

Pensieve header: Finding the most general R using low algebra - automatic error correction.

Warning: b->B, β->b !

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SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1606"];
```

```
<< OneCo.m
```

```
Errors = Get["Errors.m"]
```

$$\left\{ -2(-1 + e^x) x f_{39}[x, z], -2(-1 + e^x) x f_{48}[x, z], e^x(-1 + e^y) z f_{10}[x, z], \right.$$

$$e^y(-1 + e^x) z f_{10}[y, z], e^x(-1 + e^y) z f_{24}[x, z], \dots 186 \dots, \frac{\dots 1 \dots}{x^2},$$

$$\dots 1 \dots, \frac{\dots 1 \dots}{x y^2}, \frac{e^{-x-y} z (\dots 1 \dots)}{x^3}, \frac{1}{x^3} e^{-x-y} z \left(2(-1 + e^x)(-1 + e^y) + \right.$$

$$\left. (-1 + e^x) x^2 (\dots 1 \dots) + x^3 (\dots 1 \dots) + x \left(1 + \dots 88 \dots + e^{2x} z^2 f_{49}[y, z] \right) \right\}$$

large output | show less | show more | show all | set size limit...


```

Print["Length[E0]==", Length[E0],
"; Length[EC]==", Length[EC], "; {gn,hn}==", {gn, hn}];
For[k = 1, k ≤ Length@E0, ++k,
e1 = Factor[e0 = E0[[k]];
If[Head[e1] != Times, e2 = e1,
E0[[k]] = e2 = Select[e1, !FreeQ[#, f[_,_] | g[_] | h[[]] &]];
If[e2 == 1, Print["Panic at ", e0, "! No solutions."]; Break[]];
(*Replace[e2, f_k[_,_]→{
Print["As ",e0," adding ",f_k[_,_]→0];
done=False; EC=EC∪{f_k[_,_]→0};
Break[]
}];*)
If[!FreeQ[e2, f[_,_]] ∧ (FreeQ[e2, x] ∨ FreeQ[e2, y] ∨ FreeQ[e2, z]),
{ff} = Cases[e2, f[_,_], {0, ∞}, 1];
{{sol}} = Solve[e2 == 0, ff];
rule = ((ff /. {x → x_, y → y_, z → z_}) → (ff /. sol)) /. Rule → RuleDelayed;
AddRule[ff, rule]; Break[]
];
If[!FreeQ[e2, g[_]] ∧ (FreeQ[e2, y | z] ∨ FreeQ[e2, x | z] ∨ FreeQ[e2, z | y]),
{gg} = Cases[e2, g[_], ∞, 1];
{{sol}} = Solve[e2 == 0, gg];
rule = ((gg /. {x → x_, y → y_, z → z_}) → (gg /. sol)) /. Rule → RuleDelayed;
AddRule[gg, rule]; Break[]
];
If[Head[e2] === Plus,
s = List@@Collect[e2, f[_,_], Factor]; s1 = Select[s, FreeQ[f[_,_]]];
sxy = Cases[s, a_. * f[x, y]];
sxz = Cases[s, a_. * f[x, z]]; syz = Cases[s, a_. * f[y, z]];
Which[
sxy == {} ∧ sxz != {} ∧ syz != {}, (
{ff} = Cases[sxz, a_. * f_k[x, z] → f_k[x, z], {1}, 1];
mf = MF[First@sxz /. f[x, z] → 1, x]; mf *= MF[First@syz /. f[y, z] → 1, y];
s1 = Plus@@Simplify[s1/mf];
sxz = Plus@@Simplify[sxz/mf]; syz = Plus@@Simplify[syz/mf];
If[FreeQ[sxz, y] ∧ FreeQ[syz, x] ∧ FreeQ[s1, x | y] ∧
Simplify[(sxz /. x → y) + syz == 0],
{{sol}} = Solve[sxz == g++gn[z], ff];
rule = ((ff /. {x → x_, y → y_, z → z_}) → (ff /. sol)) /. Rule → RuleDelayed;
AddRule[ff, rule]; Break[]
]
),
syz == {} ∧ sxy != {} ∧ sxz != {}, (
{ff} = Cases[sxy, a_. * f_k[x, y] → f_k[x, y], {1}, 1];

