OneCo-1604 Implementation Showcase

In $\mathcal{U}(T)\otimes\mathcal{U}(H)$ conventions.

Generalities.

```
Simp[ε_] := Simplify[ε];
CF[ε_] :=
  ε /. λ_β | λ_a | λ_δβ | λ_δa | λ_δaa | λ_c | λ_ao | λ_ca | λ_aao ⧴
    MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
   λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f + g], r];
   λ /: g_ * λ[f_, r___] := λ[Simp[g f], r]);
AutoCollecting /@ {β, a, δβ, δa, δaa, c, ao, ca, aao};
UU /: UU[x_] + UU[y_] := UU[x + y];
UU /: a_ * UU[x_] := UU[Expand[a x]];
UU /: D[u_UU, vs__] :=
  CF[u /. λ_β | λ_a | λ_δβ | λ_δa | λ_δaa ⧴
     MapAt[D[#, vs] &, λ, 1]];
b_ç = 1;
γa[f_, j_, k_, l_, m_] :=
  δaa[f, j, k, l, m] - δaa[b_j f, ç, k, l, m];
hb[f_, i_, j_, k_] := a[b_j f, i, k] - a[b_i f, j, k]
 (*=f[a_ik,a_jk]*);
δhb[f_, i_, j_, k_] := δa[b_j f, i, k] - δa[b_i f, j, k];
δtb[f_, x_, j_, l_] := δaa[f, ç, l, x, j] - δaa[f, ç, j, x, l];
UU /: Coefficient[u_UU, λ_[js___]] :=
  Total[Cases[u, λ[f_, js] ⧴ f, ∞]];
Kδ /: Kδ_is__ := KroneckerDelta[1, Length[Union[{is}]]];
```

******

```
UU /: UU[L_] ** UU[R_] :=
  CF@UU[Expand[Distribute[pp[L, R]] /. {
       pp[0, _] → 0, pp[_, 0] → 0,
       pp[β[f_], λ_β | λ_a | λ_δβ | λ_δa | λ_δaa] ⧴
        MapAt[f # &, λ, 1],
       pp[_δβ | _δa | _δaa, _δβ | _δa | _δaa] → 0,
       pp[δa[f_, i_, j_], a[g_, k_, l_]] ⧴
        δaa[f g, i, j, k, l] - Kδ_jk δhb[f g, i, j, l],
       pp[δβ[f_], β[g_]] ⧴ δβ[f g],
       pp[δβ[f_], a[g_, i_, j_]] ⧴ δa[f g, i, j]
      }]];
```

ÅForm.

```
ÅForm = ReplaceAll[{
    δa[f_, ç, j_] ⧴ c[f, j],
    δa[f_, i_, j_] ⧴ ao[f, i, j] + c[b_i f, j],
    δaa[f_, ç, j_, k_, l_] ⧴ ca[f, j, k, l],
    δaa[f_, j_, k_, ç, l_] ⧴
     ca[f, l, j, k] + Kδ_kl ao[f, j, k],
    δaa[f_, i_, j_, k_, l_] ⧴
     aao[f, i, j, k, l] + ca[b_i f, j, k, l] +
      ca[b_k f, l, i, j] + Kδ_jl ao[b_k f, i, j] }];
δaForm = ReplaceAll[{c[f_, i_] ⧴ δa[f, ç, i],
    ao[f_, i_, j_] ⧴ δa[f, i, j] - δa[b_i f, ç, j],
    ca[f_, i_, j_, k_] ⧴ δaa[f, ç, i, j, k],
    aao[f_, i_, j_, k_, l_] ⧴
     δaa[f, i, j, k, l] - δaa[b_i f, ç, j, k, l] -
      δaa[b_k f, i, j, ç, l] }];
```

