frac{2u_1^2 w_1^2 ur_5[t_1]}{-1+t_1} - \frac{2t_1 u_1^2 w_1^2 ur_5[t_1]}{-1+t_1} -
\end{aligned}$$

**Short [E2b = CoefficientRules [Expand [E2a], {c<sub>1</sub>, u<sub>1</sub>, w<sub>1</sub>}] /. {(\_ -> c\_) :-> c} /. {t<sub>1</sub> -> x}, 9]**

$$\left\{ \frac{2 u_5[x]}{1-x} - \frac{6 x u_5[x]}{1-x} + \frac{6 x^2 u_5[x]}{1-x} - \frac{2 x^3 u_5[x]}{1-x} - \frac{2 u_5[x]}{1-x} + \frac{6 x u_5[x]}{1-x} - \frac{6 x^2 u_5[x]}{1-x} + \frac{2 x^3 u_5[x]}{1-x}, -\frac{4 u_5[x]}{1-x} + \frac{8 x u_5[x]}{1-x} - \frac{4 x^2 u_5[x]}{1-x} + \frac{4 u_5[x]}{1-x} - \frac{8 x u_5[x]}{1-x} + \frac{4 x^2 u_5[x]}{1-x}, \frac{1}{1-x} - \frac{1}{\langle\langle 1 \rangle\rangle x} + \langle\langle 38 \rangle\rangle + \frac{2 x^3 u_5[x]}{1-x}, \langle\langle 5 \rangle\rangle + \langle\langle 1 \rangle\rangle, \frac{1}{(1-x)x} - \frac{2 g_2[x]}{1-x} + \frac{g_2[x]}{(1-x)x} + \frac{x g_2[x]}{1-x} - \frac{16 g_6[x]}{1-x} + \frac{4 g_6[x]}{(1-x)x} + \frac{24 x g_6[x]}{1-x} - \frac{16 x^2 g_6[x]}{1-x} + \frac{4 x^3 g_6[x]}{1-x} + \frac{h_1[]}{2(1-x)} + \frac{h_1[]}{2(1-x)x} + \frac{2 u_3[x]}{1-x} - \frac{2 x u_3[x]}{1-x} - \frac{2 u_3[x]}{1-x} + \frac{2 x u_3[x]}{1-x}, -\frac{1}{2(1-x)} + \frac{g_5[x]}{2(1-x)} - \frac{x g_5[x]}{2(1-x)} + \frac{2 g_6[x]}{1-x} - \frac{8 x g_6[x]}{1-x} + \frac{12 x^2 g_6[x]}{1-x} - \frac{8 x^3 g_6[x]}{1-x} + \frac{2 x^4 g_6[x]}{1-x} + \frac{g_7[x]}{1-x} - \frac{x g_7[x]}{1-x} + \frac{g_8[x]}{2(1-x)} - \frac{x g_8[x]}{2(1-x)} - \frac{x h_1[]}{2(1-x)} \right\}$$

**Short [E2c = DeleteCases [Simplify [E2b], 0] // SortBy [LeafCount], 9]**

$$\left\{ 2 (u_5[x] - u_5[x]), 4 (-1+x) (u_5[x] - u_5[x]), 2 (-1+x)^2 (u_5[x] - u_5[x]), \frac{1}{2(-1+x)} (1 + (-1+x) g_5[x] - 4 (-1+x)^4 g_6[x] - 2 g_7[x] + 2 x g_7[x] - g_8[x] + x g_8[x] + x h_1[]), -\frac{1}{2(-1+x)x} (2 + 2 (-1+x)^2 g_2[x] + 8 (-1+x)^4 g_6[x] + h_1[] + x h_1[] + 4 x u_3[x] - 4 x^2 u_3[x] - 4 x u_3[x] + 4 x^2 u_3[x]), -\frac{1}{2x} (2 + 2 (-1+x)^2 g_2[x] + 8 (-1+x)^4 g_6[x] + h_1[] + x h_1[] + 4 x u_3[x] - 4 x^2 u_3[x] - 4 x u_5[x] + 8 x^2 u_5[x] - 4 x^3 u_5[x] - 4 x u_3[x] + 4 x^2 u_3[x] + 4 x u_5[x] - 8 x^2 u_5[x] + 4 x^3 u_5[x]) \right\}$$

**Short [E2d =**

$$\left\{ \text{DeleteCases} \left[ \text{Simplify} \left[ \text{E2c} /. \{u_5[x_] :-> u_5[x], g_5[x_] :-> \frac{1}{-1+x} (-1 + 4 g_6[x] - 16 x g_6[x] + 24 x^2 g_6[x] - 16 x^3 g_6[x] + 4 x^4 g_6[x] + 2 g_7[x] - 2 x g_7[x] + g_8[x] - x g_8[x] - x h_1[]), g_2[x_] :-> \frac{1}{2(-1+x)^2} (-2 - 8 g_6[x] + 32 x g_6[x] - 48 x^2 g_6[x] + 32 x^3 g_6[x] - 8 x^4 g_6[x] - h_1[] - x h_1[] - 4 x u_3[x] + 4 x^2 u_3[x] + 4 x u_3[x] - 4 x^2 u_3[x]) \right], 0 \right] // \text{SortBy} [\text{LeafCount}], 9 \right\}$$