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mf = MF[First@sxy /. f_[x, y] → 1, y]; mf *= MF[First@sxz /. f_[x, z] → 1, z];
s1 = Plus@@Simplify[s1/mf];
sxy = Plus@@Simplify[sxy/mf]; sxz = Plus@@Simplify[sxz/mf];
If[FreeQ[sxy, z] ∧ FreeQ[sxz, y] ∧ FreeQ[s1, y | z] ∧
  Simplify[(sxz /. z → y) + sxy == 0],
  {{sol}} = Solve[sxy == g++gn[x], ff];
rule = ((ff /. {x → x_, y → y_, z → z_}) → (ff /. sol)) /. Rule → RuleDelayed;
AddRule[ff, rule]; Break[]
]
),
sxy != {} ∧ sxz != {} ∧ syz != {}, (
kk = Union@Cases[e2, a_. * fk[x, y] ⇒ k, ∞];
If[Length[kk] == 1,
  {kk} = kk;
  {{sol}} = Solve[e2 == 0, fkk[x, y]];
  sol = fkk[x, y] /. sol;
  e3 = D[sol, z] // Factor;
  If[FreeQ[e3, f[_], _],
    If[Head[e3] === Times,
      e3 = Select[e3, !FreeQ[#, f(0,1)[_], _] | g[_] | h[_]] &];
      s = Collect[e3, f(0,1)[_], Factor];
      s1 = Select[s, FreeQ[f(0,1)[_], _]];
      pxz = Coefficient[s, fkk(0,1)[x, z]];
      pyz = Coefficient[s, fkk(0,1)[y, z]];
      mf = MF[pxz, x]; mf *= MF[pyz, y];
      {s1, pxz, pyz} = Simplify[{s1, pxz, pyz}/mf];
      If[FreeQ[pxz, y] ∧ FreeQ[pyz, x] ∧
        FreeQ[s1, x | y] ∧ Simplify[(pyz /. y → x) + pxz == 0],
        rule = (fkk[x_, z_] → g++gn[z] / pxz + g++gn[x]) /. Rule → RuleDelayed;
        AddRule[fkk[x, z], rule]; Break[]
      ]
    ]
  ]
)
];
If[FreeQ[e2, f[_], _] ∧ !FreeQ[e2, g[_]],
  s = List@@Collect[e2, g[_], Factor]; s1 = Select[s, FreeQ[g[_]]];
  sx = Cases[s, a_. * g[x]]; sy = Cases[s, a_. * g[y]]; sz = Cases[s, a_. * g[z]];
  Which[
    FreeQ[e2, x] ∧ sy != {} ∧ sz != {}, (
      {gg} = Cases[sy, a_. * gk[y] ⇒ gk[y], {1}, 1];
      mf = MF[First@sy /. g[y] → 1, y]; mf *= MF[First@sz /. g[z] → 1, z];
      s1 = Plus@@Simplify[s1/mf];

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sy = Plus@@Simplify[sy / mf]; sz = Plus@@Simplify[sz / mf];
If[FreeQ[sz, y] & FreeQ[sz, y] & FreeQ[s1, y | z] &
  Simplify[(sz /. z -> y) + sy == 0],
  {{sol}} = Solve[sy == h++hn[], gg];
rule = ((gg /. {x -> x_, y -> y_, z -> z_}) -> (gg /. sol)) /. Rule -> RuleDelayed;
AddRule[gg, rule]; Break[]
]
)
] (* Which *)
] (* If *)
] (* If *)
] (* For *)
]; (* While *)
E0 = Union[DeleteCases[Simplify[E0 //. EC], 0]] // SortBy[LeafCount];
Length[E0]==196; Length[EC]==0; {gn,hn}=={0, 0}
As -2 (-1 + ex) x f39[x, z], adding f39[x_, z_] -> 0
Length[E0]==190; Length[EC]==1; {gn,hn}=={0, 0}
As -2 (-1 + ex) x f48[x, z], adding f48[x_, z_] -> 0
Length[E0]==176; Length[EC]==2; {gn,hn}=={0, 0}
As ex (-1 + ey) z f10[x, z], adding f10[x_, z_] -> 0
Length[E0]==173; Length[EC]==3; {gn,hn}=={0, 0}
As (-1 + ex) y f9[y, z], adding f9[y_, z_] -> 0
Length[E0]==171; Length[EC]==4; {gn,hn}=={0, 0}
As ex (-1 + ey) z f24[x, z], adding f24[x_, z_] -> 0
Length[E0]==163; Length[EC]==5; {gn,hn}=={0, 0}
As -2 (-1 + ex) x f45[x, z], adding f45[x_, z_] -> 0
Length[E0]==148; Length[EC]==6; {gn,hn}=={0, 0}
As ex (-1 + ey) z f28[x, z], adding f28[x_, z_] -> 0
Length[E0]==140; Length[EC]==7; {gn,hn}=={0, 0}
As ex (-1 + ey) z f20[x, z], adding f20[x_, z_] -> 0
Length[E0]==132; Length[EC]==8; {gn,hn}=={0, 0}
As ex (-1 + ey) z f32[x, z], adding f32[x_, z_] -> 0
Length[E0]==129; Length[EC]==9; {gn,hn}=={0, 0}
As 2 (-1 + ex) x f30[x, z], adding f30[x_, z_] -> 0
Length[E0]==126; Length[EC]==10; {gn,hn}=={0, 0}
As 2 ex (-1 + ey) z f37[x, z], adding f37[x_, z_] -> 0
Length[E0]==116; Length[EC]==11; {gn,hn}=={0, 0}
As -2 ex (-1 + ey) y f49[x, y], adding f49[x_, y_] -> 0

