Pensieve header: Normally Ordered Exponentials at 1-Co in the t variables.

For pragmatic reasons, $\mathbb{E}[\omega, L, Q, P]$ means $\omega^{-1}(1+\epsilon\,\omega^{-4}P)\operatorname{Exp}[L+\omega^{-1}Q]$, where ω is an ϵ-free scalar, L is linear and contains only c's and b's, Q is a balanced quadratic in the u's and the w's and contains no c's and b's, and P is a balanced quartic polynomial in the c's, u's, and w's, . $\mathbb{E}$ is also a casting operator: $\mathbb{E}[\omega^{-1}(1+\epsilon\,\omega^{-4}P)\operatorname{Exp}[L+\omega^{-1}Q]]$ returns $\mathbb{E}[\omega, L, Q, P]$, meaning $\mathbb{E}[G\operatorname{Exp}[L+Q]]$ computes $\omega=(G\mid_{\epsilon=0})^{-1}$ and returns $\mathbb{E}[\omega, L, \omega Q, \omega^5\partial_\epsilon G,]$.

```
DP[P_, x_ → D_α_, y_ → D_β_][f_] :=
 Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

```
𝔼 /: Simplify[𝔼[ω_, L_, Q_, P_]] := 𝔼[Expand@Together[ω /. b_ι_ :> Log[t_ι]],
   Expand[L], Expand@Together[Q /. b_ι_ :> Log[t_ι]], Expand@Together[P /. b_ι_ :> Log[t_ι]]];
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] ≡ 𝔼[ω2_, L2_, Q2_, P2_] := (ω1 == ω2 ⋀ L1 == L2 ⋀ Q1 == Q2 ⋀ P1 == P2);
𝔼[G_. ℯ^F_] /; FreeQ[G, ℯ|-] :=
  With[{ω = (G /. ϵ → 0)^-1}, Simplify@𝔼[ω, F /. u_ | w_ | α | β → 0, ω F /. c_ → 0, ω^5 ∂_ϵ G]];
𝔼[G_] /; FreeQ[G, ℯ|-] := With[{ω = (G /. ϵ → 0)^-1}, Simplify@𝔼[ω, 0, 0, ω^5 ∂_ϵ G]];
𝔼[ℰ_] := 𝔼[Factor[ℰ]];
```

```
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] :=
  Simplify@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

```
NO[u_i_, c_j_, k_][𝔼[ω_, L_, Q_, P_]] := 𝔼[1, L /. c_j → 0, ω^-1 Q /. u_i → 0, 0] (
    𝔼[ω^-5 DP[ω^4 + ϵ P, c_j → D_γ, u_i → D_β][ℯ^(ℯ^-γ β u_k + γ c_k)] /. ℯ^-γ → Γ^-1] /.
     {γ → ∂_(c_j) L, Γ → ℯ^(∂_(c_j) L /. b_ι_ :> Log[t_ι]), β → ω^-1 ∂_(u_i) Q});
```

```
NO[w_i_, c_j_, k_][𝔼[ω_, L_, Q_, P_]] := 𝔼[1, L /. c_j → 0, ω^-1 Q /. w_i → 0, 0] (
    𝔼[ω^-5 DP[ω^4 + ϵ P, c_j → D_γ, w_i → D_β][ℯ^(ℯ^γ β w_k + γ c_k)] /. ℯ^γ → Γ] /. {γ → ∂_(c_j) L, Γ → ℯ^(∂_(c_j) L /. b_ι_ :> Log[t_ι]), β → ω^-1 ∂_(w_i) Q});
```

```
NO[w_i_, u_j_, k_][𝔼[ω_, L_, Q_, P_]] := With[{q = (1 - t_k) ν α β + ν β u_k + ν δ u_k w_k + ν α w_k},
  𝔼[1, L, ω^-1 Q /. w_i | u_j → 0, 0] (
    𝔼[ν ω^-5 DP[ω^4 + ϵ P, w_i → D_α, u_j → D_β][ℯ^q] + ϵ ν^5 ω^-1 Λ ℯ^q] /. {
      ν → (1 + (t_k - 1) δ)^-1,
      Λ → -1/2 (-1 + t_k) (α^2 β^2 + 4 α β δ (1 + (-1 + t_k) δ) + 2 δ^2 (1 + (-1 + t_k) δ)^2) +
        2 (1 + (-1 + t_k) δ)^2 (α β + δ + (-1 + t_k) δ^2) c_k - β (1 + 2 (-1 + t_k) δ) (α β + 2 δ (1 + (-1 + t_k) δ)) u_k +
        2 β δ (1 + (-1 + t_k) δ)^2 c_k u_k - 1/2 β^2 δ (2 + 3 (-1 + t_k) δ) u_k^2 + α (α β + 2 δ (1 + (-1 + t_k) δ)) w_k +
        2 α δ (1 + (-1 + t_k) δ)^2 c_k w_k - 2 (-1 + t_k) δ^2 (α β + δ (1 + (-1 + t_k) δ)) u_k w_k + 2 δ^2 (1 + (-1 + t_k) δ)^2
         c_k u_k w_k - β δ^2 (1 + 2 (-1 + t_k) δ) u_k^2 w_k + 1/2 α^2 δ (2 + (-1 + t_k) δ) w_k^2 + α δ^2 u_k w_k^2 - 1/2 (-1 + t_k) δ^4 u_k^2 w_k^2
    } /. {α → ω^-1 (∂_(w_i) Q /. u_j → 0), β → ω^-1 (∂_(u_j) Q /. w_i → 0), δ → ω^-1 ∂_(w_i, u_j) Q}));
```

```
m[i_, j_, kk_][Z_] := Module[{x, y},
  Z // ReplaceAll[{b_(i|j) → b_kk, t_(i|j) → t_kk}] // NO[w_i, c_j, x] // NO[w_x, u_j, y] //
    ReplaceAll[{c_(x|y) → c_x, w_j → w_y}] // NO[u_i, c_x, x] //
   ReplaceAll[{c_(i|x) → c_kk, u_(x|y) → u_kk, w_y → w_kk, b_(x|y) → b_kk, t_(x|y) → t_kk}] // Simplify]
```