{}

EC3 =

$$\begin{aligned}
& \text{EC2} \cup \left\{ \text{ur}_5[x_-] \Rightarrow \text{ul}_5[x], \text{g}_5[x_-] \Rightarrow \frac{1}{-1+x} (-1 + 4 \text{g}_6[x] - 16 x \text{g}_6[x] + 24 x^2 \text{g}_6[x] - 16 x^3 \text{g}_6[x] + \right. \\
& \quad \left. 4 x^4 \text{g}_6[x] + 2 \text{g}_7[x] - 2 x \text{g}_7[x] + \text{g}_8[x] - x \text{g}_8[x] - x \text{h}_1[]), \right. \\
& \quad \left. \text{g}_2[x_-] \Rightarrow \frac{1}{2(-1+x)^2} (-2 - 8 \text{g}_6[x] + 32 x \text{g}_6[x] - 48 x^2 \text{g}_6[x] + 32 x^3 \text{g}_6[x] - \right. \\
& \quad \left. 8 x^4 \text{g}_6[x] - \text{h}_1[] - x \text{h}_1[] - 4 x \text{ul}_3[x] + 4 x^2 \text{ul}_3[x] + 4 x \text{ur}_3[x] - 4 x^2 \text{ur}_3[x]) \right\} \\
& \left\{ \text{f}_1[x_-, z_-] \Rightarrow \frac{1}{2} (-2 \text{f}_6[x, z] + 2 x \text{f}_6[x, z] + \right. \\
& \quad \left. 2 \text{f}_{20}[x, z] - \text{g}_5[x] + 4 \text{g}_6[x] - 12 x \text{g}_6[x] + 12 x^2 \text{g}_6[x] - 4 x^3 \text{g}_6[x] + \text{g}_8[z]), \right. \\
& \quad \text{f}_2[x_-, y_-] \Rightarrow -\text{f}_9[x, y] + y \text{f}_9[x, y] + \text{f}_{26}[x, y] - 2 y \text{f}_{26}[x, y] + y^2 \text{f}_{26}[x, y] + \text{g}_7[x], \\
& \quad \text{f}_3[x_-, z_-] \Rightarrow \text{f}_{18}[x, z] - 2 x \text{f}_{18}[x, z] + x^2 \text{f}_{18}[x, z], \\
& \quad \text{f}_4[x_-, z_-] \Rightarrow -\text{f}_{15}[x, z] - \text{g}_1[z], \\
& \quad \text{f}_5[x_-, y_-] \Rightarrow \text{f}_{26}[x, y] - 2 y \text{f}_{26}[x, y] + y^2 \text{f}_{26}[x, y], \\
& \quad \text{f}_7[x_-, z_-] \Rightarrow \text{f}_6[x, z] - x \text{f}_6[x, z] + \text{g}_9[x] + \text{g}_{10}[z], \\
& \quad \text{f}_8[x_-, z_-] \Rightarrow 0, \text{f}_{10}[x_-, z_-] \Rightarrow 2 (-\text{f}_{18}[x, z] + x \text{f}_{18}[x, z]), \\
& \quad \text{f}_{11}[x_-, y_-] \Rightarrow -2 \text{f}_{18}[x, y] + 4 x \text{f}_{18}[x, y] - 2 x^2 \text{f}_{18}[x, y] - \text{g}_5[x], \text{f}_{12}[x_-, z_-] \Rightarrow 0, \\
& \quad \text{f}_{13}[x_-, z_-] \Rightarrow -\text{f}_{23}[x, z] - \text{g}_2[z], \text{f}_{14}[x_-, z_-] \Rightarrow \frac{-\text{f}_{15}[x, z] + \text{g}_3[z]}{-1+x}, \text{f}_{15}[y_-, z_-] \Rightarrow \frac{1}{y} \\
& \quad \left( 2 - y \text{f}_{23}[y, z] + y z \text{f}_{23}[y, z] - \text{g}_1[y] - y \text{g}_1[z] - y \text{g}_2[z] + y z \text{g}_2[z] - y \text{g}_3[y] - \text{g}_5[y] + y \text{g}_5[y] \right), \\
& \quad \text{f}_{16}[x_-, z_-] \Rightarrow 0, \text{f}_{17}[x_-, z_-] \Rightarrow 2 (-\text{f}_{26}[x, z] + z \text{f}_{26}[x, z]), \\
& \quad \text{f}_{18}[x_-, y_-] \Rightarrow \frac{-\text{f}_{19}[x, y] - 4 \text{g}_6[x] + 8 x \text{g}_6[x] - 4 x^2 \text{g}_6[x]}{2(-1+x)}, \\
& \quad \text{f}_{19}[x_-, y_-] \Rightarrow \frac{1}{(-1+x)(-1+y)} (1 + 2 \text{f}_{20}[x, y] - 2 y \text{f}_{20}[x, y] - \text{g}_5[x] + y \text{g}_5[x] - \text{g}_5[y] + \\
& \quad y \text{g}_5[y] - 4 \text{g}_6[y] + 16 y \text{g}_6[y] - 24 y^2 \text{g}_6[y] + 16 y^3 \text{g}_6[y] - 4 y^4 \text{g}_6[y] + y \text{h}_1[]), \\
& \quad \text{f}_{21}[x_-, z_-] \Rightarrow 0, \text{f}_{22}[x_-, z_-] \Rightarrow \frac{-\text{f}_{23}[x, z] + \text{g}_4[z]}{-1+x}, \text{f}_{24}[x_-, z_-] \Rightarrow 0, \text{f}_{25}[x_-, z_-] \Rightarrow 0, \\
& \quad \text{g}_1[y_-] \Rightarrow -\text{g}_2[y] + y \text{g}_2[y] - \text{g}_3[y] - \text{g}_4[y] + y \text{g}_4[y] - \text{h}_1[], \text{g}_1[x_-, y_-] \Rightarrow -\text{f}_1[x, y], \\
& \quad \text{g}_2[x_-] \Rightarrow \frac{1}{2(-1+x)^2} (-2 - 8 \text{g}_6[x] + 32 x \text{g}_6[x] - 48 x^2 \text{g}_6[x] + 32 x^3 \text{g}_6[x] - 8 x^4 \text{g}_6[x] - \text{h}_1[] - \\
& \quad x \text{h}_1[] - 4 x \text{ul}_3[x] + 4 x^2 \text{ul}_3[x] + 4 x \text{ur}_3[x] - 4 x^2 \text{ur}_3[x]), \text{g}_2[x_-, y_-] \Rightarrow -\text{f}_2[x, y], \\
& \quad \text{g}_3[y_-] \Rightarrow \frac{1}{2(-1+y)} (2 + 2 \text{g}_4[y] - 4 y \text{g}_4[y] + 2 y^2 \text{g}_4[y] - 2 \text{g}_5[y] + 2 y \text{g}_5[y] - 8 \text{g}_6[y] + \\
& \quad 32 y \text{g}_6[y] - 48 y^2 \text{g}_6[y] + 32 y^3 \text{g}_6[y] - 8 y^4 \text{g}_6[y] + \text{h}_1[] + y \text{h}_1[]), \text{g}_3[x_-, y_-] \Rightarrow -\text{f}_3[x, y], \\
& \quad \text{g}_4[x_-, y_-] \Rightarrow -\text{f}_4[x, y], \text{g}_5[x_-] \Rightarrow \frac{1}{-1+x} (-1 + 4 \text{g}_6[x] - 16 x \text{g}_6[x] + 24 x^2 \text{g}_6[x] - 16 x^3 \text{g}_6[x] + \\
& \quad 4 x^4 \text{g}_6[x] + 2 \text{g}_7[x] - 2 x \text{g}_7[x] + \text{g}_8[x] - x \text{g}_8[x] - x \text{h}_1[]), \text{g}_5[x_-, y_-] \Rightarrow -\text{f}_5[x, y], \\
& \quad \text{g}_6[x_-, y_-] \Rightarrow -\text{f}_6[x, y] + \text{f}_8[x, y] - y \text{f}_8[x, y] - \text{f}_{12}[x, y] + y \text{f}_{12}[x, y] + \text{f}_{16}[x, y] - y \text{f}_{16}[x, y], \\
& \quad \text{g}_7[x_-, y_-] \Rightarrow \\
& \quad -\frac{\text{f}_4[x, y]}{x} - \frac{1}{x} (\text{f}_1[x, y] - \text{f}_2[x, y] - \text{f}_3[x, y] - \text{f}_5[x, y] - \text{f}_6[x, y] + x \text{f}_6[x, y] + \text{f}_7[x, y]) + \\
& \quad \frac{(1-y)(-\text{f}_8[x, y] + x \text{f}_8[x, y] + \text{f}_9[x, y])}{x} + \frac{1}{x}
\end{aligned}$$