δaa relations.

```
i_ ≼ j_ := OrderedQ[{i, j}]; i_ ≺ j_ := ! OrderedQ[{j, i}];
CF[UU[ε_]] := UU[CF[ε /. δaa[f_, i_, j_, k_, l_] ⧴ Which[
      k === ç, δaa[f, ç, l, i, j] + Kδ_jl δhb[f, i, ç, j],
      (i === ç) ∨ (i ≼ k ∧ j ≼ l), δaa[f, i, j, k, l],
      k ≺ i ∧ j ≺ l, δaa[f, k, j, i, l] +
       δaa[-f b_i, ç, l, k, j] + δaa[f b_i, ç, j, k, l] +
       δaa[-f b_k, ç, j, i, l] + δaa[f b_k, ç, l, i, j],
      k ≺ i ∧ j === l, δa[-f b_i, k, j] + δa[f b_k, i, j] +
       δaa[f, k, j, i, j],
      i ≼ k ∧ l ≺ j, δaa[f, i, l, k, j] +
       δaa[-f b_i, ç, l, k, j] + δaa[f b_i, ç, j, k, l] +
       δaa[-f b_k, ç, j, i, l] + δaa[f b_k, ç, l, i, j],
      k ≺ i ∧ l ≺ j, δaa[f, k, l, i, j]
     ]]];
```

Definition of *tm*.

```
UU[ε_] // tm[x_, y_, z_] := (rr = Replace[x | y → z];
   CF[UU[Expand[ε /. {
        a[f_, x, j_] ⧴ a[f, z, j] + δhb[∂_(b_y) f, z, ç, j],
        a[f_, y, j_] ⧴ a[f, z, j],
        δa[f_, x | y, j_] ⧴ δa[f, z, j],
        δaa[f_, i_, j_, k_, l_] ⧴
         δaa[f, rr@i, j, rr@k, l]
       } /. b_(x|y) → b_z]]]);
```

Definition of *hm*.

```
UU[ε_] // hm[x_, y_, z_] := (rr = Replace[x | y → z];
   CF[UU[Expand[ε /. {
       a[f_, i_, x | y] ⧴ a[f, i, z],
       δa[f_, i_, x | y] ⧴ δa[f, i, z],
       δaa[f_, i_, y, k_, x] ⧴ δaa[f, k, z, i, z],
       δaa[f_, i_, j_, k_, l_] ⧴ δaa[f, i, rr@j, k, rr@l]
      }]]]);
```

Definition of *hts*.

```
UU[ε_] // hts[y_, x_] := CF[UU[Expand[ε /. {
      a[f_, i_, j_] ⧴ a[f, i, j] - Kδ_jy δhb[∂_(b_x) f, i, ç, y] -
        Kδ_ix Kδ_jy (β[f b_x] - δa[f, ç, y] - δβ[b_x ∂_(b_x) f]),
      δa[f_, x, y] ⧴ δa[f, x, y] - δβ[f b_x],
      δaa[f_, i_, j_, k_, l_] ⧴
       δaa[f, i, j, k, l] + Kδ_ix Kδ_jy δa[-b_x f, k, l] +
        Kδ_ix Kδ_ly (δa[b_k f, x, j] - δa[b_x f, k, j]) +
        Kδ_kx Kδ_jy (δa[b_i f, x, l] - δa[b_x f, i, l]) +
        Kδ_kx Kδ_ly δa[-b_x f, i, j] - Kδ_ix Kδ_jly δβ[b_x b_k f] +
        2 Kδ_xik Kδ_yjl δβ[b_x b_x f]
     }]]];
```

Definition of *dm*.

```
dm[x_, y_, z_][ε_] :=
 ε // hts[x, y] // tm[x, y, z] // hm[x, y, z]
```

```
tb[x_][UU[L_], UU[R_]] :=
  CF[UU[Expand[Distribute[pp[L, R]] /. {
        pp[0, _] → 0, pp[_, 0] → 0,
        pp[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
        pp[u_β | u_δβ | u_δa | u_δaa, v_a] :→ -pp[v, u]
      } /. {
       pp[a[f_, x, j_], u_] :→ (u /. {
           β[g_] :→ δhb[f ∂_{b_x} g, x, ç, j],
           a[g_, k_, l_] :→ γa[f ∂_{b_x} g, x, j, k, l] +
             Kδ_{xk} (-γa[g ∂_{b_k} f, k, l, x, j] +
                δtb[f g, x, j, l]),
           _ → 0
          }),
       pp[a[f_, j_, k_], a[g_, x, l_]] /; j =!= x :→
        -γa[g ∂_{b_x} f, x, l, j, k],
       pp[_, _] → 0
      }]]];
```