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Length[E0]==101; Length[EC]==12; {gn,hn}=={0, 0}
As  $e^{-x} (-1 + e^x) y f_{11}[y, z]$ , adding  $f_{11}[y_-, z_-] \rightarrow 0$ 
Length[E0]==99; Length[EC]==13; {gn,hn}=={0, 0}
As  $-2 (-1 + e^x) x f_{34}[x, z]$ , adding  $f_{34}[x_-, z_-] \rightarrow 0$ 
Length[E0]==95; Length[EC]==14; {gn,hn}=={0, 0}
As  $(-1 + e^x) (f_{22}[y, z] - 2 y f_{42}[y, z])$ , adding  $f_{22}[y_-, z_-] \rightarrow 2 y f_{42}[y, z]$ 
Length[E0]==93; Length[EC]==15; {gn,hn}=={0, 0}
As  $-(-1 + e^x) (y f_{14}[y, z] + z f_{15}[y, z])$ , adding  $f_{14}[y_-, z_-] \rightarrow -\frac{z f_{15}[y, z]}{y}$ 
Length[E0]==92; Length[EC]==16; {gn,hn}=={0, 0}
As  $-2 (-1 + e^x) (f_7[x, z] - x^2 f_{42}[x, z])$ , adding  $f_7[x_-, z_-] \rightarrow x^2 f_{42}[x, z]$ 
Length[E0]==82; Length[EC]==17; {gn,hn}=={0, 0}
As  $-e^x (-1 + e^y) (f_{29}[x, z] - 2 z f_{50}[x, z])$ , adding  $f_{29}[x_-, z_-] \rightarrow 2 z f_{50}[x, z]$ 
Length[E0]==80; Length[EC]==18; {gn,hn}=={0, 0}
As  $-e^{-x} (-1 + e^x) y (f_{16}[x, z] - f_{16}[y, z])$ , adding  $f_{16}[x_-, z_-] \rightarrow -g_1[z]$ 
Length[E0]==79; Length[EC]==19; {gn,hn}=={1, 0}
As  $-e^{-x} (-1 + e^x) y (f_{40}[x, z] - f_{40}[y, z])$ , adding  $f_{40}[x_-, z_-] \rightarrow -g_2[z]$ 
Length[E0]==77; Length[EC]==20; {gn,hn}=={2, 0}
As  $(-1 + e^y) x f_{47}[x, z] - (-1 + e^x) y f_{47}[y, z]$ , adding  $f_{47}[x_-, z_-] \rightarrow -\frac{(-1 + e^x) g_3[z]}{x}$ 
Length[E0]==76; Length[EC]==21; {gn,hn}=={3, 0}
As  $2 e^x (-1 + e^y) (f_{13}[x, y] - y^2 f_{50}[x, y])$ , adding  $f_{13}[x_-, y_-] \rightarrow y^2 f_{50}[x, y]$ 
Length[E0]==66; Length[EC]==22; {gn,hn}=={3, 0}
As  $(-1 + e^x) (f_{15}[y, z] - y f_{33}[y, z] - 2 z f_{38}[y, z])$ , adding  $f_{15}[y_-, z_-] \rightarrow y f_{33}[y, z] + 2 z f_{38}[y, z]$ 
Length[E0]==64; Length[EC]==23; {gn,hn}=={3, 0}
As  $e^{-x-y} (-1 + e^y) z (f_{12}[x, y] - f_{12}[x, z])$ , adding  $f_{12}[x_-, y_-] \rightarrow -g_4[x]$ 
Length[E0]==63; Length[EC]==24; {gn,hn}=={4, 0}
As  $(-1 + e^y) x (f_{35}[x, y] - f_{35}[x, z])$ , adding  $f_{35}[x_-, y_-] \rightarrow -g_5[x]$ 
Length[E0]==61; Length[EC]==25; {gn,hn}=={5, 0}
As  $e^{-x-y} (-1 + e^y) z (f_{23}[x, y] - f_{23}[x, z])$ , adding  $f_{23}[x_-, y_-] \rightarrow -g_6[x]$ 
Length[E0]==60; Length[EC]==26; {gn,hn}=={6, 0}
As  $(-1 + e^y) x (f_{43}[x, y] - f_{43}[x, z])$ , adding  $f_{43}[x_-, y_-] \rightarrow -g_7[x]$ 
Length[E0]==58; Length[EC]==27; {gn,hn}=={7, 0}
As  $e^x (-1 + e^y) x f_{33}[x, z] - e^y (-1 + e^x) y f_{33}[y, z]$ , adding  $f_{33}[x_-, z_-] \rightarrow -\frac{e^{-x} (-1 + e^x) g_8[z]}{x}$ 
Length[E0]==56; Length[EC]==28; {gn,hn}=={8, 0}