Done line

```
Q0 = 𝔼[e^(u1 w1 + u2 w3)];
t1 = Q0 // m[1, 2, 4]
```

$$\mathbb{E}\left[1,\ 0,\ 2\,u_4\,w_3 - t_4\,u_4\,w_3 + u_4\,w_4,\ -2\,u_4\,w_3 + 2\,c_4\,u_4\,w_3 - \frac{5}{2}\,u_4^2\,w_3^2 + \frac{3}{2}\,t_4\,u_4^2\,w_3^2 - u_4^2\,w_3\,w_4\right]$$

```
Q0 = 𝔼[e^(u1 w1 + u3 w2)];
```

```
Q0 = 𝔼[Exp[Sum[a_{i,j} u_i w_j + l_{i,j} b_i c_j, {i, 3}, {j, 3}]]]
```

$$\mathbb{E}[1,\ b_1\,c_1\,l_{1,1} + b_1\,c_2\,l_{1,2} + b_1\,c_3\,l_{1,3} + b_2\,c_1\,l_{2,1} + b_2\,c_2\,l_{2,2} + b_2\,c_3\,l_{2,3} + b_3\,c_1\,l_{3,1} + b_3\,c_2\,l_{3,2} + b_3\,c_3\,l_{3,3},\ u_1\,w_1\,a_{1,1} + u_1\,w_2\,a_{1,2} + u_1\,w_3\,a_{1,3} + u_2\,w_1\,a_{2,1} + u_2\,w_2\,a_{2,2} + u_2\,w_3\,a_{2,3} + u_3\,w_1\,a_{3,1} + u_3\,w_2\,a_{3,2} + u_3\,w_3\,a_{3,3},\ 0]$$

```
Q0 = 𝔼[Exp[Sum[a_{i,j} u_i w_j + l_{i,j} b_i c_j, {i, 3}, {j, 3}]]]
```

$$\mathbb{E}[1,\ b_1\,c_1\,l_{1,1} + b_1\,c_2\,l_{1,2} + b_1\,c_3\,l_{1,3} + b_2\,c_1\,l_{2,1} + b_2\,c_2\,l_{2,2} + b_2\,c_3\,l_{2,3} + b_3\,c_1\,l_{3,1} + b_3\,c_2\,l_{3,2} + b_3\,c_3\,l_{3,3},\ u_1\,w_1\,a_{1,1} + u_1\,w_2\,a_{1,2} + u_1\,w_3\,a_{1,3} + u_2\,w_1\,a_{2,1} + u_2\,w_2\,a_{2,2} + u_2\,w_3\,a_{2,3} + u_3\,w_1\,a_{3,1} + u_3\,w_2\,a_{3,2} + u_3\,w_3\,a_{3,3},\ 0]$$

```
SeedRandom[6];
Q0 = 𝔼[Exp[Sum[RandomInteger[{-2, 2}] u_i w_j + RandomInteger[{-2, 2}] b_i c_j, {i, 3}, {j, 3}]]]
t1 = Q0 // m[1, 2, 4] // m[4, 3, 5]
t2 = Q0 // m[2, 3, 4] // m[1, 4, 5]
t3 = (t1 ≡ t2)
```

