Analyzing ϵs

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OneCo Implementation Showcase

http://drorbn.net/AcademicPensieve/Projects/OneCo-1604/
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In $\mathcal{U}(T) \otimes \mathcal{U}(H)$ conventions.

SetSigns.

```
Cases[SetSigns[6], ϵ_, ∞] // Union
{ϵ5, ϵ6, ϵ9, ϵ10}
Table[i → ϵi, {i, 0, 49}]
```

$\{0 \to 1, 1 \to 1, 2 \to \epsilon_{10}, 3 \to \epsilon_5, 4 \to \epsilon_{10}, 5 \to \epsilon_5, 6 \to \epsilon_5, 7 \to -\epsilon_5, 8 \to \epsilon_5\epsilon_9, 9 \to \epsilon_9, 10 \to \epsilon_{10}, 11 \to \epsilon_{10}, 12 \to \epsilon_{10}, 13 \to -\epsilon_{10}, 14 \to \epsilon_{10}, 15 \to \epsilon_{10}, 16 \to -\epsilon_9\epsilon_{10}, 17 \to \epsilon_9\epsilon_{10}, 18 \to \frac{\epsilon_5\epsilon_{10}}{\epsilon_6}, 19 \to \epsilon_5, 20 \to \epsilon_5, 21 \to \epsilon_5, 22 \to \epsilon_5, 23 \to \epsilon_{10}, 24 \to \epsilon_{10}, 25 \to \epsilon_{10}, 26 \to \epsilon_{10}, 27 \to \epsilon_{10}, 28 \to \epsilon_{10}, 29 \to \epsilon_{10}, 30 \to \frac{\epsilon_5\epsilon_{10}}{\epsilon_6}, 31 \to \epsilon_5, 32 \to \epsilon_5\epsilon_{10}, 33 \to \epsilon_5\epsilon_{10}, 34 \to \epsilon_5\epsilon_{10}, 35 \to \epsilon_{10}, 36 \to \epsilon_5, 37 \to \epsilon_{10}, 38 \to \epsilon_{10}, 39 \to \epsilon_{10}, 40 \to \epsilon_{10}, 41 \to \epsilon_{10}^2, 42 \to \epsilon_{10}, 43 \to \epsilon_{10}, 44 \to \epsilon_{10}^2, 45 \to \epsilon_{10}, 46 \to \epsilon_{10}, 47 \to \epsilon_{10}, 48 \to \epsilon_{10}, 49 \to \epsilon_{10}\}$

Generalities.

```
Simp[expr_] := Expand[expr];
S[expr_] :=
  expr /. (λ_β | λ_a | λ_δβ | λ_δa | λ_δaa) ↦
    MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
   λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f + g], r];
   λ /: g_ * λ[f_, r___] := λ[Simp[g f], r]);
AutoCollecting /@ {β, a, δβ, δa, δaa};
UU /: UU[x_] + UU[y_] := UU[x + y];
UU /: a_ * UU[x_] := UU[Expand[a x]];
bç = 1; ca[f_, i_, j_, k_] := δaa[f, ç, i, j, k];
γ[f_, j_, k_] := δa[f, j, k] - δa[ϵ0 bj f, ç, k];
γa[f_, j_, k_, l_, m_] :=
  δaa[f, j, k, l, m] - ca[ϵ0 bj f, k, l, m];
Kδ /: Kδis__ := KroneckerDelta[1, Length[Union[{is}]]];
```

δaa relations.

```
i_ ≤ j_ := OrderedQ[{i, j}]; i_ < j_ := ! OrderedQ[{j, i}];
S[UU[expr_]] := UU[S[expr /. δaa[f_, i_, j_, k_, l_] ↦ Which[
      k === ç, δaa[f, ç, l, i, j] + Kδjl γ[ϵ49 f, i, j],
      (i === ç) ⋁ (i ≤ k ⋀ j ≤ l), δaa[f, i, j, k, l],
      k < i ⋀ j < l, δaa[f, k, j, i, l] + ca[-f bi ϵ1, l, k, j] +
       ca[f bi ϵ1, j, k, l] + ca[-f bk ϵ1, j, i, l] +
       ca[f bk ϵ1, l, i, j],
      k < i ⋀ j === l, δa[-f bi ϵ2, k, j] + δa[f bk ϵ2, i, j] +
       δaa[f, k, j, i, j],
      i ≤ k ⋀ l < j, δaa[f, i, l, k, j] + ca[-f bi ϵ1, l, k, j] +
       ca[f bi ϵ1, j, k, l] + ca[-f bk ϵ1, j, i, l] +
       ca[f bk ϵ1, l, i, j],
      k < i ⋀ l < j, δaa[f, k, l, i, j]
     ]]];
```