$$\begin{aligned}
 & (-1 + y) \left( -f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y] \right) + \frac{(1 - y) \left( -f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y] \right)}{x}, \\
 g_8[x_-, y_-] & \Rightarrow -x f_8[x, y], \quad g_9[x_-, y_-] \Rightarrow f_8[x, y] - x f_8[x, y] - f_9[x, y], \quad g_{10}[y_-] \Rightarrow \\
 & \frac{1}{2(1 - 2y + y^2)} \left( 1 + g_5[y] - 3y g_5[y] + 2y^2 g_5[y] - 4g_6[y] + 16y g_6[y] - 24y^2 g_6[y] + 16y^3 g_6[y] - \right. \\
 & \quad \left. 4y^4 g_6[y] - 2y g_7[y] + 2y^2 g_7[y] - g_8[y] + y g_8[y] - 2g_9[y] + 4y g_9[y] - 2y^2 g_9[y] + y h_1[] \right), \\
 g_{10}[x_-, y_-] & \Rightarrow -f_{10}[x, y] + f_{12}[x, y] - y f_{12}[x, y], \quad g_{11}[x_-, y_-] \Rightarrow \\
 & -\frac{1}{x} \left( 2f_3[x, y] - f_4[x, y] - f_{10}[x, y] + x f_{10}[x, y] + f_{11}[x, y] \right) + \\
 & \frac{(1 - y) \left( -f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y] \right)}{x}, \\
 g_{12}[x_-, y_-] & \Rightarrow -x f_{12}[x, y], \quad g_{13}[x_-, y_-] \Rightarrow f_{12}[x, y] - x f_{12}[x, y] - f_{13}[x, y], \\
 g_{14}[x_-, y_-] & \Rightarrow -f_{14}[x, y] + f_{16}[x, y] - y f_{16}[x, y], \\
 g_{15}[x_-, y_-] & \Rightarrow -\frac{1}{x} \left( f_4[x, y] - 2f_5[x, y] - f_{14}[x, y] + x f_{14}[x, y] + f_{15}[x, y] \right) + \\
 & \frac{(1 - y) \left( -f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y] \right)}{x}, \\
 g_{16}[x_-, y_-] & \Rightarrow -x f_{16}[x, y], \quad g_{17}[x_-, y_-] \Rightarrow f_{16}[x, y] - x f_{16}[x, y] - f_{17}[x, y], \\
 g_{18}[x_-, y_-] & \Rightarrow -f_{18}[x, y] + f_{21}[x, y] - y f_{21}[x, y] + (-1 + 2y - y^2) f_{24}[x, y], \\
 g_{19}[x_-, y_-] & \Rightarrow -\frac{1}{x} \left( f_{10}[x, y] - f_{12}[x, y] + y f_{12}[x, y] - f_{14}[x, y] + f_{16}[x, y] - y f_{16}[x, y] - \right. \\
 & \quad \left. 2f_{18}[x, y] + 2x f_{18}[x, y] + f_{19}[x, y] + 2f_{21}[x, y] - 2x f_{21}[x, y] - 2y f_{21}[x, y] + \right. \\
 & \quad \left. 2xy f_{21}[x, y] - f_{22}[x, y] + y f_{22}[x, y] - 2f_{24}[x, y] + 2x f_{24}[x, y] + 4y f_{24}[x, y] - \right. \\
 & \quad \left. 4xy f_{24}[x, y] - 2y^2 f_{24}[x, y] + 2xy^2 f_{24}[x, y] + f_{25}[x, y] - 2y f_{25}[x, y] + y^2 f_{25}[x, y] \right), \\
 g_{20}[x_-, y_-] & \Rightarrow \frac{f_4[x, y]}{x^2} + \frac{(1 - y) \left( -f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y] \right)}{x^2} + \frac{1}{x^2} \\
 & (-1 + y) \left( -f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y] \right) - \\
 & \frac{1}{x^2} \left( f_3[x, y] + f_5[x, y] - f_{10}[x, y] + x f_{10}[x, y] + f_{11}[x, y] + f_{14}[x, y] - x f_{14}[x, y] - f_{15}[x, y] + \right. \\
 & \quad \left. f_{18}[x, y] - 2x f_{18}[x, y] + x^2 f_{18}[x, y] - f_{19}[x, y] + x f_{19}[x, y] + f_{20}[x, y] \right) + \frac{1}{x^2} \\
 & (1 - y) \left( f_{21}[x, y] - 2x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y] \right) + \\
 & \frac{1}{x^2} (-1 + 2y - y^2) \left( f_{24}[x, y] - 2x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y] \right), \\
 g_{21}[x_-, y_-] & \Rightarrow -x f_{21}[x, y] + 2x f_{24}[x, y] - 2xy f_{24}[x, y], \\
 g_{22}[x_-, y_-] & \Rightarrow -f_{12}[x, y] + f_{16}[x, y] + 2f_{21}[x, y] - 2x f_{21}[x, y] - f_{22}[x, y] - \\
 & 4f_{24}[x, y] + 4x f_{24}[x, y] + 4y f_{24}[x, y] - 4xy f_{24}[x, y] + 2f_{25}[x, y] - 2y f_{25}[x, y], \\
 g_{23}[x_-, y_-] & \Rightarrow -\frac{-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y]}{x} + \frac{-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y]}{x} - \\
 & \frac{1}{x} \left( f_{21}[x, y] - 2x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y] \right) + \\
 & \left( \frac{2}{x} - \frac{2y}{x} \right) \left( f_{24}[x, y] - 2x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y] \right), \\
 g_{24}[x_-, y_-] & \Rightarrow -x^2 f_{24}[x, y], \quad g_{25}[x_-, y_-] \Rightarrow 2x f_{24}[x, y] - 2x^2 f_{24}[x, y] - x f_{25}[x, y], \\
 g_{26}[x_-, y_-] & \Rightarrow -f_{24}[x, y] + 2x f_{24}[x, y] - x^2 f_{24}[x, y] + f_{25}[x, y] - x f_{25}[x, y] - f_{26}[x, y], \\
 ul_1[x_-] & \Rightarrow -h_1[] - ul_3[x] + x ul_3[x] + ul_5[x] - 2x ul_5[x] + \\
 & x^2 ul_5[x] + ur_1[x] + ur_3[x] - x ur_3[x] - ur_5[x] + 2x ur_5[x] - x^2 ur_5[x],
 \end{aligned}$$

