

Pensieve header: OneCo, the local picture. Continues pensieve://2016-03/SnG.nb.

```
Print["In the U(T)⊗U(H) conventions. Internal use symbols: ", {rr, pp}]
```

```
In the U(T)⊗U(H) conventions. Internal use symbols: {rr, pp}
```

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"]
```

```
C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604
```

## Generalities

Generalities

```
Simp[ε_] := Expand[ε];
CF[ε_] := ε /. λ_β | λ_a | λ_δβ | λ_δa | λ_δaa => MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
  λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f+g], r];
  λ /: g_*λ[f_, r___] := λ[Simp[gf], r]);
AutoCollecting /@ {β, a, δβ, δa, δaa};
UU /: UU[x_] + UU[y_] := UU[x+y];
UU /: a_*UU[x_] := UU[Expand[a x]];
UU /: D[u_UU, vs_] :=
  CF[u /. λ_β | λ_a | λ_δβ | λ_δa | λ_δaa => MapAt[D[#, vs] &, λ, 1]];
UU /: β[f_] ** u_UU := CF[u /. λ_β | λ_a | λ_δβ | λ_δa | λ_δaa => MapAt[f# &, λ, 1]];
b_φ = 1; ca[f_, i_, j_, k_] := δaa[f, φ, i, j, k];
γ[f_, j_, k_] := δa[f, j, k] - δa[b_j f, φ, k];
γa[f_, j_, k_, l_, m_] := δaa[f, j, k, l, m] - ca[b_j f, k, l, m];
Kδ /: Kδ_{is__} := KroneckerDelta[1, Length[Union[{is}]]];
```

## δaa relations

delta-aa

```
i_ ≤ j_ := OrderedQ[{i, j}]; i_ < j_ := ! OrderedQ[{j, i}];
CF[UU[ε_]] := UU[CF[ε /. δaa[f_, i_, j_, k_, l_] => Which[
  k === φ, δaa[f, φ, l, i, j] + Kδ_{jl} γ[f, i, j],
  (i === φ) ∨ (i ≤ k ∧ j ≤ l), δaa[f, i, j, k, l],
  k < i ∧ j < l, δaa[f, k, j, i, l] + ca[-f b_i, l, k, j] +
  ca[f b_i, j, k, l] + ca[-f b_k, j, i, l] + ca[f b_k, l, i, j],
  k < i ∧ j === l, δa[-f b_i, k, j] + δa[f b_k, i, j] + δaa[f, k, j, i, j],
  i ≤ k ∧ l < j, δaa[f, i, l, k, j] + ca[-f b_i, l, k, j] +
  ca[f b_i, j, k, l] + ca[-f b_k, j, i, l] + ca[f b_k, l, i, j],
  k < i ∧ l < j, δaa[f, k, l, i, j]
]]];
```

## Bases

```

UUBasis[T_List, H_List, f_] := Module[
  {ff, n = 0, h, t, h1, h2},
  ff := f_{++n} @@ Table[b_t, {t, T}];
  CF /@ UU /@ Flatten@{
     $\beta$ [ff],  $\delta\beta$ [ff],
    Table[a[ff, t, h], {t, T}, {h, H}],
    Table[ $\delta a$ [ff, t, h], {t, T  $\cup$  { $\zeta$ }}, {h, H}],
    Table[ca[ff, h1, t, h2], {t, T}, {h1, H}, {h2, H}],
    Table[ $\delta aa$ [ff, T[[i]], H[[j]], T[[k]], H[[l]],
      {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
  ] /. 1_[___]  $\rightarrow$  1
];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];

```

## tm, hm, hts, dm

tm-def

```

UU[ $\varepsilon$ ] // tm[x_, y_, z_] := (rr = Replace[x | y  $\rightarrow$  z];
  CF[UU[Expand[ $\varepsilon$  /. {
    a[f_, x, j_]  $\Rightarrow$  a[f, z, j] +  $\gamma$ [ $\delta_{b_y} f, z, j$ ],
    a[f_, y, j_]  $\Rightarrow$  a[f, z, j],
     $\delta a$ [f_, x | y, j_]  $\Rightarrow$   $\delta a$ [f, z, j],
     $\delta aa$ [f_, i_, j_, k_, l_]  $\Rightarrow$   $\delta aa$ [f, rr@i, j, rr@k, l]
  } /. b_x|y  $\rightarrow$  b_z]]]);

```

hm-def