Definition of *tm*.

```
UU[expr_] // tm[x_, y_, z_] := S[UU[Expand[expr /. {
      a[f_, x, j_] ↦ a[f, z, j] + ϵ3 γ[∂by f, z, j],
      a[f_, y, j_] ↦ a[f, z, j],
      δa[f_, x | y, j_] ↦ δa[f, z, j],
      δaa[f_, i_, j_, k_, l_] ↦
       δaa[f, i // Replace[x | y → z], j,
        k // Replace[x | y → z], l]
     } /. bx|y → bz]]];
```

Definition of *hm*.

```
UU[expr_] // hm[x_, y_, z_] := S[UU[Expand[expr /. {
      a[f_, i_, x | y] ↦ a[f, i, z],
      δa[f_, i_, x | y] ↦ δa[f, i, z],
      δaa[f_, i_, y, k_, x] ↦ δaa[f, k, z, i, z],
      δaa[f_, i_, j_, k_, l_] ↦
       δaa[f, i, j // Replace[x | y → z], k,
        l // Replace[x | y → z]]
     }]]];
```

ϵ_6 appears to be "rescaling β".

Definition of *hts*.

```
UU[expr_] // hts[y_, x_] := S[UU[Expand[expr /. {
      a[f_, i_, j_] ↦ a[f, i, j] - ϵ5 Kδjy γ[∂bx f, i, y] -
        Kδix Kδjy (ϵ6 β[f bx] + ϵ7 δa[f, ç, y] -
        ϵ8 δβ[bx ∂bx f]),
      δa[f_, x, y] ↦ δa[f, x, y] - ϵ9 δβ[f bx],
      δaa[f_, i_, j_, k_, l_] ↦
       δaa[f, i, j, k, l] + ϵ12 Kδix Kδjy δa[-bx f, k, l] +
        ϵ13 Kδix Kδjy (-δa[bk f, x, j] + δa[bx f, k, j]) +
        ϵ14 Kδkx Kδjy (δa[bi f, x, l] - δa[bx f, i, l]) +
        ϵ15 Kδkx Kδly δa[-bx f, i, j] +
        ϵ16 Kδix Kδjly δβ[bx bk f] + 2 ϵ17 Kδxik Kδyjl δβ[bx bx f]
     }]]];
```

$\epsilon_5\epsilon_9$ ϵ_8 seems to be "rescaling fβ".

Definition of *dm*.

```
dm[x_, y_, z_][expr_] :=
 expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]
```

Renaming operations.

```
tσ[x_List, y_List][expr_] := Module[{r = Thread[x → y]},
   S[expr /. bi_ ↦ bi /. r /. {
      a[f_, i_, j_] ↦ a[f, i /. r, j],
      δa[f_, i_, j_] ↦ δa[f, i /. r, j],
      δaa[f_, i_, j_, k_, l_] ↦ δaa[f, i /. r, j, k /. r, l]
     }]];
tσ[x_, y_][expr_] := tσ[{x}, {y}][expr];
hσ[x_List, y_List][expr_] :=
  Module[{r = Thread[x → y]},
   S[expr /. {
      a[f_, i_, j_] ↦ a[f, i, j /. r],
      δa[f_, i_, j_] ↦ δa[f, i, j /. r],
      δaa[f_, i_, j_, k_, l_] ↦ δaa[f, i, j /. r, k, l /. r]
     }]];
hσ[x_, y_][expr_] := hσ[{x}, {y}][expr];
tσ[x_, y_][expr_] := expr // tσ[x, y] // hσ[x, y];
```

Definition of *tb*.

```
tb[x_][UU[L_], UU[R_]] :=
  Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
        p[u_β | u_δβ | u_δa | u_δaa, v_a] ↦ -p[v, u]
       } /. {
       p[a[f_, x, j_], u_] ↦ (u /. {
           β[g_] ↦ ϵ18 γ[f ∂bx g, x, j],
           a[g_, k_, l_] ↦ ϵ19 γa[f ∂bx g, x, j, k, l] +
             Kδxk (-γa[ϵ20 g ∂bk f, k, l, x, j] +
               ca[ϵ21 f g, l, x, j] - ca[ϵ21 f g, j, k, l]),
           _ → 0
          }),
       p[a[f_, j_, k_], a[g_, x, l_]] /; j =!= x ↦
        -γa[ϵ22 g ∂bx f, x, l, j, k],
       p[_, _] → 0
      }]]]];
```