Definition of *tb*.

```
thb[x_, y_][UU[L_], UU[R_]] :=
  CF[UU[Expand[Distribute[pp[L, R]] /. {
       pp[0, _] → 0, pp[_, 0] → 0,
       pp[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
       pp[_a, _β | _δβ] → 0,
       pp[β[f_], a[g_, i_, j_]] :→
        Kδ_{yj} δhb[g ∂_{b_x} f, i, ç, y],
       pp[a[f_, i_, j_], a[g_, k_, l_]] :→ Kδ_{yl} (
          γa[g ∂_{b_x} f, k, l, i, j] + Kδ_{xi} (
             δhb[-b_k g ∂_{b_x} f, i, ç, j] + δa[b_k g ∂_{b_x} f, i, j] -
              δa[b_i g ∂_{b_x} f, k, j] + hb[f g, k, i, j] +
              δaa[f g, ç, j, k, l] - δaa[f g, ç, l, k, j])),
       pp[a[f_, i_, j_], δa[g_, k_, l_]] :→
        Kδ_{xi} Kδ_{yl} (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
       pp[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] :→ Kδ_{xi} (
          Kδ_{yl} (-δaa[b_k f g, i, j, m, n] +
              δaa[b_i f g, k, j, m, n]) +
           Kδ_{yn} (-δaa[b_m f g, k, l, i, j] +
              δaa[b_i f g, k, l, m, j]) +
           Kδ_{yln} (δa[b_x b_m f g, k, j] - δa[b_k b_m f g, x, j])),
       pp[_δβ, _a] → 0,
       pp[δa[f_, i_, j_], a[g_, k_, l_]] :→
        Kδ_{xi} Kδ_{yl} (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
       pp[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] :→
        Kδ_{xi} Kδ_{yl} (-δaa[b_k f g, i, j, m, n] +
            δaa[b_i f g, k, j, m, n]) +
         Kδ_{xm} Kδ_{yl} (-δaa[b_k f g, i, j, m, n] +
            δaa[b_m f g, i, j, k, n])}]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
```

Definition of *thb*.

```
hb[y_][UU[L_], UU[R_]] :=
  CF[UU[Expand[Distribute[pp[L, R]] /. {
        pp[0, _] → 0, pp[_, 0] → 0,
        pp[_β | _δβ, _] → 0,
        pp[_, _β | _δβ] → 0,
        pp[_δa | _δaa, _δa | _δaa] → 0,
        pp[u_δa | u_δaa, v_a] :→ -pp[v, u]
       } /. {
       pp[a[f_, i_, y], u_] :→ (u /. {
            a[g_, j_, k_] :→ Kδ_{yk} hb[f g, i, j, k],
            δa[g_, j_, k_] :→ Kδ_{yk} δhb[f g, i, j, k],
            δaa[g_, j_, k_, l_, m_] :→
             Kδ_{yk} (δaa[b_j f g, i, y, l, m] -
                 δaa[b_i f g, j, k, l, m]) +
              Kδ_{ym} (δaa[b_l f g, j, k, i, y] -
                 δaa[b_i f g, j, k, l, m])
           }),
       _pp → 0
      }]]];
```

Definition of *hb*.