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As $(-1 + e^x) (-f_{25}[x, z] + f_{25}[y, z] + x f_{46}[x, z] - y f_{46}[y, z])$
, adding $f_{25}[x_, z_] \mapsto x f_{46}[x, z] - g_9[z]$

Length[E0]==54; Length[EC]==29; {gn,hn}=={9, 0}

As $\frac{(-1 + e^y) x (g_1[y] - g_1[z] - y g_2[y] + z g_2[z])}{y}$, adding $g_1[y_] \mapsto y g_2[y] + h_1[]$

Length[E0]==52; Length[EC]==30; {gn,hn}=={9, 1}

As $-(-1 + e^x) (f_8[x, z] - f_8[y, z] - x f_{26}[x, z] + y f_{26}[y, z])$
, adding $f_8[x_, z_] \mapsto x f_{26}[x, z] - g_{10}[z]$

Length[E0]==48; Length[EC]==31; {gn,hn}=={10, 1}

As $\frac{(-1 + e^y) (y g_9[y] - z g_9[z] - g_{10}[y] + g_{10}[z])}{y}$, adding $g_9[y_] \mapsto \frac{g_{10}[y] - h_2[]}{y}$

Length[E0]==46; Length[EC]==32; {gn,hn}=={10, 2}

As $-2 z^2 (e^x (-1 + e^y) f_{38}[x, z] - e^y (-1 + e^x) f_{38}[y, z])$, adding $f_{38}[x_, z_] \mapsto -e^{-x} (-1 + e^x) g_{11}[z]$

Length[E0]==43; Length[EC]==33; {gn,hn}=={11, 2}

As $-e^{-x-y} z ((-1 + e^y) f_{17}[x, z] - (-1 + e^x) f_{17}[y, z])$, adding $f_{17}[x_, z_] \mapsto -(-1 + e^x) g_{12}[z]$

Length[E0]==42; Length[EC]==34; {gn,hn}=={12, 2}

As $-2 e^{-x-y} z ((-1 + e^y) f_{41}[x, z] - (-1 + e^x) f_{41}[y, z])$, adding $f_{41}[x_, z_] \mapsto -(-1 + e^x) g_{13}[z]$

Length[E0]==41; Length[EC]==35; {gn,hn}=={13, 2}

As $e^x (-1 + e^y) (f_{26}[x, y] - f_{26}[x, z] - y f_{46}[x, y] + z f_{46}[x, z])$
, adding $f_{26}[x_, y_] \mapsto y f_{46}[x, y] - g_{14}[x]$