Definition of *thb*.

```
thb[x_, y_][UU[L_], UU[R_]] :=
  Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
        p[_a, _β | _δβ] → 0,
        p[β[f_], a[g_, i_, j_]] ↦
         Kδyj γ[ϵ30 g ∂bx f, i, y],
        p[a[f_, i_, j_], a[g_, k_, l_]] ↦ Kδyl (
           γa[ϵ31 g ∂bx f, k, l, i, j] + Kδxi (
              γ[-ϵ32 bk g ∂bx f, i, j] + δa[ϵ33 bk g ∂bx f,
                i, j] - δa[ϵ34 bi g ∂bx f, k, j] - a[
                ϵ35 bk f g, i, j] + a[ϵ35 bi f g, k,
                j] + ca[ϵ36 f g, j, k, l] - ca[ϵ36 f g,
                l, k, j]))),
        p[a[f_, i_, j_], δa[g_, k_, l_]] ↦
         ϵ38 Kδxi Kδyl (-δa[bk f g, i, j] + δa[bi f g, k, j]),
        p[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] ↦ Kδxi (
           ϵ42 Kδyl (-δaa[bk f g, i, j, m, n] + δaa[
                bi f g, k, j, m, n]) +
             ϵ43 Kδyn (-δaa[bm f g, k, l, i, j] + δaa[
                bi f g, k, l, m, j]) +
             ϵ44 Kδyln (δa[bx bm f g, k, j] - δa[bk bm f g,
                x, j])),
        p[_δβ, _a] → 0,
        p[δa[f_, i_, j_], a[g_, k_, l_]] ↦
         ϵ45 Kδxi Kδyl (-δa[bk f g, i, j] + δa[bi f g, k, j]),
        p[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] ↦
         ϵ47 Kδxi Kδyl (-δaa[bk f g, i, j, m, n] +
             δaa[bi f g, k, j, m, n]) +
           ϵ48 Kδxm Kδyl (-δaa[bk f g, i, j, m, n] +
             δaa[bm f g, i, j, k, n])
       }]]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
```

Definition of *hb*.

```
hb[y_][UU[L_], UU[R_]] :=
  Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ, _] → 0,
        p[_, _β | _δβ] → 0,
        p[_δa | _δaa, _δa | _δaa] → 0,
        p[u_δa | u_δaa, v_a] ↦ -p[v, u]
       } /. {
       p[a[f_, i_, y], u_] ↦ (u /. {
           a[g_, j_, k_] ↦
            ϵ23 Kδyk (a[bj f g, i, y] - a[bi f g, j, k]),
           δa[g_, j_, k_] ↦
            ϵ25 Kδyk (δa[bj f g, i, y] - δa[bi f g, j, k]),
           δaa[g_, j_, k_, l_, m_] ↦
            ϵ28 Kδyk (δaa[bj f g, i, y, l, m] -
                δaa[bi f g, j, k, l, m]) +
              ϵ29 Kδym (δaa[bi f g, j, k, i, y] -
                δaa[bi f g, j, k, l, m])
          }),
       _p → 0
      }]]]];
```

Definition of *db*.

Using $h_1h_2t_1t_2 \to h_1h_2t_1t_2 \to h_1h_2t_2t_1 \to h_2h_1t_2t_1 \to h_2h_1t_2t_1$:

```
db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
    htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] //
     hm[x, h, x],
    tb[x][u, v // hσ[x, h]] // hm[x, h, x],
    hb[x][u, v // tσ[x, t]] // tm[t, x, x],
    thb[x, x][u // hσ[x, h], v // tσ[x, t]] //
      tm[t, x, x] // hm[x, h, x]
   ]];
```

The bracket.

```
bb[S_List] := Module[{w, bar, t, n = 0},
   bar[x_] := -x;
   w = #2 // dσ[S, bar /@ S];
   Sum[
    t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]]];
    Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]]], S[[i]]],
     {i, k + 1, Length@S}];
    t,
    {k, Length@S}
   ]
  ] &
bb[S___] := bb[{S}]
```

$\epsilon_5=\epsilon_{21}=1$ is the coefficient of the 2-cocycle: $[a_{kj}, a_{kl}] = c_q a_{kj} - c_j a_{kl}$

Fixes $\epsilon_{23}=1$ hence $\epsilon_{10}=1$: $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$