$$\begin{aligned} ul_2[x_] &\Rightarrow ul_5[x] - 2x ul_5[x] + x^2 ul_5[x] + ur_2[x] - ur_5[x] + 2x ur_5[x] - x^2 ur_5[x], \\ ul_4[x_] &\Rightarrow -2 ul_5[x] + 2x ul_5[x] + ur_4[x] + 2 ur_5[x] - 2x ur_5[x], \quad ur_5[x_] \Rightarrow ul_5[x] \end{aligned}$$

`Simplify[(kpr[1] kmr[2] // m[1, 2, 1]) // . EC3]`

`E[1, 0, 0, 0]`

`Simplify[(kpr[1] kmr[2] // m[2, 1, 1]) // . EC3]`

`E[1, 0, 0, 0]`

`Simplify[(kpl[1] kml[2] // m[1, 2, 1]) // . EC3]`

`E[1, 0, 0, 0]`

`Simplify[(kpl[1] kml[2] // m[2, 1, 1]) // . EC3]`

`E[1, 0, 0, 0]`

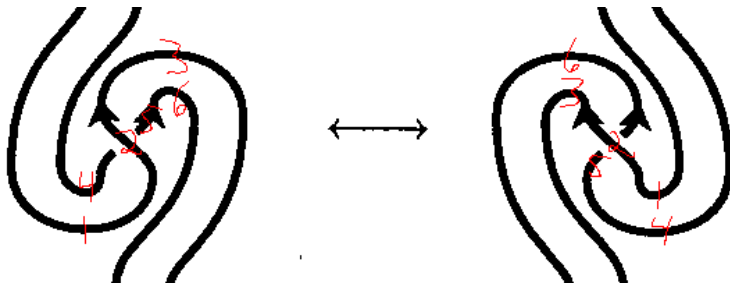
`Simplify[kpr[1] == kpl[1] // . EC3]`

`True`

`Simplify[kmr[1] == kml[1] // . EC3]`

`True`

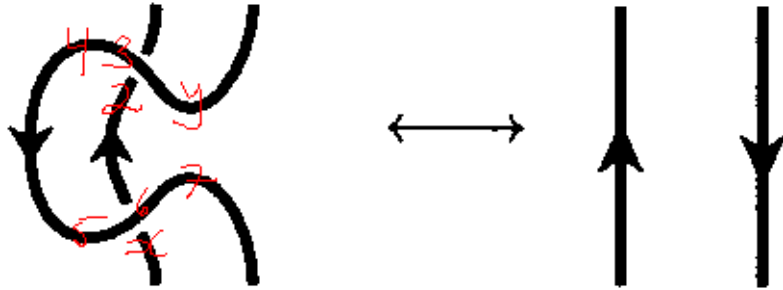
### The Swirl Equation



```
Simplify[
(ur[x] Rm[2, 5] nr[3] ur[y] nr[6] // m[x, 2, x] // m[x, 3, x] // m[y, 5, y] // m[y, 6, y]) ==
(ul[x] Rm[2, 5] nl[3] ul[y] nl[6] // m[x, 2, x] // m[x, 3, x] // m[y, 5, y] //
m[y, 6, y]) // . EC3]
```

`True`

### The Cyclic R2 Equation



```
E3a = Simplify[
  Last[Rp[6, x] Rm[3, 2] ul[y] nl[4] ur[5] nr[7] // m[x, 2, x] // m[y, 3, y] // m[y, 4, y] //
    m[y, 5, y] // m[y, 6, y] // m[y, 7, y]] // EC3]
0
```