Length[E0]==35; Length[EC]==36; {gn,hn}=={14, 2}

As $-e^{-x-y} (-1 + e^x) z (e^y y g_2[y] + g_4[y] - g_{12}[y] + e^y g_{12}[y] + e^y h_1[])$
, adding $g_2[y_] \mapsto -\frac{e^{-y} (g_4[y] - g_{12}[y] + e^y g_{12}[y] + e^y h_1[])}{y}$

Length[E0]==34; Length[EC]==37; {gn,hn}=={14, 2}

As $-(-1 + e^x) (f_1[x, z] - f_1[y, z] - x f_{18}[x, z] + y f_{18}[y, z] - x^2 f_{42}[x, z] + y^2 f_{42}[y, z])$
, adding $f_1[x_, z_] \mapsto x f_{18}[x, z] + x^2 f_{42}[x, z] - g_{15}[z]$

Length[E0]==32; Length[EC]==38; {gn,hn}=={15, 2}

As $e^x (-1 + e^y) (f_2[x, y] - f_2[x, z] - y f_{21}[x, y] + z f_{21}[x, z] - y^2 f_{50}[x, y] + z^2 f_{50}[x, z])$
, adding $f_2[x_, y_] \mapsto y f_{21}[x, y] + y^2 f_{50}[x, y] - g_{16}[x]$

Length[E0]==30; Length[EC]==39; {gn,hn}=={16, 2}

As $-(-1 + e^y) x f_{19}[x, y] + (-1 + e^y) x f_{19}[x, z] - (-1 + e^x) (y f_{19}[y, z] - y g_6[y] + e^y g_{15}[y] + g_{16}[y])$
, adding $f_{19}[x_, z_] \mapsto \frac{(1 - e^x) g_{17}[z]}{x} + g_{18}[x]$

Length[E0]==30; Length[EC]==40; {gn,hn}=={18, 2}

As $(-1 + e^x) (y g_6[y] - e^y g_{15}[y] - g_{16}[y] - g_{17}[y] + e^y g_{17}[y] - y g_{18}[y])$
, adding $g_6[y_] \mapsto \frac{e^y g_{15}[y] + g_{16}[y] + g_{17}[y] - e^y g_{17}[y] + y g_{18}[y]}{y}$

Length[E0]==28; Length[EC]==41; {gn,hn}=={18, 2}

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As e-x (-1 + ex) (g4[y] - y g5[y] - 2 g12[y] + 2 ey g12[y] + 2 y g13[y] - 2 ey y g13[y] + ey h1[])
, adding g4[y_] := y g5[y] + 2 g12[y] - 2 ey g12[y] - 2 y g13[y] + 2 ey y g13[y] - ey h1[]
Length[E0]==26; Length[EC]==42; {gn,hn}=={18, 2}

As  $\frac{1}{x} 2 e^{-x} (-1 + e^y) y$ 
(x f36[x, y] - (-1 + ex) ((-1 + ex) g12[x] + ex g12[y] + x g13[x] - ex x g13[x] - ex y g13[y] + ex h1[])
, adding f36[x_, y_] :=  $\frac{1}{x} (-1 + e^x)$ 
(-g12[x] + ex g12[x] + ex g12[y] + x g13[x] - ex x g13[x] - ex y g13[y] + ex h1[])
Length[E0]==22; Length[EC]==43; {gn,hn}=={18, 2}

As  $\frac{1}{y} 2 (-1 + e^x)$ 
(y2 f31[y, z] - e-y (-1 + ey) (ey y g8[y] - (-1 + ey) z g8[z] + ey y2 g11[y] + z2 g11[z] - ey z2 g11[z]))
, adding f31[y_, z_] :=  $\frac{1}{y^2}$ 
e-y (-1 + ey) (ey y g8[y] + z g8[z] - ey z g8[z] + ey y2 g11[y] + z2 g11[z] - ey z2 g11[z])
Length[E0]==18; Length[EC]==44; {gn,hn}=={18, 2}