$h_1h_2t_1t_2 \to h_1h_2t_1t_2 \to h_1h_2t_2t_1 \to h_2h_1t_2t_1 \to h_2h_1t_2t_1$: *db*.

```
db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
    htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] //
     hm[x, h, x],
    tb[x][u, v // hσ[x, h]] // hm[x, h, x],
    hb[x][u, v // tσ[x, t]] // tm[t, x, x],
    thb[x, x][u // hσ[x, h], v // tσ[x, t]] //
      tm[t, x, x] // hm[x, h, x] ]];
```

```
bb[S_List] := Module[{w, bar, t, n = 0, i, k},
   w = #2 // dσ[S, bar /@ S];
   Sum[t = db[S⟦k⟧][#1, w // dσ[bar[S⟦k⟧], S⟦k⟧]];
    Do[t = t // dm[bar[S⟦i⟧], S⟦i⟧, S⟦i⟧], {i, 1, k - 1}];
    Do[t = t // dm[S⟦i⟧, bar[S⟦i⟧], S⟦i⟧],
     {i, k + 1, Length@S}];
    t, {k, Length@S}] ] &
bb[S___] := bb[{S}]
```

The bracket.

```
σ[γ_TSD0] := Keys @@ γ;
TSD0[λ_]_{j_} := Lookup[λ, j, UU@a[1, j, h∞]];
UU[u_] // γ_TSD0 := CF[u /. λ_a :→ γ@λ];
TSD0 /: (γ_TSD0)^-1 := Module[{S = σ@γ, m},
   m = Table[Coefficient[γ_i , a[j, h∞]], {i, S}, {j, S}] //
     Inverse;
   TSD0@<|Table[S⟦α⟧ →
       CF@UU@Sum[a[m⟦α, β⟧, S⟦β⟧, h∞], {β, Length@S}],
      {α, Length@S}]|> ];
a[f_, j_, k_] // γ_TSD0 := Module[{S = Keys @@ γ, γi},
   Switch[{MemberQ[S, j], MemberQ[S, k]},
    {False, False}, UU@a[f, j, k],
    {True, False}, γ_j /. a[g_, i_, h∞] :→ a[f g, i, k],
    {False, True}, (γi = γ^-1;
     CF@Sum[
       γ[bb[S ⋃ {j}][γi_i, UU@a[f, j, k]] /.
           _δβ | _δa | _δaa → 0] /. {
          a[_, i, h∞] :→ 0, a[g_, l_, h∞] :→ a[g / b_i, l, i]},
       {i, S}]),
    {True, True}, ct[h∞, t∞][γ@a[f, j, h∞],
     γ@a[1, t∞, k]] ]];
```

0-Co.

Definition of *ct*.

```
ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_][UU[L_], UU[R_]] := CF[UU[Distribute[pp[L, R]] /. {
    pp[_β | _δβ, _] → 0,
    pp[a[f_, i_, h], β[g_]] :→ β[f b_i ((∂_{b_t} g) /. b_t → 0)],
    pp[a[f_, i_, h], a[g_, t, j_]] :→ a[f (g /. b_t → 0), i, j],
    pp[a[f_, i_, h], a[g_, j_, k_]] :→ a[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    pp[a[f_, i_, h], δa[g_, t, j_]] :→ δa[f (g /. b_t → 0), i, j],
    pp[a[f_, i_, h], δa[g_, j_, k_]] :→ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    pp[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
    pp[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] :→ δaa[f (g /. b_t → 0), i, j, k, l],
    pp[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] :→ δaa[f (g /. b_t → 0), j, k, i, l],
    pp[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] :→ δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
    pp[a[__], _] → 0, pp[_δa | _δaa, _δβ | _δa | _δaa] → 0,
    pp[δa[f_, i_, h], β[g_]] :→ δβ[f b_i ((∂_{b_t} g) /. b_t → 0)],
    pp[δa[f_, i_, h], a[g_, t, j_]] :→ δa[f (g /. b_t → 0), i, j],
    pp[δa[f_, i_, h], a[g_, j_, k_]] :→ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    pp[_δa, _] → 0, pp[δaa[_, _, h, _, h], _] → 0,
    pp[δaa[f_, i_, h, j_, k_], β[g_]] :→ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    pp[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] :→ δaa[f (g /. b_t → 0), i, l, j, k],
    pp[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :→ δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
    pp[δaa[f_, i_, j_, k_, h], β[g_]] :→ δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
    pp[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] :→ δaa[f (g /. b_t → 0), i, j, k, l],
    pp[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :→ δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
    pp[_δaa, _] → 0 }]];
```

