

**Warning: b->B !**

$$e2 = (-1 + e^y) x f_{19}[x, y] - (-1 + e^y) x f_{19}[x, z] + (-1 + e^x) (y f_{19}[y, z] - y g_6[y] + e^y g_{15}[y] + g_{16}[y])$$

$$(-1 + e^y) x f_{19}[x, y] - (-1 + e^y) x f_{19}[x, z] + (-1 + e^x) (y f_{19}[y, z] - y g_6[y] + e^y g_{15}[y] + g_{16}[y])$$

**kk = Union@Cases[e2, a\_. \* f\_k\_[x, y] :-> k, ∞]**

{19}

**Length[kk] == 1**

True

**{kk} = kk**

{19}

**{{sol}} = Solve[e2 == 0, f\_kk[x, y]];**

**sol = f\_kk[x, y] /. sol**

$$\frac{1}{(-1 + e^y) x} (-x f_{19}[x, z] + e^y x f_{19}[x, z] + y f_{19}[y, z] - e^x y f_{19}[y, z] - y g_6[y] + e^x y g_6[y] + e^y g_{15}[y] - e^{x+y} g_{15}[y] + g_{16}[y] - e^x g_{16}[y])$$

**e3 = D[sol, z] // Factor**

$$\frac{1}{(-1 + e^y) x} (-x f_{19}^{(0,1)}[x, z] + e^y x f_{19}^{(0,1)}[x, z] + y f_{19}^{(0,1)}[y, z] - e^x y f_{19}^{(0,1)}[y, z])$$

**FreeQ[e3, f\_[\_, \_]]**

True

**If[Head[e3] === Times, e3 = Select[e3, !FreeQ[#, f\_^{(0,1)}[\_, \_] | g\_[\_] | h\_[\_]] &]]**

$$-x f_{19}^{(0,1)}[x, z] + e^y x f_{19}^{(0,1)}[x, z] + y f_{19}^{(0,1)}[y, z] - e^x y f_{19}^{(0,1)}[y, z]$$

**s = Collect[e3, f\_^{(0,1)}[\_, \_], Factor]**

$$(-1 + e^y) x f_{19}^{(0,1)}[x, z] - (-1 + e^x) y f_{19}^{(0,1)}[y, z]$$

**s1 = Select[s, FreeQ[f\_^{(0,1)}[\_, \_]]]**

0

**{pxz = Coefficient[s, f\_kk^{(0,1)}[x, z]], pyz = Coefficient[s, f\_kk^{(0,1)}[y, z]]}**

$$\{(-1 + e^y) x, -(-1 + e^x) y\}$$

**mf = MF[pxz, x]**

$$-1 + e^y$$

**mf \*= MF[pyz, y]**

$$(1 - e^x) (-1 + e^y)$$

$$\{s1, pxz, pyz\} = \text{Simplify}[\{s1, pxz, pyz\} / mf]$$

$$\left\{0, \frac{x}{1 - e^x}, \frac{y}{-1 + e^y}\right\}$$

$$\text{FreeQ}[pxz, y] \wedge \text{FreeQ}[pyz, x] \wedge \text{FreeQ}[s1, x | y]$$

True

$$\text{Simplify}[(pyz / . y \rightarrow x) + pxz == 0]$$

True

$$f_{kk}^{(0,1)}[x, z] == g_{++gn}[z] / pxz$$

$$f_{19}^{(0,1)}[x, z] == \frac{(1 - e^x) g_3[z]}{x}$$

$$f_{kk}[x, z] == g_{++gn}[z] / pxz + g_{++gn}[x]$$

$$f_{19}[x, z] == \frac{(1 - e^x) g_4[z]}{x} + g_5[x]$$