As -1 + ex+y -  $\frac{1}{x} + \frac{e^x}{x} + \frac{e^y}{x} - \frac{e^{x+y}}{x} - \frac{1}{y} + \frac{e^x}{y} + \frac{e^y}{y} - \frac{e^{x+y}}{y} - (-1 + e^y) x f_{27}[x, y] + e^y g_{10}[y] - e^{x+y} g_{10}[y] +$ 
y g14[y] - ex y g14[y] + (-1 + ex) (ey g15[y] + g16[y] + g17[y] - ey g17[y] + y g18[y]) + h2[] - ex h2[]
, adding f27[x_, y_] :=  $\frac{1}{(-1 + e^y) x^2 y}$ 
(-x + ex x + ey x - ex+y x - y + ex y + ey y - ex+y y - x y + ex+y x y + ey x y g10[y] - ex+y x y g10[y] +
x y2 g14[y] - ex x y2 g14[y] - ey x y g15[y] + ex+y x y g15[y] - x y g16[y] + ex x y g16[y] - x y g17[y] +
ex x y g17[y] + ey x y g17[y] - ex+y x y g17[y] - x y2 g18[y] + ex x y2 g18[y] + x y h2[] - ex x y h2[])
Length[E0]==12; Length[EC]==45; {gn,hn}=={18, 2}

As  $\frac{1}{y^2} (-1 + e^x)$ 
(2 - 2 ey + y + ey y + 2 (-1 + ey) y2 g3[y] + 2 y3 g7[y] - 2 ey y g10[y] - 2 y2 g14[y] - y h2[] + ey y h2[])
, adding g3[y_] :=  $\frac{1}{2 (-1 + e^y) y^2}$ 
(-2 + 2 ey - y - ey y - 2 y3 g7[y] + 2 ey y g10[y] + 2 y2 g14[y] + y h2[] - ey y h2[])
Length[E0]==8; Length[EC]==46; {gn,hn}=={18, 2}

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$$\begin{aligned}
 \text{As } & -\frac{1}{x^3} e^{-x} y (-2 + 2 e^x + 2 e^y - 2 e^{x+y} - x - e^x x + e^y x + e^{x+y} x - 2 (-1 + e^y) x^3 f_{44}[x, y] + \\
 & x^3 g_7[x] - e^x x^3 g_7[x] - e^y x^3 g_7[x] + e^{x+y} x^3 g_7[x] + x y^2 g_7[y] - 2 e^x x y^2 g_7[y] + \\
 & e^{2x} x y^2 g_7[y] - (-1 + e^x) (-1 + e^y) x (e^x g_{15}[x] + g_{16}[x] + g_{17}[x] - e^x g_{17}[x] + x g_{18}[x]) - \\
 & x (e^y g_{15}[y] + g_{16}[y] + g_{17}[y] - e^y g_{17}[y] + y g_{18}[y]) + \\
 & 2 e^x x (e^y g_{15}[y] + g_{16}[y] + g_{17}[y] - e^y g_{17}[y] + y g_{18}[y]) - \\
 & e^{2x} x (e^y g_{15}[y] + g_{16}[y] + g_{17}[y] - e^y g_{17}[y] + y g_{18}[y]) + x h_2[] - 2 e^x x h_2[] + e^{2x} x h_2[] \\
 , \text{ adding } & f_{44}[x_, y_] := \frac{1}{2 (-1 + e^y) x^3} (-2 + 2 e^x + 2 e^y - 2 e^{x+y} - x - e^x x + e^y x + e^{x+y} x + \\
 & x^3 g_7[x] - e^x x^3 g_7[x] - e^y x^3 g_7[x] + e^{x+y} x^3 g_7[x] + x y^2 g_7[y] - 2 e^x x y^2 g_7[y] + \\
 & e^{2x} x y^2 g_7[y] - e^x x g_{15}[x] + e^{2x} x g_{15}[x] + e^{x+y} x g_{15}[x] - e^{2x+y} x g_{15}[x] - e^y x g_{15}[y] + \\
 & 2 e^{x+y} x g_{15}[y] - e^{2x+y} x g_{15}[y] - x g_{16}[x] + e^x x g_{16}[x] + e^y x g_{16}[x] - e^{x+y} x g_{16}[x] - \\
 & x g_{16}[y] + 2 e^x x g_{16}[y] - e^{2x} x g_{16}[y] - x g_{17}[x] + 2 e^x x g_{17}[x] - e^{2x} x g_{17}[x] + \\
 & e^y x g_{17}[x] - 2 e^{x+y} x g_{17}[x] + e^{2x+y} x g_{17}[x] - x g_{17}[y] + 2 e^x x g_{17}[y] - e^{2x} x g_{17}[y] + \\
 & e^y x g_{17}[y] - 2 e^{x+y} x g_{17}[y] + e^{2x+y} x g_{17}[y] - x^2 g_{18}[x] + e^x x^2 g_{18}[x] + e^y x^2 g_{18}[x] - \\
 & e^{x+y} x^2 g_{18}[x] - x y g_{18}[y] + 2 e^x x y g_{18}[y] - e^{2x} x y g_{18}[y] + x h_2[] - 2 e^x x h_2[] + e^{2x} x h_2[])
 \end{aligned}$$