```

UU[ $\varepsilon$ ] // hm[x_, y_, z_] := (rr = Replace[x | y  $\rightarrow$  z];
  CF[UU[Expand[ $\varepsilon$  /. {
    a[f_, i_, x | y]  $\Rightarrow$  a[f, i, z],
     $\delta a$ [f_, i_, x | y]  $\Rightarrow$   $\delta a$ [f, i, z],
     $\delta aa$ [f_, i_, y, k_, x]  $\Rightarrow$   $\delta aa$ [f, k, z, i, z],
     $\delta aa$ [f_, i_, j_, k_, l_]  $\Rightarrow$   $\delta aa$ [f, i, rr@j, k, rr@l]
  }]]]);

```

hts-def

```

UU[ε_] // hts[y_, x_] := CF[UU[Expand[ε /. {
  a[f_, i_, j_] =>
    a[f, i, j] - Kδjy γ[∂bx f, i, y] - Kδix Kδjy (β[f bx] - δa[f, c, y] - δβ[bx ∂bx f]),
  δa[f_, x, y] => δa[f, x, y] - δβ[f bx],
  δaa[f_, i_, j_, k_, l_] => δaa[f, i, j, k, l] +
    Kδix Kδjy δa[-bx f, k, l] + Kδix Kδly (δa[bk f, x, j] - δa[bx f, k, j]) +
    Kδkx Kδjy (δa[bi f, x, l] - δa[bx f, i, l]) + Kδkx Kδly δa[-bx f, i, j] -
    Kδix Kδjly δβ[bx bk f] + 2 Kδxik Kδyjl δβ[bx bx f]
}]]];

```

dm-def

```

dm[x_, y_, z_][ε_] := ε // hts[x, y] // tm[x, y, z] // hm[x, y, z]

```

## $t\sigma, h\sigma, d\sigma$ on $\{\beta, a, \delta\beta, \delta a, \delta aa\}$

sigma-def

```

tσ[x_List, y_List][ε_] := (rr = Replace[Thread[x → y]]];
CF[ε /. bi => brr@i /. {
  a[f_, i_, j_] => a[f, rr@i, j],
  δa[f_, i_, j_] => δa[f, rr@i, j],
  δaa[f_, i_, j_, k_, l_] => δaa[f, rr@i, j, rr@k, l]
}]);
tσ[x_, y_][ε_] := tσ[{x}, {y}][ε];
hσ[x_List, y_List][ε_] := (rr = Replace[Thread[x → y]]];
CF[ε /. {
  a[f_, i_, j_] => a[f, i, rr@j],
  δa[f_, i_, j_] => δa[f, i, rr@j],
  δaa[f_, i_, j_, k_, l_] => δaa[f, i, rr@j, k, rr@l]
}]);
hσ[x_, y_][ε_] := hσ[{x}, {y}][ε];
dσ[x_, y_][ε_] := ε // tσ[x, y] // hσ[x, y];

```

## tb, hb, thb, htb, db, bb on $\{\beta, a, \delta\beta, \delta a, \delta aa\}$

tb-def

```

tb[x_][UU[L_], UU[R_]] := CF[UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _] → 0, pp[_ , 0] → 0,
  pp[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
  pp[u_β | u_δβ | u_δa | u_δaa, v_a] ⇒ -pp[v, u]
} /. {
  pp[a[f_, x, j_], u_] ⇒ (u /. {
    β[g_] ⇒ γ[f ∂bxg, x, j],
    a[g_, k_, l_] ⇒ γa[f ∂bxg, x, j, k, l] +
      Kδxk (-γa[g ∂bxf, k, l, x, j] + ca[fg, l, x, j] - ca[fg, j, k, l]),
    _ → 0
  }),
  pp[a[f_, j_, k_], a[g_, x, l_]] /; j != x ⇒ -γa[g ∂bxf, x, l, j, k],
  pp[_ , _] → 0
}]]];

```

hb-def

```

hb[y_][UU[L_], UU[R_]] := CF[UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _] → 0, pp[_ , 0] → 0,
  pp[_β | _δβ, _] → 0,
  pp[_ , _β | _δβ] → 0,
  pp[_δa | _δaa, _δa | _δaa] → 0,
  pp[u_δa | u_δaa, v_a] ⇒ -pp[v, u]
} /. {
  pp[a[f_, i_, y], u_] ⇒ (u /. {
    a[g_, j_, k_] ⇒ Kδyk (a[bjfg, i, y] - a[bifg, j, k]),
    δa[g_, j_, k_] ⇒ Kδyk (δa[bjfg, i, y] - δa[bifg, j, k]),
    δaa[g_, j_, k_, l_, m_] ⇒ Kδyk (δaa[bjfg, i, y, l, m] - δaa[bifg, j, k,
      l, m]) + Kδym (δaa[bifg, j, k, i, y] - δaa[bifg, j, k, l, m])
  }),
  _pp → 0
}]]];

```

thb-def

