

Pensieve header: Local testing notebook for Scatter and Glow in ZeroCo.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604\\Zero"];
<< ZeroLocal.m
```

In "T before H" conventions. Internal use symbols: {rr, pp}

Export

## Bracket comparisons with OneCo.pdf

```
B = bb[i, j, k];
Column@{B[UU@a[1, i, j], UU@a[1, i, k]],
  B[UU@a[1, i, k], UU@a[1, j, k]],
  B[UU@a[1, i, j], UU@a[1, j, k]],
  B[UU@a[1, i, j], UU@a[1, j, i]]}
UU[0]
UU[a[-bi, j, k] + a[bj, i, k]]
UU[a[bi, j, k] + a[-bj, i, k]]
UU[a[bi, j, i] + a[bi, j, j] + a[-bj, i, i] + a[-bj, i, j]]
```

## Bases

```
UUBasis[{1, 2}, {3, 4}, f]
{UU[β[f1[b1, b2]]], UU[a[f2[b1, b2], 1, 3]],
  UU[a[f3[b1, b2], 1, 4]], UU[a[f4[b1, b2], 2, 3]], UU[a[f5[b1, b2], 2, 4]]}
```

## Functoriality of tm

```
tmto12[u_] := (u // tm[1, 2, 1]) - (u // to[{1, 2}, {2, 1}] // tm[2, 1, 1]);
DeleteCases[
  {#} → tmto12[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}

tmto23[u_] :=
  (u // tm[1, 2, 1]) - (u // to[{2, 3}, {3, 2}] // tm[1, 3, 1] // to[2, 3]);
DeleteCases[
  {#} → tmto23[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}

```

## Meta-associativity for tm

```
((# // tm[1, 2, 1] // tm[1, 3, 1]) == (# // tm[2, 3, 2] // tm[1, 2, 1])) & /@
  UUBasis[{1, 2, 3, 4}, {5, 6}, f] // Union
{True}
```

## Functoriality of hm

```
hmho12[u_] := (u // hm[1, 2, 1]) - (u // ho[{1, 2}, {2, 1}] // hm[2, 1, 1]);
DeleteCases[
  {#} → hmho12[#] & /@ UUBasis[{1, 2}, {3, 4, 5}, f],
  _ → UU[0]
]
{}

hmho23[u_] :=
  (u // hm[1, 2, 1]) - (u // ho[{2, 3}, {3, 2}] // hm[1, 3, 1] // ho[2, 3]);
DeleteCases[
  {#} → hmho23[#] & /@ UUBasis[{1, 2}, {3, 4, 5}, f],
  _ → UU[0]
]
{}

```

## Meta-associativity for hm

```
((# // hm[1, 2, 1] // hm[1, 3, 1]) == (# // hm[2, 3, 2] // hm[1, 2, 1])) & /@
  UUBasis[{5, 6}, {1, 2, 3, 4}, f] // Union
{True}
```

## Compatibility between tm and hts

```
tmhts[u_] :=
  (u // tm[1, 2, 1] // hts[4, 1]) - (u // hts[4, 1] // hts[4, 2] // tm[1, 2, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}

```

```

tmhts[u_] :=
  (u // tm[1, 2, 1] // hts[5, 1]) - (u // hts[5, 1] // hts[5, 2] // tm[1, 2, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}

tmhts[u_] :=
  (u // tm[2, 1, 1] // hts[4, 1]) - (u // hts[4, 2] // hts[4, 1] // tm[2, 1, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}

tmhts[u_] :=
  (u // tm[2, 1, 1] // hts[5, 1]) - (u // hts[5, 2] // hts[5, 1] // tm[2, 1, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}

```

## Compatibility between hm and hts

```

hmhts[u_] :=
  (u // hm[1, 2, 1] // hts[1, 4]) - (u // hts[2, 4] // hts[1, 4] // hm[1, 2, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

hmhts[u_] :=
  (u // hm[1, 2, 1] // hts[1, 5]) - (u // hts[2, 5] // hts[1, 5] // hm[1, 2, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

```

```

hmhts[u_] :=
  (u // hm[2, 1, 1] // hts[1, 4]) - (u // hts[1, 4] // hts[2, 4] // hm[2, 1, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

hmhts[u_] :=
  (u // hm[2, 1, 1] // hts[1, 5]) - (u // hts[1, 5] // hts[2, 5] // hm[2, 1, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

```

## Functoriality of dm