$$\mathbb{E}[1,\ -2\,b_1\,c_1 - b_2\,c_1 + 2\,b_3\,c_1 + b_2\,c_2 - b_3\,c_2 + 2\,b_1\,c_3 + b_2\,c_3 - b_3\,c_3,\ -u_3\,w_1 + 2\,u_2\,w_2 + u_3\,w_2 + u_1\,w_3 - u_2\,w_3,\ 0]$$

$$\mathbb{E}\Big[2 + \frac{1}{t_5^4} + \frac{2}{t_5^2} - \frac{4}{t_5},\ -4\,b_5 + b_5\,c_5,\ -8\,\text{Log}[t_5] - \frac{4\,\text{Log}[t_5]}{t_5^4} - \frac{8\,\text{Log}[t_5]}{t_5^2} + \frac{16\,\text{Log}[t_5]}{t_5} + \frac{5\,u_5\,w_5}{t_5^4} - \frac{5\,u_5\,w_5}{t_5^3} + \frac{2\,u_5\,w_5}{t_5},$$
$$-\frac{8\,c_5}{t_5^{14}} + \frac{8\,c_5}{t_5^{13}} + \frac{12}{t_5^{12}} - \frac{48\,c_5}{t_5^{12}} - \frac{32}{t_5^{11}} + \frac{144\,c_5}{t_5^{11}} + \frac{76}{t_5^{10}} - \frac{240\,c_5}{t_5^{10}} - \frac{232}{t_5^9} + \frac{528\,c_5}{t_5^9} + \frac{464}{t_5^8} - \frac{1024\,c_5}{t_5^8} - \frac{704}{t_5^7} + \frac{1408\,c_5}{t_5^7} +$$
$$\frac{1088}{t_5^6} - \frac{1824\,c_5}{t_5^6} - \frac{1472}{t_5^5} + \frac{2336\,c_5}{t_5^5} + \frac{1360}{t_5^4} - \frac{2240\,c_5}{t_5^4} - \frac{768}{t_5^3} + \frac{1344\,c_5}{t_5^3} + \frac{240}{t_5^2} - \frac{448\,c_5}{t_5^2} - \frac{32}{t_5} + \frac{64\,c_5}{t_5} - \frac{4\,u_5\,w_5}{t_5^{16}} +$$
$$\frac{10\,c_5\,u_5\,w_5}{t_5^{16}} + \frac{4\,u_5\,w_5}{t_5^{15}} - \frac{8\,c_5\,u_5\,w_5}{t_5^{15}} - \frac{6\,u_5\,w_5}{t_5^{14}} + \frac{16\,c_5\,u_5\,w_5}{t_5^{14}} + \frac{42\,u_5\,w_5}{t_5^{13}} - \frac{88\,c_5\,u_5\,w_5}{t_5^{13}} + \frac{108\,u_5\,w_5}{t_5^{12}} + \frac{52\,c_5\,u_5\,w_5}{t_5^{12}} -$$
$$\frac{472\,u_5\,w_5}{t_5^{11}} + \frac{48\,c_5\,u_5\,w_5}{t_5^{11}} + \frac{864\,u_5\,w_5}{t_5^{10}} + \frac{8\,c_5\,u_5\,w_5}{t_5^{10}} - \frac{1912\,u_5\,w_5}{t_5^9} + \frac{128\,c_5\,u_5\,w_5}{t_5^9} + \frac{3728\,u_5\,w_5}{t_5^8} - \frac{600\,c_5\,u_5\,w_5}{t_5^8} -$$
$$\frac{4528\,u_5\,w_5}{t_5^7} + \frac{672\,c_5\,u_5\,w_5}{t_5^7} + \frac{3192\,u_5\,w_5}{t_5^6} - \frac{64\,c_5\,u_5\,w_5}{t_5^6} - \frac{1128\,u_5\,w_5}{t_5^5} - \frac{480\,c_5\,u_5\,w_5}{t_5^5} + \frac{16\,u_5\,w_5}{t_5^4} + \frac{464\,c_5\,u_5\,w_5}{t_5^4} +$$
$$\frac{128\,u_5\,w_5}{t_5^3} - \frac{192\,c_5\,u_5\,w_5}{t_5^3} - \frac{32\,u_5\,w_5}{t_5^2} + \frac{32\,c_5\,u_5\,w_5}{t_5^2} - \frac{3\,u_5^2\,w_5^2}{t_5^{18}} + \frac{2\,u_5^2\,w_5^2}{t_5^{17}} - \frac{51\,u_5^2\,w_5^2}{2\,t_5^{16}} + \frac{127\,u_5^2\,w_5^2}{2\,t_5^{15}} - \frac{76\,u_5^2\,w_5^2}{t_5^{14}} + \frac{190\,u_5^2\,w_5^2}{t_5^{13}} -$$
$$\frac{163\,u_5^2\,w_5^2}{t_5^{12}} - \frac{446\,u_5^2\,w_5^2}{t_5^{11}} + \frac{1102\,u_5^2\,w_5^2}{t_5^{10}} - \frac{800\,u_5^2\,w_5^2}{t_5^9} - \frac{194\,u_5^2\,w_5^2}{t_5^8} + \frac{722\,u_5^2\,w_5^2}{t_5^7} - \frac{496\,u_5^2\,w_5^2}{t_5^6} + \frac{128\,u_5^2\,w_5^2}{t_5^5} + \frac{8\,u_5^2\,w_5^2}{t_5^4} - \frac{8\,u_5^2\,w_5^2}{t_5^3}\Big]$$