```

thb[x_, y_][UU[L_], UU[R_]] := CF[UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _] → 0, pp[_ , 0] → 0,
  pp[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
  pp[_a, _β | _δβ] → 0,
  pp[β[f_], a[g_, i_, j_]] ⇒ Kδyj γ[g ∂bx f, i, y],
  pp[a[f_, i_, j_], a[g_, k_, l_]] ⇒ Kδyl (
    γa[g ∂bx f, k, l, i, j] + Kδxi (
      γ[-bk g ∂bx f, i, j] + δa[bk g ∂bx f, i, j] - δa[bi g ∂bx f, k, j] - a[bk f g, i, j] +
      a[bi f g, k, j] + ca[f g, j, k, l] - ca[f g, l, k, j]),
  pp[a[f_, i_, j_], δa[g_, k_, l_]] ⇒ Kδxi Kδyl
    (-δa[bk f g, i, j] + δa[bi f g, k, j]),
  pp[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] ⇒ Kδxi (
    Kδyl (-δaa[bk f g, i, j, m, n] + δaa[bi f g, k, j, m, n]) +
    Kδyn (-δaa[bm f g, k, l, i, j] + δaa[bi f g, k, l, m, j]) +
    Kδyln (δa[bx bm f g, k, j] - δa[bk bm f g, x, j]),
  pp[_δβ, _a] → 0,
  pp[δa[f_, i_, j_], a[g_, k_, l_]] ⇒
    Kδxi Kδyl (-δa[bk f g, i, j] + δa[bi f g, k, j]),
  pp[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] ⇒
    Kδxi Kδyl (-δaa[bk f g, i, j, m, n] + δaa[bi f g, k, j, m, n]) +
    Kδxm Kδyl (-δaa[bk f g, i, j, m, n] + δaa[bm f g, i, j, k, n])
}]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
t1 h1 t2 h2 → t1 t2 h1 h2 → t2 t1 h1 h2 → t2 t1 h2 h1 → t2 h2 t1 h1 :

```

db-def

```

db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
  htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] // hm[x, h, x],
  tb[x][u, v // hσ[x, h]] // hm[x, h, x],
  hb[x][u, v // tσ[x, t]] // tm[t, x, x],
  thb[x, x][u // hσ[x, h], v // tσ[x, t]] // tm[t, x, x] // hm[x, h, x] ]];

```

bb-def

```

bb[S_List] := Module[{w, bar, t, n = 0, i, k},
  w = #2 // dσ[S, bar /@ S];
  Sum[t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]]];
  Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]], {i, 1, k - 1}];
  Do[t = t // dm[S[[i]], bar[S[[i]], S[[i]], {i, k + 1, Length@S}];
  t, {k, Length@S}] &
bb[S___] := bb[{S}]

```

## ct (contract)

`ct::usage =`

"ct[h,t][L,R] contracts the head h in L with the tail t in R. ct[s][L,R] takes h=t=s, and ct[][L,R] takes s=0. When ambiguous, L is placed below R.";

ct-def