## Finding The “Best” Solution

$$\text{BestEC} = \text{EC3} \cup \{f_6[\_] \rightarrow 0, f_9[\_] \rightarrow 0, f_{26}[\_] \rightarrow 0, f_{23}[x, y] \Rightarrow g_4[y] + (x-1) h_1[x, y],$$

$$h_1[\_] \rightarrow -1, f_{20}[x, y] \Rightarrow x - \frac{x^2}{2} - \frac{1}{2} \left( -8(-1+x)^3 g_6[x] + 2g_7[x] + g_8[x] + g_8[y] \right),$$

$$g_7[\_] \Rightarrow 0, ul_3[x] \Rightarrow \frac{-x}{4} + ur_3[x], g_{4|6|8|9}[\_] \Rightarrow 0, h_1[\_, \_] \Rightarrow 0,$$

$$ul_5[\_] \Rightarrow 0, ur_{2|4}[\_] \Rightarrow 0, ur_3[x] \Rightarrow \frac{x}{8}, ur_1[x] \Rightarrow \frac{x^2}{4}\};$$

```
FromCoefficientRules[
  CoefficientRules[Simplify[Rp[1, 2] // BestEC /. {t1 -> x, t2 -> y}] // Last,
    {c1, u1, w1, c2, u2, w2}] /. {(e -> c_) => (e -> Simplify[c])},
  {c1, u1, w1, c2, u2, w2}
]
-1/2 (-1+x)^2 c1 - 1/2 (-1+x)^2 c1^2 + 1/2 (-2-x+y^2) c1 c2 + (1-x) c1 u1 w1 - 1/2 c2 u1 w1 - 1/2 u1^2 w1^2 + u1 w2 +
(-2+x) x c1 u1 w2 + 1/2 x c2 u1 w2 + 1/2 (1+y) c1 u2 w2 + (-1+x) u1^2 w1 w2 - 1/2 (-2+x) x u1^2 w2^2
```

```
FromCoefficientRules[
  CoefficientRules[Simplify[ur[1] // BestEC /. {t1 -> x, t2 -> y}] // Last,
    {c1, u1, w1, c2, u2, w2}] /. {(e -> c_) => (e -> FullSimplify[c])},
  {c1, u1, w1, c2, u2, w2}
]
x c1 / 4 + u1 w1 / 8
```



```

FromCoefficientRules [
CoefficientRules[Simplify[ul[1] //. BestEC /. {t1 -> x, t2 -> y}] // Last,
  {c1, u1, w1, c2, u2, w2}] /. {(e_ -> c_) := (e -> FullSimplify[c])},
  {c1, u1, w1, c2, u2, w2}
]

$$\frac{1}{4} x (4 + x) c_1 - \frac{1}{8} x^2 u_1 w_1$$


```

```

FromCoefficientRules [
CoefficientRules[Simplify[Rm[1, 2] //. BestEC /. {t1 -> x, t2 -> y}] // Last,
  {c1, u1, w1, c2, u2, w2}] /. {(e_ -> c_) := (e -> Simplify[c])},
  {c1, u1, w1, c2, u2, w2}
]

$$\frac{1}{2} (-1 + x)^2 c_1 + \frac{1}{2} (-1 + x)^2 c_1^2 + \frac{1}{2} (2 + x - y^2) c_1 c_2 + (-1 + x) c_1 u_1 w_1 +$$


$$\frac{1}{2} c_2 u_1 w_1 + \frac{1}{2} u_1^2 w_1^2 + \frac{(-1 + x) u_1 w_2}{2 x} + \left(-\frac{5}{2} + \frac{3}{2 x} + x\right) c_1 u_1 w_2 + \frac{(1 + x - y^2) c_2 u_1 w_2}{2 x} +$$


$$\frac{1}{2} (-1 - y) c_1 u_2 w_2 + \left(1 - \frac{3}{2 x}\right) u_1^2 w_1 w_2 + \frac{(2 - 3 x + x^2) u_1^2 w_2^2}{2 x^2} - \frac{(1 + y) u_1 u_2 w_2^2}{2 x}$$


```

### BestEC

```

{f6[___] -> 0, f9[___] -> 0, f26[___] -> 0, h1[] -> -1,
f1[x_, z_] :=  $\frac{1}{2} (-2 f_6[x, z] + 2 x f_6[x, z] + 2 f_{20}[x, z] -$ 
   $g_5[x] + 4 g_6[x] - 12 x g_6[x] + 12 x^2 g_6[x] - 4 x^3 g_6[x] + g_8[z])$ ,
f2[x_, y_] :=  $-f_9[x, y] + y f_9[x, y] + f_{26}[x, y] - 2 y f_{26}[x, y] + y^2 f_{26}[x, y] + g_7[x]$ ,
f3[x_, z_] :=  $f_{18}[x, z] - 2 x f_{18}[x, z] + x^2 f_{18}[x, z]$ ,
f4[x_, z_] :=  $-f_{15}[x, z] - g_1[z]$ , f5[x_, y_] :=  $f_{26}[x, y] - 2 y f_{26}[x, y] + y^2 f_{26}[x, y]$ ,
f7[x_, z_] :=  $f_6[x, z] - x f_6[x, z] + g_9[x] + g_{10}[z]$ ,
f8[x_, z_] := 0, f10[x_, z_] :=  $2 (-f_{18}[x, z] + x f_{18}[x, z])$ ,
f11[x_, y_] :=  $-2 f_{18}[x, y] + 4 x f_{18}[x, y] - 2 x^2 f_{18}[x, y] - g_5[x]$ , f12[x_, z_] := 0,
f13[x_, z_] :=  $-f_{23}[x, z] - g_2[z]$ , f14[x_, z_] :=  $\frac{-f_{15}[x, z] + g_3[z]}{-1 + x}$ , f15[y_, z_] :=  $\frac{1}{y}$ 
   $(-2 y f_{23}[y, z] + y z f_{23}[y, z] - g_1[y] - y g_1[z] - y g_2[z] + y z g_2[z] - y g_3[y] - g_5[y] + y g_5[y])$ ,
f16[x_, z_] := 0, f17[x_, z_] :=  $2 (-f_{26}[x, z] + z f_{26}[x, z])$ ,
f18[x_, y_] :=  $\frac{-f_{19}[x, y] - 4 g_6[x] + 8 x g_6[x] - 4 x^2 g_6[x]}{2 (-1 + x)}$ ,
f19[x_, y_] :=  $\frac{1}{(-1 + x) (-1 + y)} (1 + 2 f_{20}[x, y] - 2 y f_{20}[x, y] - g_5[x] + y g_5[x] - g_5[y] +$ 
   $y g_5[y] - 4 g_6[y] + 16 y g_6[y] - 24 y^2 g_6[y] + 16 y^3 g_6[y] - 4 y^4 g_6[y] + y h_1[])$ ,
f20[x_, y_] :=  $x - \frac{x^2}{2} - \frac{1}{2} (-8 (-1 + x)^3 g_6[x] + 2 g_7[x] + g_8[x] + g_8[y])$ ,
f21[x_, z_] := 0, f22[x_, z_] :=  $\frac{-f_{23}[x, z] + g_4[z]}{-1 + x}$ ,
f23[x_, y_] :=  $g_4[y] + (x - 1) h_1[x, y]$ , f24[x_, z_] := 0, f25[x_, z_] := 0,
g1[y_] :=  $-g_2[y] + y g_2[y] - g_3[y] - g_4[y] + y g_4[y] - h_1[]$ , g1[x_, y_] :=  $-f_1[x, y]$ ,

