Pensieve header: The OneCo propject using the (b-ϵc)-scapegoated low algebra.

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ϵ /: ϵ^2 = 0;
PBWBasis = {c, u, w};
FullPBWBasis = PBWBasis ⋃ (#_ & /@ PBWBasis) ⋃ {ϵ};
IndexedScalars = {b};
b[c, w] = -(b[w, c] = w);
b[u, c] = -(b[c, u] = u);
b[w, u] = -(b[u, w] = b - 2 ϵ c);

b[x_, x_] = 0;
b[(x_)_i_, (y_)_i_] :=
  b[x_i, y_i] = b[x, y] /. Table[z → z_i, {z, PBWBasis ⋃ IndexedScalars}];
b[(x_)_i_, (y_)_j_] /; i =!= j := 0;
b[x_, y_] := x ** y - y ** x;

Simp[ℰ_] := Collect[ℰ, FullPBWBasis, Simplify]

ScalarQ = FreeQ[Alternatives @@ PBWBasis];
AtomicQ = MatchQ[Alternatives @@ Flatten[{#, #_} & /@ PBWBasis]];
DecomposableQ[x_Plus] = False; DecomposableQ[x_] := ! (ScalarQ[x] ∨ AtomicQ[x]);
Type[(x_)_] := Type[x];
Type /: Type[x_] < Type[y_] :=
  Position[PBWBasis, x]⟦1, 1⟧ < Position[PBWBasis, y]⟦1, 1⟧;
Type /: Type[x_] ≤ Type[y_] :=
  Position[PBWBasis, x]⟦1, 1⟧ ≤ Position[PBWBasis, y]⟦1, 1⟧;
CleaveLeft = Replace[Flatten[{
     {{x_. * #^n_. :> {#, x #^(n-1)}, x_. * #_i_^n_. :> {#_i, x #_i^(n-1)}} & /@ PBWBasis},
     x_ :> {x, 1} }]];
CleaveRight = Replace[Flatten[{
     {{x_. * #^n_. :> {x #^(n-1), #}, x_. * #_i_^n_. :> {x #_i^(n-1), #_i}} & /@ Reverse@PBWBasis},
     x_ :> {1, x} }]];

Unprotect[NonCommutativeMultiply];
0 ** _ = _ ** 0 = 0;
x_ ** 1 := x; 1 ** x_ := x;
(x_Plus) ** y_ := (# ** y) & /@ x;
x_ ** (y_Plus) := (x ** #) & /@ y;
x_ ** y_ /; ScalarQ[x] ∨ ScalarQ[y] := x * y;
x_ ** y_ /; AtomicQ[x] ⋀ AtomicQ[y] := x * y + If[Type[x] < Type[y], 0, b[x, y]];
x_ ** y_ /; AtomicQ[x] ⋀ DecomposableQ[y] := (
   CleaveLeft[y] /. {y1_, y2_} :> If[Type[x] ≤ Type[y1], x * y, (x ** y1) ** y2]);
x_ ** y_ /; DecomposableQ[x] := (CleaveRight[x] /. {x1_, x2_} :> x1 ** (x2 ** y));
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LBasis[n_Integer] := LBasis[Range[n]];
LBasis[S_] := Module[{i, j, k, l}, SortBy[(# /. {ϵ → 2, c_ → 2, u_ → 2, w_ → 2}) &][
   Union@Flatten[{{1, ϵ},
      Table[{c_i, u_i, w_i, ϵ c_i, ϵ u_i, ϵ w_i}, {i, S}],
      Table[{u_i w_j, ϵ u_i w_j, ϵ c_i c_j, ϵ c_i u_j, ϵ c_i w_j}, {i, S}, {j, S}],
      Table[{ϵ c_i u_j w_k, ϵ u_i u_j w_k, ϵ u_i w_j w_k}, {i, S}, {j, S}, {k, S}],
      Table[ϵ u_i u_j w_k w_l, {i, S}, {j, S}, {k, S}, {l, S}] }]
  ]]