$$\mathbb{E}\Big[2 + \frac{1}{t_5^4} + \frac{2}{t_5^2} - \frac{4}{t_5},\ -4\,b_5 + b_5\,c_5,\ -8\,\text{Log}[t_5] - \frac{4\,\text{Log}[t_5]}{t_5^4} - \frac{8\,\text{Log}[t_5]}{t_5^2} + \frac{16\,\text{Log}[t_5]}{t_5} + \frac{5\,u_5\,w_5}{t_5^4} - \frac{5\,u_5\,w_5}{t_5^3} + \frac{2\,u_5\,w_5}{t_5},$$
$$-\frac{8\,c_5}{t_5^{14}} + \frac{8\,c_5}{t_5^{13}} + \frac{12}{t_5^{12}} - \frac{48\,c_5}{t_5^{12}} - \frac{32}{t_5^{11}} + \frac{144\,c_5}{t_5^{11}} + \frac{76}{t_5^{10}} - \frac{240\,c_5}{t_5^{10}} - \frac{232}{t_5^9} + \frac{528\,c_5}{t_5^9} + \frac{464}{t_5^8} - \frac{1024\,c_5}{t_5^8} - \frac{704}{t_5^7} + \frac{1408\,c_5}{t_5^7} +$$
$$\frac{1088}{t_5^6} - \frac{1824\,c_5}{t_5^6} - \frac{1472}{t_5^5} + \frac{2336\,c_5}{t_5^5} + \frac{1360}{t_5^4} - \frac{2240\,c_5}{t_5^4} - \frac{768}{t_5^3} + \frac{1344\,c_5}{t_5^3} + \frac{240}{t_5^2} - \frac{448\,c_5}{t_5^2} - \frac{32}{t_5} + \frac{64\,c_5}{t_5} - \frac{4\,u_5\,w_5}{t_5^{16}} +$$
$$\frac{10\,c_5\,u_5\,w_5}{t_5^{16}} + \frac{4\,u_5\,w_5}{t_5^{15}} - \frac{8\,c_5\,u_5\,w_5}{t_5^{15}} - \frac{6\,u_5\,w_5}{t_5^{14}} + \frac{16\,c_5\,u_5\,w_5}{t_5^{14}} + \frac{42\,u_5\,w_5}{t_5^{13}} - \frac{88\,c_5\,u_5\,w_5}{t_5^{13}} + \frac{108\,u_5\,w_5}{t_5^{12}} + \frac{52\,c_5\,u_5\,w_5}{t_5^{12}} -$$
$$\frac{472\,u_5\,w_5}{t_5^{11}} + \frac{48\,c_5\,u_5\,w_5}{t_5^{11}} + \frac{864\,u_5\,w_5}{t_5^{10}} + \frac{8\,c_5\,u_5\,w_5}{t_5^{10}} - \frac{1912\,u_5\,w_5}{t_5^9} + \frac{128\,c_5\,u_5\,w_5}{t_5^9} + \frac{3728\,u_5\,w_5}{t_5^8} - \frac{600\,c_5\,u_5\,w_5}{t_5^8} -$$
$$\frac{4528\,u_5\,w_5}{t_5^7} + \frac{672\,c_5\,u_5\,w_5}{t_5^7} + \frac{3192\,u_5\,w_5}{t_5^6} - \frac{64\,c_5\,u_5\,w_5}{t_5^6} - \frac{1128\,u_5\,w_5}{t_5^5} - \frac{480\,c_5\,u_5\,w_5}{t_5^5} + \frac{16\,u_5\,w_5}{t_5^4} + \frac{464\,c_5\,u_5\,w_5}{t_5^4} +$$
$$\frac{128\,u_5\,w_5}{t_5^3} - \frac{192\,c_5\,u_5\,w_5}{t_5^3} - \frac{32\,u_5\,w_5}{t_5^2} + \frac{32\,c_5\,u_5\,w_5}{t_5^2} - \frac{3\,u_5^2\,w_5^2}{t_5^{18}} + \frac{2\,u_5^2\,w_5^2}{t_5^{17}} - \frac{51\,u_5^2\,w_5^2}{2\,t_5^{16}} + \frac{127\,u_5^2\,w_5^2}{2\,t_5^{15}} - \frac{76\,u_5^2\,w_5^2}{t_5^{14}} + \frac{190\,u_5^2\,w_5^2}{t_5^{13}} -$$
$$\frac{163\,u_5^2\,w_5^2}{t_5^{12}} - \frac{446\,u_5^2\,w_5^2}{t_5^{11}} + \frac{1102\,u_5^2\,w_5^2}{t_5^{10}} - \frac{800\,u_5^2\,w_5^2}{t_5^9} - \frac{194\,u_5^2\,w_5^2}{t_5^8} + \frac{722\,u_5^2\,w_5^2}{t_5^7} - \frac{496\,u_5^2\,w_5^2}{t_5^6} + \frac{128\,u_5^2\,w_5^2}{t_5^5} + \frac{8\,u_5^2\,w_5^2}{t_5^4} - \frac{8\,u_5^2\,w_5^2}{t_5^3}\Big]$$