Global Generalities.

```
TSD[λ_List] := TSD[Association @@ λ];
σ[γ_TSD] := Keys @@ γ;
TSD[λ_]_j_ := Lookup[λ, j, UU@a[1, j, h∞]];
UU[u_] // γ_TSD :=
  CF[u /. λ_β | λ_a | λ_δa | λ_δaa :→ γ@λ];
TSD0[γ_TSD] :=
  TSD0[Table[k → CF[γ_k /. _δβ | _δa | _δaa → 0], {k, σ@γ}]];
TSD /: (γ1_TSD) ** (γ2_TSD) :=
  TSD[Table[k → γ2[γ1_k], {k, σ[γ1] ⋃ σ[γ2]}]];
```

h∞ Scattering.

```
β[f_] // γ_TSD := UU[β[f]] + Sum[
    ct[h∞, t∞][γ_k - UU[a[1, k, h∞]], UU[β[b_t∞ ∂_{b_k} f]]],
    {k, σ@γ}];
a[f_, j_, h∞] // γ_TSD := (β[f] // γ) ** γ_j;
δa[f_, ç, k_] // γ_TSD :=
  ct[h∞, t∞][UU@δa[1, ç, h∞], TSD0[γ][a[f, t∞, k]]];
δa[f_, j_, k_] // γ_TSD :=
  UU[δβ[1]] ** CF[a[f, j, k] // TSD0[γ]];
δaa[f_, i_, j_, k_, h∞] // γ_TSD :=
  (δa[f, i, j] // γ) ** TSD0[γ]_k +
   Kδ_jk γ[UU@δhb[f, i, j, h∞]];
δaa[f_, k_, h∞, i_, j_] // γ_TSD := δaa[f, i, j, k, h∞] // γ;
```

Exponentiating an arrow.

```
ϕ_k_[x_] := x^-k (e^x - Sum[x^α / α!, {α, 0, k - 1}]);
Ea[t_, j_, k_] := TSD[{
    j → UU[a[1, j, h∞] + δaa[t, ç, h∞, j, k] +
        δaa[-t ϕ_1[-t b_j], ç, k, j, h∞] +
        δaa[-t^2 ϕ_2[-t b_j], j, k, j, h∞]],
    k → UU[a[1, k, h∞] + hb[-t ϕ_1[t b_j], j, k, h∞] +
        δhb[t^2 b_k e^(t b_j) ϕ_2[-t b_j], ç, j, h∞] +
        δaa[-t^2 b_j e^(t b_j) ϕ_2[-t b_j], ç, k, k, h∞] +
        δaa[t^2 e^(t b_j) ϕ_2[-t b_j], j, k, k, h∞] +
        δaa[-t ϕ_1[t b_j] - t^2 b_k ϕ_2[t b_j], ç, h∞, j, k] +
        δaa[t ϕ_1[-t b_j] + t^2 b_k e^(t b_j) ϕ_2[-t b_j], ç, k, j, h∞] +
        δaa[t (ϕ_1[t b_j] - ϕ_1[-t b_j]) / b_j -
          (2 - 2 e^(t b_j) + (1 + e^(t b_j)) t b_j) b_k / b_j^3, j, k, j, h∞]]}];
Ea[j_, k_] := Ea[1, j, k];
```

The best available *R*.

```
Table[
  CF[R[1, 2]_i /. {gg_{2|4|5|7}[_] → 0, gg_6[x_] :→ (2-x)/(2 x^2),
    gg_8[x_] :→ 1/x, cc_1 → 0}], {i, 2}] // ÅForm
```

```
{UU[a[1, 1, h∞] + aao[-(-1+e^(-b_1+b_1))/b_1^2, 1, 2, 1, h∞] +
   aao[1/b_2, 1, 2, 2, h∞] + ca[(1-e^(-b_1))/b_1, h∞, 1, 2]],
 UU[a[e^(b_1), 2, h∞] + a[-((-1+e^(b_1)) b_2)/b_1, 1, h∞] +
   aao[(e^(-b_1) (-1+e^(b_1))^2)/b_1^2, 1, 2, 1, h∞] + aao[(1-e^(b_1))/(b_1 b_2), 1, 2, 2, h∞] +
   ao[(-1+e^(b_1)-e^(b_1) b_2)/b_1, 1, h∞] + ca[-(1-e^(-b_1))/b_1, h∞, 1, 2]]}
```