```
ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_][UU[L_], UU[R_]] := CF[UU[Distribute[pp[L, R]] /. {
  pp[_β | _δβ, _] → 0,
  pp[a[f_, i_, h], β[g_]] ⇒ β[f bi ((∂btg) /. bt → 0)],
  pp[a[f_, i_, h], a[g_, t, j_]] ⇒ a[f (g /. bt → 0), i, j],
  pp[a[f_, i_, h], a[g_, j_, k_]] ⇒ a[f bi ((∂btg) /. bt → 0), j, k],
  pp[a[f_, i_, h], δa[g_, t, j_]] ⇒ δa[f (g /. bt → 0), i, j],
  pp[a[f_, i_, h], δa[g_, j_, k_]] ⇒ δa[f bi ((∂btg) /. bt → 0), j, k],
  pp[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
  pp[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] ⇒ δaa[f (g /. bt → 0), i, j, k, l],
  pp[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] ⇒ δaa[f (g /. bt → 0), j, k, i, l],
  pp[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] ⇒
    δaa[f bi ((∂btg) /. bt → 0), j, k, l, m],
  pp[a[_], _] → 0, pp[_δa | _δaa, _δβ | _δa | _δaa] → 0,
  pp[δa[f_, i_, h], β[g_]] ⇒ δβ[f bi ((∂btg) /. bt → 0)],
  pp[δa[f_, i_, h], a[g_, t, j_]] ⇒ δa[f (g /. bt → 0), i, j],
  pp[δa[f_, i_, h], a[g_, j_, k_]] ⇒ δa[f bi ((∂btg) /. bt → 0), j, k],
  pp[_δa, _] → 0, pp[δaa[_ , _ , h, _ , h], _] → 0,
  pp[δaa[f_, i_, h, j_, k_], β[g_]] ⇒ δa[f bi ((∂btg) /. bt → 0), j, k],
  pp[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] ⇒ δaa[f (g /. bt → 0), i, l, j, k],
  pp[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] ⇒
    δaa[f bi ((∂btg) /. bt → 0), j, k, l, m],
  pp[δaa[f_, i_, j_, k_, h], β[g_]] ⇒ δa[f bk ((∂btg) /. bt → 0), i, j],
  pp[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] ⇒ δaa[f (g /. bt → 0), i, j, k, l],
  pp[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] ⇒
    δaa[f bk ((∂btg) /. bt → 0), i, j, l, m],
  pp[_δaa, _] → 0 }]]];
```

## dect (de-contract)

```
dect::usage =
  "dect[h,t][uu] returns a pair {L,R} such that ct[h,t][L,R]=uu. Similarly
  for dect[s] and dect[]. uu is assumed to be atomic.";
dect[s_] := dect[s, s];
dect[] = dect[0, 0];
dect[h_, t_][β[f_]] := {};
dect[h_, t_][δβ[f_]] := TBD;
```

## Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"];

ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
    DeleteFile[temp],
    (* else *) Print["Exporting " <> fname <> "..."];
    If[exists, DeleteFile[fname]];
    RenameFile[temp, fname]
  ];
  fname
]

Button["Export",
  SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
  TagProperties[_] := {};
  TagProperties["ct-def"] = {PageWidth → 6/0.65};
  Options[CellExport] = {
    PageWidth → 4/0.65, CellFilter → Identity,
    ExportDirectory → "Snips", ExportBaseFilename → Automatic,
    ExportFormat → ".pdf", ExportOptions → {}, Split → False
  };
  CellExport[tag_String, opts___Rule] := CellExport[
    NotebookGet[EvaluationNotebook[]],
    tag, opts
  ];
```

```

CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{
    OptionValue[ExportDirectory] /. Automatic → Directory[],
    OptionValue[ExportBaseFilename] /. Automatic → tag
  }];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
    nb, c_Cell /; FreeQ[List@@c, Cell] && !FreeQ[c, CellTags → tag],
    Infinity
  ]];
  If[!OptionValue[Split],
  If[Length[cells] ≥ 1,
  If[Length[cells] == 1,
    cells = Append[First[cells], PageWidth → 1.2 × 72 OptionValue[PageWidth]],
    cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
  ];
  ConditionalExport[
    filename <> format, cells,
    ImageResolution → 300,
    OptionValue[ExportOptions]
  ]
  ],
  k = 0;
  Table[
    ++k;
    ConditionalExport[
      filename <> "-" <> ToString[k] <> format,
      Append[cell, PageWidth → 72 OptionValue[PageWidth]],
      ImageResolution → 300,
      OptionValue[ExportOptions]
    ],
    {cell, cells}
  ]
  ];
  nb = NotebookGet[EvaluationNotebook[]];
  tags = Cases[nb, (CellTags → tag_) ⇒ tag, Infinity] // Union;
  CellExport /@ tags;
  Print["Done."]
]

```



Export

Exporting Snips\bb-def.pdf...  
Exporting Snips\ct-def.pdf...  
Exporting Snips\db-def.pdf...  
Exporting Snips\delta-aa.pdf...  
Exporting Snips\dm-def.pdf...  
Exporting Snips\Generalities.pdf...  
Exporting Snips\hb-def.pdf...  
Exporting Snips\hm-def.pdf...  
Exporting Snips\hts-def.pdf...  
Exporting Snips\sigma-def.pdf...  
Exporting Snips\tb-def.pdf...  
Exporting Snips\thb-def.pdf...  
Exporting Snips\tm-def.pdf...  
Done.