```

dm $\sigma$ 12[u_] := (u // dm[1, 2, 1]) - (u // d $\sigma$ [{1, 2}, {2, 1}] // dm[2, 1, 1]);
DeleteCases[
  {#} → dm $\sigma$ 12[#] & /@ UUBasis[{1, 2, 3}, f],
  _ → UU[0]
]
{}

dm $\sigma$ 23[u_] :=
  (u // dm[1, 2, 1]) - (u // d $\sigma$ [{2, 3}, {3, 2}] // dm[1, 3, 1] // d $\sigma$ [2, 3]);
DeleteCases[
  {#} → dm $\sigma$ 23[#] & /@ UUBasis[{1, 2, 3}, f],
  _ → UU[0]
]
{}

```

## Meta-associativity for dm

```

((# // dm[1, 2, 1] // dm[1, 3, 1]) == (# // dm[2, 3, 2] // dm[1, 2, 1])) & /@
  UUBasis[4, f] // Union
{True}

```

## Functoriality of tb

```

tbto01[u_, v_] :=
  tb[0][u, v] - (tb[1][u // tσ[{0, 1}, {1, 0}], v // tσ[0, 1]] // tσ[{1, 0}, {0, 1}]);
DeleteCases[
  Flatten[Table[
    {u, v} → tbto01[u, v],
    {u, UUBasis[{0, 1, 2}, {1, 2}, f]}, {v, UUBasis[{0, 3}, {3, 4}, g]}
  ]],
  _ → UU[0]
]
{}

tbto12[u_, v_] :=
  tb[0][u, v] - (tb[0][u // tσ[{1, 2}, {2, 1}], v] // tσ[{1, 2}, {2, 1}]);
DeleteCases[
  Flatten[Table[
    {u, v} → tbto12[u, v],
    {u, UUBasis[{0, 1, 2}, {1, 2}, f]}, {v, UUBasis[{0, 3}, {3, 4}, g]}
  ]],
  _ → UU[0]
]
{}

```

## Leibnitz for tm/tb

Following <http://drorbn.net/bbs/show?shot=VanDerVeen-160304-100231.jpg>

```

tmtb[u_, v_] := tb[3][u // tm[1, 2, 3], v] -
  tm[3, 2, 3][tb[3][u // tσ[1, 3], v]] - tm[1, 3, 3][tb[3][u // tσ[2, 3], v]];
DeleteCases[
  Flatten[Table[
    {u, v} → tmtb[u, v],
    {u, UUBasis[{1, 2, 4}, {1, 2}, f]}, {v, UUBasis[{3, 5}, {3, 4}, g]}
  ]],
  _ → UU[0]
]
{}

```

## Meta-AS and meta-Jacobi for tb

```

tbAS[u_, v_] := tb[0][u, v] + tb[0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → tbAS[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{0, 2}, {3, 4}, g]
  ]],
  _ → UU[0]
]
{}

tbJacobi[u_, v_, w_] :=
  tb[0][u, tb[0][v, w]] + tb[0][v, tb[0][w, u]] + tb[0][w, tb[0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → tbJacobi[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{0, 2}, {3, 4}, g], UUBasis[{0, 3}, {5, 6}, h]
  ]],
  _ → UU[0]
]
{}

```

## Compatibility of tb and tm

```

tbtm[u_, v_] := Plus[
  tb[1][u, tσ[2, -2][v]] // tm[2, -2, 2],
  -tb[1][u, tσ[2, -2][v]] // tm[-2, 2, 2],
  -tb[2][u, tσ[1, -1][v]] // tm[1, -1, 1],
  tb[2][u, tσ[1, -1][v]] // tm[-1, 1, 1]
];
DeleteCases[
  Flatten[Outer[
    {##} → tbtm[##] &,
    UUBasis[{1, 2, 3}, {1, 2}, f], UUBasis[{1, 2, 4}, {3, 4}, f]
  ]],
  _ → UU[0]
]
{}

```

## Functoriality of hb