```

$$\begin{aligned}
g_2[x_-] &\Rightarrow \frac{1}{2(-1+x)^2} \left( -2 - 8g_6[x] + 32xg_6[x] - 48x^2g_6[x] + 32x^3g_6[x] - 8x^4g_6[x] - h_1[] - \right. \\
&\quad \left. xh_1[] - 4xul_3[x] + 4x^2ul_3[x] + 4xur_3[x] - 4x^2ur_3[x] \right), \quad g_2[x_-, y_-] \Rightarrow -f_2[x, y], \\
g_3[y_-] &\Rightarrow \frac{1}{2(-1+y)} \left( 2 + 2g_4[y] - 4yg_4[y] + 2y^2g_4[y] - 2g_5[y] + 2yg_5[y] - 8g_6[y] + \right. \\
&\quad \left. 32yg_6[y] - 48y^2g_6[y] + 32y^3g_6[y] - 8y^4g_6[y] + h_1[] + yh_1[] \right), \quad g_3[x_-, y_-] \Rightarrow -f_3[x, y], \\
g_4[x_-, y_-] &\Rightarrow -f_4[x, y], \quad g_5[x_-] \Rightarrow \frac{1}{-1+x} \left( -1 + 4g_6[x] - 16xg_6[x] + 24x^2g_6[x] - 16x^3g_6[x] + \right. \\
&\quad \left. 4x^4g_6[x] + 2g_7[x] - 2xg_7[x] + g_8[x] - xg_8[x] - xh_1[] \right), \quad g_5[x_-, y_-] \Rightarrow -f_5[x, y], \\
g_6[x_-, y_-] &\Rightarrow -f_6[x, y] + f_8[x, y] - yg_8[x, y] - f_{12}[x, y] + yf_{12}[x, y] + f_{16}[x, y] - yf_{16}[x, y], \\
g_7[-] &\Rightarrow 0, \quad g_7[x_-, y_-] \Rightarrow \\
&\quad -\frac{f_4[x, y]}{x} - \frac{1}{x} \left( f_1[x, y] - f_2[x, y] - f_3[x, y] - f_5[x, y] - f_6[x, y] + xf_6[x, y] + f_7[x, y] \right) + \\
&\quad \frac{(1-y)(-f_8[x, y] + xf_8[x, y] + f_9[x, y])}{x} + \frac{1}{x} \\
&\quad (-1+y)(-f_{12}[x, y] + xf_{12}[x, y] + f_{13}[x, y]) + \frac{(1-y)(-f_{16}[x, y] + xf_{16}[x, y] + f_{17}[x, y])}{x}, \\
g_8[x_-, y_-] &\Rightarrow -xf_8[x, y], \quad g_9[x_-, y_-] \Rightarrow f_8[x, y] - xf_8[x, y] - f_9[x, y], \quad g_{10}[y_-] \Rightarrow \\
&\quad \frac{1}{2(1-2y+y^2)} \left( 1 + g_5[y] - 3yg_5[y] + 2y^2g_5[y] - 4g_6[y] + 16yg_6[y] - 24y^2g_6[y] + 16y^3g_6[y] - \right. \\
&\quad \left. 4y^4g_6[y] - 2yg_7[y] + 2y^2g_7[y] - g_8[y] + yg_8[y] - 2g_9[y] + 4yg_9[y] - 2y^2g_9[y] + yh_1[] \right), \\
g_{10}[x_-, y_-] &\Rightarrow -f_{10}[x, y] + f_{12}[x, y] - yf_{12}[x, y], \quad g_{11}[x_-, y_-] \Rightarrow \\
&\quad -\frac{1}{x} \left( 2f_3[x, y] - f_4[x, y] - f_{10}[x, y] + xf_{10}[x, y] + f_{11}[x, y] \right) + \\
&\quad \frac{(1-y)(-f_{12}[x, y] + xf_{12}[x, y] + f_{13}[x, y])}{x}, \\
g_{12}[x_-, y_-] &\Rightarrow -xf_{12}[x, y], \quad g_{13}[x_-, y_-] \Rightarrow f_{12}[x, y] - xf_{12}[x, y] - f_{13}[x, y], \\
g_{14}[x_-, y_-] &\Rightarrow -f_{14}[x, y] + f_{16}[x, y] - yf_{16}[x, y], \\
g_{15}[x_-, y_-] &\Rightarrow -\frac{1}{x} \left( f_4[x, y] - 2f_5[x, y] - f_{14}[x, y] + xf_{14}[x, y] + f_{15}[x, y] \right) + \\
&\quad \frac{(1-y)(-f_{16}[x, y] + xf_{16}[x, y] + f_{17}[x, y])}{x}, \\
g_{16}[x_-, y_-] &\Rightarrow -xf_{16}[x, y], \quad g_{17}[x_-, y_-] \Rightarrow f_{16}[x, y] - xf_{16}[x, y] - f_{17}[x, y], \\
g_{18}[x_-, y_-] &\Rightarrow -f_{18}[x, y] + f_{21}[x, y] - yf_{21}[x, y] + (-1+2y-y^2)f_{24}[x, y], \\
g_{19}[x_-, y_-] &\Rightarrow -\frac{1}{x} \left( f_{10}[x, y] - f_{12}[x, y] + yf_{12}[x, y] - f_{14}[x, y] + f_{16}[x, y] - yf_{16}[x, y] - \right. \\
&\quad \left. 2f_{18}[x, y] + 2xf_{18}[x, y] + f_{19}[x, y] + 2f_{21}[x, y] - 2xf_{21}[x, y] - 2yf_{21}[x, y] + \right. \\
&\quad \left. 2xyf_{21}[x, y] - f_{22}[x, y] + yf_{22}[x, y] - 2f_{24}[x, y] + 2xf_{24}[x, y] + 4yf_{24}[x, y] - \right. \\
&\quad \left. 4xyf_{24}[x, y] - 2y^2f_{24}[x, y] + 2xy^2f_{24}[x, y] + f_{25}[x, y] - 2yf_{25}[x, y] + y^2f_{25}[x, y] \right), \\
g_{20}[x_-, y_-] &\Rightarrow \frac{f_4[x, y]}{x^2} + \frac{(1-y)(-f_{12}[x, y] + xf_{12}[x, y] + f_{13}[x, y])}{x^2} + \frac{1}{x^2} \\
&\quad (-1+y)(-f_{16}[x, y] + xf_{16}[x, y] + f_{17}[x, y]) - \\
&\quad \frac{1}{x^2} \left( f_3[x, y] + f_5[x, y] - f_{10}[x, y] + xf_{10}[x, y] + f_{11}[x, y] + f_{14}[x, y] - xf_{14}[x, y] - f_{15}[x, y] + \right. \\
&\quad \left. f_{18}[x, y] - 2xf_{18}[x, y] + x^2f_{18}[x, y] - f_{19}[x, y] + xf_{19}[x, y] + f_{20}[x, y] \right) + \frac{1}{x^2}
\end{aligned}$$