Length[E0]==0; Length[EC]==47; {gn,hn}=={18, 2}

EC

$$\begin{aligned}
 \{ & f_1[x_, z_] := x f_{18}[x, z] + x^2 f_{42}[x, z] - g_{15}[z], \\
 & f_2[x_, y_] := y f_{21}[x, y] + y^2 f_{50}[x, y] - g_{16}[x], f_7[x_, z_] := x^2 f_{42}[x, z], \\
 & f_8[x_, z_] := x f_{26}[x, z] - g_{10}[z], f_9[y_, z_] := 0, f_{10}[x_, z_] := 0, f_{11}[y_, z_] := 0, \\
 & f_{12}[x_, y_] := -g_4[x], f_{13}[x_, y_] := y^2 f_{50}[x, y], f_{14}[y_, z_] := -\frac{z f_{15}[y, z]}{y}, \\
 & f_{15}[y_, z_] := y f_{33}[y, z] + 2 z f_{38}[y, z], f_{16}[x_, z_] := -g_1[z], \\
 & f_{17}[x_, z_] := -(-1 + e^x) g_{12}[z], f_{19}[x_, z_] := \frac{(1 - e^x) g_{17}[z]}{x} + g_{18}[x], \\
 & f_{20}[x_, z_] := 0, f_{22}[y_, z_] := 2 y f_{42}[y, z], f_{23}[x_, y_] := -g_6[x], f_{24}[x_, z_] := 0, \\
 & f_{25}[x_, z_] := x f_{46}[x, z] - g_9[z], f_{26}[x_, y_] := y f_{46}[x, y] - g_{14}[x], \\
 & f_{27}[x_, y_] := \frac{1}{(-1 + e^y) x^2 y} (-x + e^x x + e^y x - e^{x+y} x - y + e^x y + e^y y - e^{x+y} y - \\
 & x y + e^{x+y} x y + e^y x y g_{10}[y] - e^{x+y} x y g_{10}[y] + x y^2 g_{14}[y] - e^x x y^2 g_{14}[y] - \\
 & e^y x y g_{15}[y] + e^{x+y} x y g_{15}[y] - x y g_{16}[y] + e^x x y g_{16}[y] - x y g_{17}[y] + e^x x y g_{17}[y] + \\
 & e^y x y g_{17}[y] - e^{x+y} x y g_{17}[y] - x y^2 g_{18}[y] + e^x x y^2 g_{18}[y] + x y h_2[] - e^x x y h_2[]), \\
 & f_{28}[x_, z_] := 0, f_{29}[x_, z_] := 2 z f_{50}[x, z], f_{30}[x_, z_] := 0, f_{31}[y_, z_] := \frac{1}{y^2} \\
 & e^{-y} (-1 + e^y) (e^y y g_8[y] + z g_8[z] - e^y z g_8[z] + e^y y^2 g_{11}[y] + z^2 g_{11}[z] - e^y z^2 g_{11}[z]), \\
 & f_{32}[x_, z_] := 0, f_{33}[x_, z_] := -\frac{e^{-x} (-1 + e^x) g_8[z]}{x}, f_{34}[x_, z_] := 0, \\
 & f_{35}[x_, y_] := -g_5[x], f_{36}[x_, y_] := \frac{1}{x} \\
 & (-1 + e^x) (-g_{12}[x] + e^x g_{12}[x] + e^x g_{12}[y] + x g_{13}[x] - e^x x g_{13}[x] - e^x y g_{13}[y] + e^x h_1[]), \\
 & f_{37}[x_, z_] := 0, f_{38}[x_, z_] := -e^{-x} (-1 + e^x) g_{11}[z], f_{39}[x_, z_] := 0, \\
 & f_{40}[x_, z_] := -g_2[z], f_{41}[x_, z_] := -(-1 + e^x) g_{13}[z], f_{43}[x_, y_] := -g_7[x],
 \end{aligned}$$