```

hbhσ01[u_, v_] :=
  hb[0][u, v] - (hb[1][u // hσ[{0, 1}, {1, 0}], v // hσ[0, 1]] // hσ[{1, 0}, {0, 1}]);
DeleteCases[
  Flatten[Table[
    {u, v} → hbhσ01[u, v],
    {u, UUBasis[{1, 2}, {0, 1, 2}, f]}, {v, UUBasis[{3, 4}, {0, 3}, g]}
  ]],
  _ → UU[0]
]
{}

hbhσ12[u_, v_] :=
  hb[0][u, v] - (hb[0][u // hσ[{1, 2}, {2, 1}], v] // hσ[{1, 2}, {2, 1}]);
DeleteCases[
  Flatten[Table[
    {u, v} → hbhσ12[u, v],
    {u, UUBasis[{1, 2}, {0, 1, 2}, f]}, {v, UUBasis[{3, 4}, {0, 3}, g]}
  ]],
  _ → UU[0]
]
{}

```

## Leibnitz for hm/hb

Following <http://drorbn.net/bbs/show?shot=VanDerVeen-160304-100231.jpg>

```

hmhb[u_, v_] := hb[3][u // hm[1, 2, 3], v] -
  hm[3, 2, 3][hb[3][u // hσ[1, 3], v]] - hm[1, 3, 3][hb[3][u // hσ[2, 3], v]];
DeleteCases[
  Flatten[Table[
    {u, v} → hmhb[u, v],
    {u, UUBasis[{1, 2}, {1, 2, 4}, f]}, {v, UUBasis[{3, 4}, {3, 5}, g]}
  ]],
  _ → UU[0]
]
{}

```

## Meta-AS and meta-Jacobi for hb

```

hbAS[u_, v_] := hb[0][u, v] + hb[0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → hbAS[##] &,
    UUBasis[{1, 2}, {0, 1}, f], UUBasis[{3, 4}, {0, 2}, g]
  ]],
  _ → UU[0]
]
{}

hbJacobi[u_, v_, w_] :=
  hb[0][u, hb[0][v, w]] + hb[0][v, hb[0][w, u]] + hb[0][w, hb[0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → hbJacobi[##] &,
    UUBasis[{1, 2}, {0, 1}, f], UUBasis[{3, 4}, {0, 2}, g], UUBasis[{5, 6}, {0, 3}, h]
  ]],
  _ → UU[0]
]
{}

```

## Compatibility of hb and hm

```

hbhm[u_, v_] := Plus[
  hb[1][u, hσ[2, -2][v]] // hm[2, -2, 2],
  -hb[1][u, hσ[2, -2][v]] // hm[-2, 2, 2],
  -hb[2][u, hσ[1, -1][v]] // hm[1, -1, 1],
  hb[2][u, hσ[1, -1][v]] // hm[-1, 1, 1]
];
DeleteCases[
  Flatten[Outer[
    {##} → hbhm[##] &,
    UUBasis[{1, 2}, {1, 2, 3}, f], UUBasis[{3, 4}, {1, 2, 4}, g]
  ]],
  _ → UU[0]
]
{}

```



## Meta-Jacobi for thb/tb and for thb/hb

```

tthJacobi[u_, v_, w_] := Plus[
  -thb[0, 0][tb[0][u, v], w] + tb[0][thb[0, 0][u, w], v] - thb[0, 0][v,
    thb[0, 0][u, w]] + tb[0][u, thb[0, 0][v, w]] + thb[0, 0][u, thb[0, 0][v, w]]
];
DeleteCases[
  Flatten[Outer[
    {##} → tthJacobi[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{0, 2}, {3, 4}, g], UUBasis[{3, 4}, {0, 5}, h]
  ]],
  _ → UU[0]
]
{}

thhJacobi[u_, v_, w_] := Plus[
  -thb[0, 0][u, hb[0][v, w]] + hb[0][thb[0, 0][u, v], w] + thb[0, 0][thb[0, 0][u, v],
    w] + hb[0][v, thb[0, 0][u, w]] - thb[0, 0][thb[0, 0][u, w], v]
];
DeleteCases[
  Flatten[Outer[
    {##} → thhJacobi[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{2, 3}, {0, 3}, g], UUBasis[{4, 5}, {0, 4}, h]
  ]],
  _ → UU[0]
]
{}

```

## Leibnitz for dm/db

Following <http://drorbn.net/bbs/show?shot=VanDerVeen-160304-100231.jpg>

```

dmdb[u_, v_] := db[3][u // dm[1, 2, 3], v] -
  dm[3, 2, 3][db[3][u // dσ[1, 3], v]] - dm[1, 3, 3][db[3][u // dσ[2, 3], v]];
DeleteCases[
  Flatten[Table[
    {u, v} → dmdb[u, v],
    {u, UUBasis[{1, 2, 4}, f]}, {v, UUBasis[{3, 5}, g]}
  ]],
  _ → UU[0]
]
{}

```

## Meta-AS and meta-Jacobi for db