Recovery challenges.

```
AutoAd[bb[j, k], UU@a[1, j, k]][UU@a[1, 0, j]]
```

$$
\begin{aligned}
&\text{UU}\Big[a[1, 0, j] + a\left[1 - e^{-b_j}, 0, k\right] + \\
&\quad a\left[\frac{\left(-1+e^{-b_j}\right) b_0}{b_j}, j, k\right] + \delta a\left[-1 - e^{-b_j} + \frac{1-e^{-2 b_j}}{b_j}, 0, k\right] + \\
&\quad \delta a\left[b_0 \left(1 + \frac{-1+e^{-b_j}}{b_j}\right), ç, k\right] + \delta a\left[\frac{e^{-2 b_j} b_0 \left(1+e^{b_j} \left(-1+b_j\right)\right)}{b_j^2}, j, k\right] + \\
&\quad \delta aa\left[\frac{2 e^{-b_j} b_0 \left(\text{Sinh}[b_j]-b_j\right)}{b_j^3}, j, k, j, k\right] + \\
&\quad \delta aa\left[\frac{b_0 \left(1-e^{-b_j}-b_j\right)}{b_j^2}, ç, j, j, k\right] + \\
&\quad \delta aa\left[\frac{1-e^{-b_j}}{b_j}, ç, k, 0, j\right] + \delta aa\left[\frac{-1+e^{-b_j}}{b_j}, ç, j, 0, k\right] + \\
&\quad \delta aa\left[\frac{e^{-2 b_j} \left(-1-e^{b_j} \left(-1+b_j\right)\right)}{b_j}, ç, k, 0, k\right] + \\
&\quad \delta aa\left[\frac{-1+e^{-b_j}+b_j}{b_j^2}, 0, j, j, k\right] + \\
&\quad \delta aa\left[\frac{e^{-2 b_j} \left(1+e^{2 b_j} \left(-1+b_j\right)+e^{b_j} b_j\right)}{b_j^2}, 0, k, j, k\right] + \\
&\quad \delta aa\left[-\frac{e^{-2 b_j} b_0 \left(-1+e^{b_j}+e^{b_j} \left(-2+e^{b_j}\right) b_j\right)}{b_j^2}, ç, k, j, k\right]\Big]
\end{aligned}
$$

```
AutoAd[bb[j, k], UU@a[1, j, k]][UU@a[1, 0, k]]
```

$$
\begin{aligned}
&\text{UU}\Big[a\left[e^{-b_j}, 0, k\right] + a\left[\frac{\left(1-e^{-b_j}\right) b_0}{b_j}, j, k\right] + \\
&\quad \delta a\left[\frac{e^{-2 b_j} b_0 \left(-1-e^{b_j} \left(-1+b_j\right)\right)}{b_j^2}, j, k\right] + \\
&\quad \delta a\left[\frac{e^{-2 b_j} \left(1+e^{b_j} \left(-1+b_j\right)\right)}{b_j}, 0, k\right] + \\
&\quad \delta aa\left[\frac{e^{-2 b_j} \left(-1-e^{b_j} \left(-1+b_j\right)\right)}{b_j^2}, 0, k, j, k\right] + \\
&\quad \delta aa\left[\frac{2 e^{-b_j} b_0 \left(\text{Sinh}[b_j]-b_j\right)}{b_j^2}, ç, k, j, k\right] + \\
&\quad \delta aa\left[\frac{e^{-2 b_j} \left(1+e^{b_j} \left(-1+b_j\right)\right)}{b_j}, ç, k, 0, k\right] + \\
&\quad \delta aa\left[\frac{2 e^{-b_j} b_0 \left(-\text{Sinh}[b_j]+b_j\right)}{b_j^3}, j, k, j, k\right]\Big]
\end{aligned}
$$