Comment: ϵ_{10} can be set to any scalar by rescaling b

AutoAd.

```
AutoAd[B_, x_][y_] :=
  Module[{pows, states, i, s, seq, sh = 5, dseq,
    sf1, sf2, sf, t1, n},
   pows = NestList[B[x, #] &, y, 20];
   states =
    Union[Cases[pows, s_β | s_δβ | s_a | s_δa | s_δaa ↦
       ReplacePart[s, 1 → _], ∞]];
   UU@Sum[
     seq = Cases[{#}, states[[i]], ∞] & /@ pows;
     seq = Replace[seq, {{_[f_, ___]} ↦ f, {} → 0}, {1}];
     dseq = Drop[seq, sh];
     If[Union[Length[MonomialList[#]] & /@ dseq] === {1} ∧
       Union[Length[FactorTermsList[#]] & /@ dseq] ===
        {2},
      sf1 = FindSequenceFunction[
        FactorTermsList[#][[1]] & /@ dseq];
      sf2 = FindSequenceFunction[
        FactorTermsList[#][[2]] & /@ dseq];
      sf = (sf1[#] sf2[#] &),
      (*Else*)
      sf = FindSequenceFunction[dseq,
        FunctionSpace → {"ConstantRecursive",
          "HolonomicSequence", "Polynomial",
          "RationalFunction", "HypergeometricTerm"}]];
     ReplacePart[states[[i]],
      1 → Simplify[∑_{n=0}^{sh-1} seq[[n+1]]/n! + ∑_{n=sh}^{∞} sf[n+1-sh]/n!]],
     {i, Length@states}]];
```

The scattering of a tail by an exponential of tails.

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 1, 0]]
```

$$UU\left[a[1,1,0] + \delta aa\left[\epsilon_5, \varsigma, 0, 1, 2\right] + \delta aa\left[-\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}}, \varsigma, 2, 1, 0\right] + \delta aa\left[-\frac{\epsilon_5\left(-1+e^{-b_1\epsilon_{10}}+b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}}, 1, 0, 1, 2\right]\right]$$

The scattering of a tail by an exponential of heads.

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 2, 0]]
```

$$UU\left[a\left[e^{b_1\epsilon_{10}}, 2, 0\right] + a\left[-\frac{\left(-1+e^{b_1\epsilon_{10}}\right)b_2}{b_1}, 1, 0\right] + \delta a\left[-\frac{b_2\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2}, 1, 0\right] + \delta a\left[\frac{b_2\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1}, \varsigma, 0\right] + \delta aa\left[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}}, \varsigma, 2, 1, 0\right] + \delta aa\left[\frac{\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}}, 1, 0, 2, 2\right] + \delta aa\left[-\frac{\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}}, \varsigma, 0, 2, 2\right] + \delta aa\left[\frac{\epsilon_5\left(-2\left(-1+e^{b_1\epsilon_{10}}\right)b_2+b_1\left(1-e^{b_1\epsilon_{10}}+\left(1+e^{b_1\epsilon_{10}}\right)b_2\epsilon_{10}\right)\right)}{b_1^2\epsilon_{10}}, \varsigma, 0, 1, 2\right] + \delta aa\left[-\frac{1}{b_1^2\epsilon_{10}} e^{-b_1\epsilon_{10}}\epsilon_5\left(-2e^{b_1\epsilon_{10}}\left(-1+e^{b_1\epsilon_{10}}\right)b_2+b_1\left(-\left(-1+e^{b_1\epsilon_{10}}\right)^2+e^{b_1\epsilon_{10}}\left(1+e^{b_1\epsilon_{10}}\right)b_2\epsilon_{10}\right)\right), 1, 0, 1, 2\right]\right]$$

Definition of *ct*.

```
ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_][UU[L_], UU[R_]] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
    p[_β | _δβ, _] → 0,
    p[a[f_, i_, h], β[g_]] ↦ β[f b_i ((∂_{b_t} g) /. b_t → 0)],
    p[a[f_, i_, h], a[g_, t, j_]] ↦ a[f (g /. b_t → 0), i, j],
    p[a[f_, i_, h], a[g_, j_, k_]] ↦ a[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[a[f_, i_, h], δa[g_, t, j_]] ↦ δa[f (g /. b_t → 0), i, j],
    p[a[f_, i_, h], δa[g_, j_, k_]] ↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
    p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] ↦ δaa[f (g /. b_t → 0), i, j, k, l],
    p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] ↦ δaa[f (g /. b_t → 0), j, k, i, l],
    p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] ↦ δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
    p[a[__], _] → 0, p[_δa | _δaa, _δβ | _δa | _δaa] → 0,
    p[δa[f_, i_, h], β[g_]] ↦ δβ[f b_i ((∂_{b_t} g) /. b_t → 0)],
    p[δa[f_, i_, h], a[g_, t, j_]] ↦ δa[f (g /. b_t → 0), i, j],
    p[δa[f_, i_, h], a[g_, j_, k_]] ↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[_δa, _] → 0, p[δaa[_, _, h, _, h], _] → 0,
    p[δaa[f_, i_, h, j_, k_], β[g_]] ↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] ↦ δaa[f (g /. b_t → 0), i, l, j, k],
    p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] ↦ δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
    p[δaa[f_, i_, j_, k_, h], β[g_]] ↦ δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
    p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] ↦ δaa[f (g /. b_t → 0), i, j, k, l],
    p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] ↦ δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
    p[_δaa, _] → 0 }]]];
```