True

```
Simplify[t3 /. b_5 → Log[t_5]]
```

True

```
Rp[1, 2] Rm[4, 3]
```

$$\mathbb{E}\Big[b_1\,c_2 - b_4\,c_3 + \frac{\left(-1 + e^{b_1}\right)\,u_1\,w_2}{b_1} + \frac{\left(-1 + e^{-b_4}\right)\,u_4\,w_3}{b_4}\Big]$$

```
Rp[1, 2] Rm[4, 3] // m[1, 4, 1] // m[2, 3, 2]
```

$$\mathbb{E}[0]$$

```
Rm[1, 2] Rp[4, 3] // m[4, 1, 4] // m[2, 3, 2]
```

$$\mathbb{E}[0]$$

```
t1 = Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[3, 5, a] // m[1, 6, b] // m[2, 4, c]
```

$$\mathbb{E}\left[\frac{b_a^2\, b_b\, (c_b + c_c) + \left(-1 + e^{b_a}\right) b_b\, u_a\, (w_b + w_c) + b_a \left(b_b^2\, c_c + \left(-1 + e^{b_b}\right) u_b\, w_c\right)}{b_a\, b_b}\right]$$

```
t2 = Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[1, 3, a] // m[2, 5, b] // m[4, 6, c]
```

$$\mathbb{E}\left[\frac{b_a^2\, b_b\, (c_b + c_c) + \left(-1 + e^{b_a}\right) b_b\, u_a\, (w_b + w_c) + b_a \left(b_b^2\, c_c + \left(-1 + e^{b_b}\right) u_b\, w_c\right)}{b_a\, b_b}\right]$$

```
t1 ≡ t2
```