```

dbAS[u_, v_] := db[0][u, v] + db[0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → dbAS[##] &,
    UUBasis[{0, 1, 2}, f], UUBasis[{0, 3, 4}, g]
  ]],
  _ → UU[0]
]
{}

dbJacobi[u_, v_, w_] :=
  db[0][u, db[0][v, w]] + db[0][v, db[0][w, u]] + db[0][w, db[0][u, v]];
Module[{len, bas1, bas2, bas3, test},
  len = Length[bas1 = UUBasis[{0, 1, 2}, f]];
  bas2 = UUBasis[{0, 3, 4}, g];
  bas3 = UUBasis[{0, 5, 6}, h];
  DeleteCases[
    Flatten[Table[
      test = {bas1[[i]], bas2[[j]], bas3[[k]]};
      test → dbJacobi @@ test,
      {i, 1, len - 2}, {j, i + 1, len - 1}, {k, j + 1, len}
    ]],
    _ → UU[0]
  ]
]
{}

```

## Compatibility of db and dm

```

dbdm[u_, v_] := Plus[
  db[1][u, dσ[2, -2][v]] // dm[2, -2, 2],
  -db[1][u, dσ[2, -2][v]] // dm[-2, 2, 2],
  -db[2][u, dσ[1, -1][v]] // dm[1, -1, 1],
  db[2][u, dσ[1, -1][v]] // dm[-1, 1, 1]
];
DeleteCases[
  Flatten[Outer[
    {##} → dbdm[##] &,
    UUBasis[{1, 2, 3}, f], UUBasis[{1, 2, 4}, g]
  ]],
  _ → UU[0]
]
{}

```

## AS and Jacobi for bb

```

bb4 = bb[1, 2, 3, 4];
bbAS[u_, v_] := bb4[u, v] + bb4[v, u];
DeleteCases[
  Flatten[Outer[
    {##} → bbAS[##] &,
    UUBasis[2, f], UUBasis[2, g]
  ]],
  _ → UU[0]
]
{}

```

```

bb3 = bb[1, 2, 3];
bbJacobi[u_, v_, w_] := bb3[u, bb3[v, w]] + bb3[v, bb3[w, u]] + bb3[w, bb3[u, v]];
Module[{bas1, bas2, bas3, len, test},
  {bas1, bas2, bas3} = {UUBasis[3, f], UUBasis[3, g], UUBasis[3, h]};
  len = Length[bas1];
  DeleteCases[
    Flatten[Table[
      test = {bas1[[i]], bas2[[j]], bas3[[k]]};
      test → bbJacobi @@ test,
      {i, 1, len - 2}, {j, i + 1, len - 1}, {k, j + 1, len}
    ]],
    _ → UU[0]
  ]
]
{}

```

## ct (contract) poperties

```

cttm[u_, v_] := ct[][u // tm[1, 2, 1], v] - (ct[][u, v] // tm[1, 2, 1]);
DeleteCases[
  Flatten[Table[
    {u, v} → cttm[u, v],
    {u, UUBasis[{1, 2, 3}, {0, 1}, f]}, {v, UUBasis[{0, 4}, {2, 3}, g]}
  ]],
  _ → UU[0]
]
{}

Outer[
  {#1, #2} → Equal[
    ct[][#1 // hm[1, 2, 1], #2],
    ct[][#1, #2] // hm[1, 2, 1]
  ] &,
  UUBasis[{1, 2}, {0, 1, 2, 3}, f], UUBasis[{0, 4}, {4, 5}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

```

```
Outer[
  {#1, #2} → Equal[
    ct[][#1, #2 // hm[2, 3, 2]],
    ct[][#1, #2] // hm[2, 3, 2]
  ] &,
  UUBasis[{1, 2}, {0, 1}, f], UUBasis[{0, 3}, {2, 3, 4}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

Outer[
  {#1, #2} → Equal[
    ct[][#1, #2 // tm[3, 4, 3]],
    ct[][#1, #2] // tm[3, 4, 3]
  ] &,
  UUBasis[{1, 2}, {0, 1}, f], UUBasis[{0, 3, 4, 5}, {2, 3}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

```