$$\begin{aligned}
& (1-y) (f_{21}[x, y] - 2x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y]) + \\
& \frac{1}{x^2} (-1 + 2y - y^2) (f_{24}[x, y] - 2x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y]), \\
g_{21}[x_, y_] & \Rightarrow -x f_{21}[x, y] + 2x f_{24}[x, y] - 2xy f_{24}[x, y], \\
g_{22}[x_, y_] & \Rightarrow -f_{12}[x, y] + f_{16}[x, y] + 2f_{21}[x, y] - 2x f_{21}[x, y] - f_{22}[x, y] - \\
& 4f_{24}[x, y] + 4x f_{24}[x, y] + 4y f_{24}[x, y] - 4xy f_{24}[x, y] + 2f_{25}[x, y] - 2y f_{25}[x, y], \\
g_{23}[x_, y_] & \Rightarrow -\frac{-f_{12}[x, y] + x f_{12}[x, y] + f_{13}[x, y]}{x} + \frac{-f_{16}[x, y] + x f_{16}[x, y] + f_{17}[x, y]}{x} - \\
& \frac{1}{x} (f_{21}[x, y] - 2x f_{21}[x, y] + x^2 f_{21}[x, y] - f_{22}[x, y] + x f_{22}[x, y] + f_{23}[x, y]) + \\
& \left(\frac{2}{x} - \frac{2y}{x}\right) (f_{24}[x, y] - 2x f_{24}[x, y] + x^2 f_{24}[x, y] - f_{25}[x, y] + x f_{25}[x, y] + f_{26}[x, y]), \\
g_{24}[x_, y_] & \Rightarrow -x^2 f_{24}[x, y], \quad g_{25}[x_, y_] \Rightarrow 2x f_{24}[x, y] - 2x^2 f_{24}[x, y] - x f_{25}[x, y], \\
g_{26}[x_, y_] & \Rightarrow -f_{24}[x, y] + 2x f_{24}[x, y] - x^2 f_{24}[x, y] + f_{25}[x, y] - x f_{25}[x, y] - f_{26}[x, y], \\
g_{4|6|8|9}[_] & \Rightarrow \theta, \quad h_{1}[_] \Rightarrow \theta, \\
ul_1[x_] & \Rightarrow -h_1[_] - ul_3[x] + x ul_3[x] + ul_5[x] - 2x ul_5[x] + \\
& x^2 ul_5[x] + ur_1[x] + ur_3[x] - x ur_3[x] - ur_5[x] + 2x ur_5[x] - x^2 ur_5[x], \\
ul_2[x_] & \Rightarrow ul_5[x] - 2x ul_5[x] + x^2 ul_5[x] + ur_2[x] - ur_5[x] + 2x ur_5[x] - x^2 ur_5[x], \\
ul_3[x_] & \Rightarrow -\frac{x}{4} + ur_3[x], \quad ul_4[x_] \Rightarrow -2 ul_5[x] + 2x ul_5[x] + ur_4[x] + 2 ur_5[x] - 2x ur_5[x], \\
ul_5[_] & \Rightarrow \theta, \quad ur_1[x_] \Rightarrow \frac{x^2}{4}, \quad ur_3[x_] \Rightarrow \frac{x}{8}, \quad ur_5[x_] \Rightarrow ul_5[x], \quad ur_{2|4}[_] \Rightarrow \theta
\end{aligned}$$