True

```
t3 = Rm[12, 1] Rm[2, 7] Rm[8, 3] Rm[4, 11] Rp[16, 5] Rp[6, 13] Rp[14, 9] Rp[10, 15]
```

$$\mathbb{E}\left[-b_{12}\, c_1 - b_8\, c_3 + b_{16}\, c_5 - b_2\, c_7 + b_{14}\, c_9 - b_4\, c_{11} + b_6\, c_{13} + b_{10}\, c_{15} + \frac{\left(-1 + e^{-b_{12}}\right) u_{12}\, w_1}{b_{12}} + \frac{\left(-1 + e^{-b_8}\right) u_8\, w_3}{b_8} + \frac{\left(-1 + e^{b_{16}}\right) u_{16}\, w_5}{b_{16}} + \frac{\left(-1 + e^{-b_2}\right) u_2\, w_7}{b_2} + \frac{\left(-1 + e^{b_{14}}\right) u_{14}\, w_9}{b_{14}} + \frac{\left(-1 + e^{-b_4}\right) u_4\, w_{11}}{b_4} + \frac{\left(-1 + e^{b_6}\right) u_6\, w_{13}}{b_6} + \frac{\left(-1 + e^{b_{10}}\right) u_{10}\, w_{15}}{b_{10}}\right]$$

```
t3 // m[1, 2, 1]
```

$$\mathbb{E}\left[-b_{12}\, c_1 - b_8\, c_3 + b_{16}\, c_5 - b_1\, c_7 + b_{14}\, c_9 - b_4\, c_{11} + b_6\, c_{13} + b_{10}\, c_{15} + \frac{\left(-1 + e^{-b_{12}}\right) u_{12}\, w_1}{b_{12}} + \frac{\left(-1 + e^{-b_8}\right) u_8\, w_3}{b_8} + \frac{\left(-1 + e^{b_{16}}\right) u_{16}\, w_5}{b_{16}} + \frac{\left(-1 + e^{-b_1}\right) u_1\, w_7}{b_1} - \frac{\left(-1 + e^{-b_1}\right)\left(-1 + e^{-b_{12}}\right) u_{12}\, w_7}{b_{12}} + \frac{\left(-1 + e^{b_{14}}\right) u_{14}\, w_9}{b_{14}} + \frac{\left(-1 + e^{-b_4}\right) u_4\, w_{11}}{b_4} + \frac{\left(-1 + e^{b_6}\right) u_6\, w_{13}}{b_6} + \frac{\left(-1 + e^{b_{10}}\right) u_{10}\, w_{15}}{b_{10}}\right]$$

```
t3 // m[1, 2, 1] // m[1, 3, 1]
```

$$\mathbb{E}\left[-b_8\, c_1 - b_{12}\, c_1 + b_{16}\, c_5 - b_1\, c_7 + b_{14}\, c_9 - b_4\, c_{11} + b_6\, c_{13} + b_{10}\, c_{15} + \frac{\left(-1 + e^{-b_8}\right) u_8\, w_1}{b_8} + \frac{e^{-b_8}\left(-1 + e^{-b_{12}}\right) u_{12}\, w_1}{b_{12}} + \frac{\left(-1 + e^{b_{16}}\right) u_{16}\, w_5}{b_{16}} + \frac{e^{b_8}\left(-1 + e^{-b_1}\right) u_1\, w_7}{b_1} - \frac{\left(-1 + e^{-b_1}\right)\left(-1 + e^{-b_{12}}\right) u_{12}\, w_7}{b_{12}} + \frac{\left(-1 + e^{b_{14}}\right) u_{14}\, w_9}{b_{14}} + \frac{\left(-1 + e^{-b_4}\right) u_4\, w_{11}}{b_4} + \frac{\left(-1 + e^{b_6}\right) u_6\, w_{13}}{b_6} + \frac{\left(-1 + e^{b_{10}}\right) u_{10}\, w_{15}}{b_{10}}\right]$$

```
Do[t3 = t3 // m[1, kk, 1], {kk, 2, 16}]; t3
```

$$-\frac{e^{3\, b_1}\, \mathbb{E}[0]}{1 - 4\, e^{b_1} + 8\, e^{2\, b_1} - 11\, e^{3\, b_1} + 8\, e^{4\, b_1} - 4\, e^{5\, b_1} + e^{6\, b_1}}$$

```
Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[1, 4, 4] // m[4, 5, 5] // m[5, 2, 2] // m[2, 3, 3] // m[3, 6, 6] // m[6, 1, 1]
```

$$\frac{\mathbb{E}\left[3\, b_1\, c_1 + \frac{\left(1 - e^{-3\, b_1}\right) u_1\, w_1}{b_1}\right]}{1 + e^{b_1}\left(-1 + e^{b_1}\right)}$$