a_{i_,j_} := b_i c_j - ϵ c_i c_j + u_i w_j;
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R0[___][y_] /; ScalarQ[y] := y;
R0[x___][y_Plus] := R0[x] /@ y;
R0[τ_, i_, j_][c_i_] :=
  c_i - (ⅇ^(-τ b_i) (-1 + ⅇ^(τ b_i)) u_i w_j)/b_i + (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) - τ b_i) u_i w_j)/b_i^2 - (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) - τ b_i) c_i u_i w_j)/b_i^2 +
   (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) - τ b_i) c_j u_i w_j)/b_i^2 + (ⅇ^(-2 τ b_i) ϵ (-1 + ⅇ^(2 τ b_i) - 2 ⅇ^(τ b_i) τ b_i) u_i^2 w_j^2)/b_i^3;
R0[τ_, i_, j_][c_j_] := c_j + (ⅇ^(-τ b_i) (-1 + ⅇ^(τ b_i)) u_i w_j)/b_i - (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) - τ b_i) u_i w_j)/b_i^2 +
   (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) - τ b_i) c_i u_i w_j)/b_i^2 - (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) - τ b_i) c_j u_i w_j)/b_i^2 -
   (ⅇ^(-2 τ b_i) ϵ (-1 + ⅇ^(2 τ b_i) - 2 ⅇ^(τ b_i) τ b_i) u_i^2 w_j^2)/b_i^3;
R0[τ_, i_, j_][c_k_] /; (k =!= i) ∧ (k =!= j) := c_k;
R0[τ_, i_, j_][u_i_] := u_i - ϵ τ c_j u_i - (ⅇ^(-τ b_i) ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) u_i^2 w_j)/b_i^2;
R0[τ_, i_, j_][u_j_] :=
  -((-1 + ⅇ^(τ b_i)) b_j u_i)/b_i - (ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) b_j u_i)/b_i^2 + (ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) b_j c_i u_i)/b_i^2 +
   (ϵ (-2 b_i + 2 ⅇ^(τ b_i) b_i - b_j + ⅇ^(τ b_i) b_j - τ b_i b_j) c_j u_i)/b_i^2 + ⅇ^(τ b_i) u_j - ⅇ^(τ b_i) ϵ τ c_i u_j - 1/b_i^3
   ⅇ^(-τ b_i) ϵ (-b_i + 2 ⅇ^(τ b_i) b_i - ⅇ^(2 τ b_i) b_i + 2 ⅇ^(τ b_i) b_j - 2 ⅇ^(2 τ b_i) b_j + ⅇ^(τ b_i) τ b_i b_j + ⅇ^(2 τ b_i) τ b_i b_j) u_i^2 w_j +
   (ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) u_i u_j w_j)/b_i^2;
R0[τ_, i_, j_][u_k_] /; (k =!= i) ∧ (k =!= j) := u_k;
R0[τ_, i_, j_][w_i_] := w_i + ϵ τ c_j w_i + ⅇ^(-τ b_i) (-1 + ⅇ^(τ b_i)) w_j -
   (ⅇ^(-τ b_i) ϵ (-1 + ⅇ^(τ b_i) + τ b_i) c_i w_j)/b_i + (ⅇ^(-τ b_i) ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) c_j w_j)/b_i +
   (ⅇ^(-τ b_i) ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) u_i w_i w_j)/b_i^2 + (ⅇ^(-2 τ b_i) (1 + ⅇ^(τ b_i)) ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) u_i w_j^2)/b_i^2;
R0[τ_, i_, j_][w_j_] := ⅇ^(-τ b_i) w_j + ⅇ^(-τ b_i) ϵ τ c_i w_j - (ⅇ^(-2 τ b_i) ϵ (1 - ⅇ^(τ b_i) + ⅇ^(τ b_i) τ b_i) u_i w_j^2)/b_i^2;
R0[τ_, i_, j_][w_k_] /; (k =!= i) ∧ (k =!= j) := w_k;
R0[x___][y_] := CleaveLeft[y] /. {y1_, y2_} :→ R0[x][y1] ** R0[x][y2];
R0[i_, j_] := R0[-1, i, j];
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