## The “Best” Solution

```

ReplacePart[Rp[i, j], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[Rp[i, j] //. BestEC] // Last, {c_i, u_i, w_i, c_j, u_j, w_j}] /.
  {(e_ -> c_) -> (e -> Simplify[c])},
  {c_i, u_i, w_i, c_j, u_j, w_j}]]]

```

$$\begin{aligned}
& E[1, b_i c_j, u_i w_j, -\frac{1}{2} c_i (-1 + t_i)^2 - \frac{1}{2} c_i^2 (-1 + t_i)^2 + \\
& \frac{1}{2} c_i c_j (-2 - t_i + t_j^2) - \frac{1}{2} c_j u_i w_i + c_i (1 - t_i) u_i w_i - \frac{1}{2} u_i^2 w_i^2 + u_i w_j + \frac{1}{2} c_j t_i u_i w_j + \\
& c_i (-2 + t_i) t_i u_i w_j + \frac{1}{2} c_i (1 + t_j) u_j w_j + (-1 + t_i) u_i^2 w_i w_j - \frac{1}{2} (-2 + t_i) t_i u_i^2 w_j^2]
\end{aligned}$$

```

ReplacePart[Rm[i, j], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[Rm[i, j] //. BestEC] // Last, {c_i, u_i, w_i, c_j, u_j, w_j}] /.
  {(e_ -> c_) -> (e -> Simplify[c])},
  {c_i, u_i, w_i, c_j, u_j, w_j}]]]

```

$$\begin{aligned}
& E[1, -b_i c_j, -\frac{u_i w_j}{t_i}, \frac{1}{2} c_i (-1 + t_i)^2 + \frac{1}{2} c_i^2 (-1 + t_i)^2 + \frac{1}{2} c_i c_j (2 + t_i - t_j^2) + \frac{1}{2} c_j u_i w_i + \\
& c_i (-1 + t_i) u_i w_i + \frac{1}{2} u_i^2 w_i^2 + \left(\frac{1}{2} - \frac{1}{2 t_i}\right) u_i w_j + c_i \left(-\frac{5}{2} + \frac{3}{2 t_i} + t_i\right) u_i w_j + \frac{c_j (1 + t_i - t_j^2) u_i w_j}{2 t_i} + \\
& \frac{1}{2} c_i (-1 - t_j) u_j w_j + \left(1 - \frac{3}{2 t_i}\right) u_i^2 w_i w_j + \left(\frac{1}{2} + \frac{1}{t_i^2} - \frac{3}{2 t_i}\right) u_i^2 w_j^2 - \frac{(1 + t_j) u_i u_j w_j^2}{2 t_i}]
\end{aligned}$$

```

ReplacePart[ur[i], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[ur[i] //. BestEC] // Last, {c_i, u_i, w_i}] /.
  {(e_ -> c_) :-> (e -> Simplify[c])},
  {c_i,
   u_i,
   w_i}]]]

```

$$\mathbb{E}\left[\frac{1}{t_i^{1/4}}, 0, 0, \frac{c_i t_i}{4} + \frac{u_i w_i}{8}\right]$$

```

ReplacePart[nr[i], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[nr[i] //. BestEC] // Last, {c_i, u_i, w_i}] /.
  {(e_ -> c_) :-> (e -> Simplify[c])},
  {c_i,
   u_i,
   w_i}]]]

```

$$\mathbb{E}\left[t_i^{1/4}, 0, 0, -\frac{1}{4} c_i t_i^3 - \frac{1}{8} t_i^2 u_i w_i\right]$$

```

ReplacePart[ul[i], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[ul[i] //. BestEC] // Last, {c_i, u_i, w_i}] /.
  {(e_ -> c_) :-> (e -> Simplify[c])},
  {c_i,
   u_i,
   w_i}]]]

```

$$\mathbb{E}\left[t_i^{1/4}, 0, 0, \frac{1}{4} c_i t_i (4 + t_i) - \frac{1}{8} t_i^2 u_i w_i\right]$$

```

ReplacePart[nl[i], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[nl[i] //. BestEC] // Last, {c_i, u_i, w_i}] /.
  {(e_ -> c_) :-> (e -> Simplify[c])},
  {c_i,
   u_i,
   w_i}]]]

```

$$\mathbb{E}\left[\frac{1}{t_i^{1/4}}, 0, 0, c_i \left(-\frac{1}{4} - \frac{1}{t_i}\right) + \frac{u_i w_i}{8}\right]$$

```

ReplacePart[kpr[i], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[kpr[i] //. BestEC] // Last, {c_i, u_i, w_i}] /.
  {(e_ -> c_) :-> (e -> Simplify[c])},
  {c_i,
   u_i,
   w_i}]]]

```

$$\mathbb{E}\left[\sqrt{t_i}, b_i c_i, \frac{u_i w_i}{\sqrt{t_i}}, \frac{1}{2} c_i^2 (-3 + t_i) t_i^2 + \frac{1}{4} c_i t_i^2 (2 + 5 t_i - 3 t_i^2) + c_i t_i u_i w_i - \frac{3}{4} t_i^2 u_i w_i\right]$$

```

ReplacePart[kmr[i], 4 -> FromCoefficientRules[
  CoefficientRules[Simplify[kmr[i] //. BestEC] // Last, {ci, ui, wi}] /.
  {(e- -> c-) -> (e -> Simplify[c])},
  {ci,
   ui,
   wi}]]]

```

$$E\left[\frac{1}{\sqrt{t_i}}, -b_i c_i, -\frac{u_i w_i}{\sqrt{t_i}}, \frac{1}{4} c_i \left(3 - \frac{2}{t_i^2} - \frac{5}{t_i}\right) - \frac{c_i^2 (-3 + t_i)}{2 t_i^2} + \left(\frac{3}{4} - \frac{2}{t_i^2}\right) u_i w_i + \frac{c_i (2 - t_i) u_i w_i}{t_i^2} - \frac{(-1 + t_i) u_i^2 w_i^2}{2 t_i^2}\right]$$